Integrated Algebra

In implementing the Algebra process and content performance indicators, it is expected that students will identify and justify mathematical relationships. The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is

therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible patterns and generalizations that result from investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems.

Apply - Students will use a theorem or concept to solve an algebraic or numerical problem.

Problem Solving Strand

Students willbuild new mathematical knowledge through problem solving.

A.PS.1 Use a variety of problem solving strategies to understand new mathematical content

A.PS.2 Recognize and understand equivalent representations of a problem situation or a mathematical concept

Students will solve problems that arise in mathematics and in other contexts.

A.PS.3 Observe and explain patterns to formulate generalizations and conjectures

A.PS.4 Use multiple representations to represent and explain problem situations (e.g., verbally, numerically, algebraically, graphically)

Students will apply and adapt a variety of appropriate strategies to solve problems.

A.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

A.PS.6 Use a variety of strategies to extend solution methods to other problems

A.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

Students will monitor and reflect on the process of mathematical problem solving.

A.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

A.PS.9 Interpret solutions within the given constraints of a problem

A.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

A.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

Students will make and investigate mathematical conjectures.

A.RP.2 Use mathematical strategies to reach a conclusion and provide supportive arguments for a conjecture

A.RP.3 Recognize when an approximation is more appropriate than an exact answer

Students will develop and evaluate mathematical arguments and proofs.

A.RP.4 Develop, verify, and explain an argument, using appropriate mathematical ideas and language

A.RP.5 Construct logical arguments that verify claims or counterexamples that refute them

A.RP.6 Present correct mathematical arguments in a variety of forms

A.RP.7 Evaluate written arguments for validity

Students will select and use various types of reasoning and methods of proof.

A.RP.8 Support an argument by using a systematic approach to test more than one case

A.RP.9 Devise ways to verify results or use counterexamples to refute incorrect statements

A.RP.10 Extend specific results to more general cases

A.RP.11 Use a Venn diagram to support a logical argument

A.RP.12 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

A.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

A.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, Venn diagrams, and other diagrams

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

A.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

A.CM.4 Explain relationships among different representations of a problem

A.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

A.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

Students will analyze and evaluate the mathematical thinking and strategies of others.

A.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

A.CM.8 Reflect on strategies of others in relation to one's own strategy

A.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

Students will use the language of mathematics to express mathematical ideas precisely.

A.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

A.CM.11 Represent word problems using standard mathematical notation

A.CM.12 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and rationale

A.CM.13 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

Students will recognize and use connections among mathematical ideas.

A.CN.1 Understand and make connections among multiple representations of the same mathematical idea

A.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

A.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

A.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

A.CN.5 Understand how quantitative models connect to various physical models and representations

Students will recognize and apply mathematics in contexts outside of mathematics.

A.CN.6 Recognize and apply mathematics to situations in the outside world

A.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

A.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

A.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts

A.R.2 Recognize, compare, and use an array of representational forms

A.R.3 Use representation as a tool for exploring and understanding mathematical ideas

Students will select, apply, and translate among mathematical representations to solve problems.

A.R.4 Select appropriate representations to solve problem situations

A.R.5 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

A.R.6 Use mathematics to show and understand physical phenomena (e.g., find the height of a building if a ladder of a given length forms a given angle of elevation with the ground)

A.R.7 Use mathematics to show and understand social phenomena (e.g., determine profit from student and adult ticket sales)

A.R.8 Use mathematics to show and understand mathematical phenomena (e.g., compare the graphs of the functions represented by the equations $y = x^2$ and $y = -x^2$)

Number Sense and Operations Strand

Students will understand numbers, multiple ways of representing numbers, relationships among numbers, and number systems.

Number Theory

A.N.1 Identify and apply the properties of real numbers (closure, commutative, associative, distributive, identity, inverse) *Note: Students do not need to identify groups and fields, but students should be engaged in the ideas.*

Students will understand meanings of operations and procedures, and how they relate to one another.

Operations

A.N.2 Simplify radical terms (no variable in the radicand)

A.N.3 Perform the four arithmetic operations using like and unlike radical terms and express the result in simplest form

A.N.4 Understand and use scientific notation to compute products and quotients of numbers

A.N.5 Solve algebraic problems arising from situations that involve fractions, decimals, percents (decrease/increase and discount), and proportionality/direct variation

A.N.6 Evaluate expressions involving factorial(s), absolute value(s), and exponential expression(s)

	A.N.7	Determine the number of possible events, using counting techniques or the Fundamental Principle of Counting
	A.N.8	Determine the number of possible arrangements (permutations) of a list of items

Algebra Strand

Students will represent and analyze algebraically a wide variety of problem solving situations.

Variables and Expressions	A.A.1	Translate a quantitative verbal phrase into an algebraic expression
	A.A.2	Write a verbal expression that matches a given mathematical expression
Equations and Inequalities	A.A.3	Distinguish the difference between an algebraic expression and an algebraic equation
	A.A.4	Translate verbal sentences into mathematical equations or inequalities
	A.A.5	Write algebraic equations or inequalities that represent a situation
	A.A.6	Analyze and solve verbal problems whose solution requires solving a linear equation in one variable or linear inequality in one variable
	A.A.7	Analyze and solve verbal problems whose solution requires solving systems of linear equations in two variables
	A.A.8	Analyze and solve verbal problems that involve quadratic equations
	A.A.9	Analyze and solve verbal problems that involve exponential growth and decay
	A.A.10	Solve systems of two linear equations in two variables algebraically (See A.G.7)
	A.A.11	Solve a system of one linear and one quadratic equation

		in two variables, where only factoring is required *Note: The quadratic equation should represent a parabola and the solution(s) should be integers.*

Students will perform algebraic procedures accurately.

Variables and Expressions	A.A.12	Multiply and divide monomial expressions with a common base, using the properties of exponents *Note: Use integral exponents only.*
	A.A.13	Add, subtract, and multiply monomials and polynomials
	A.A.14	Divide a polynomial by a monomial or binomial, where the quotient has no remainder
	A.A.15	Find values of a variable for which an algebraic fraction is undefined
	A.A.16	Simplify fractions with polynomials in the numerator and denominator by factoring both and renaming them to lowest terms
	A.A.17	Add or subtract fractional expressions with monomial or like binomial denominators
	A.A.18	Multiply and divide algebraic fractions and express the product or quotient in simplest form
	A.A.19	Identify and factor the difference of two perfect squares
	A.A.20	Factor algebraic expressions completely, including trinomials with a lead coefficient of one (after factoring a GCF)
Equations and Inequalities	A.A.21	Determine whether a given value is a solution to a given linear equation in one variable or linear inequality in one variable
	A.A.22	Solve all types of linear equations in one variable
	A.A.23	Solve literal equations for a given variable
	A.A.24	Solve linear inequalities in one variable
	A.A.25	Solve equations involving fractional expressions *Note: Expressions which result in linear equations in one variable.*

	A.A.26	Solve algebraic proportions in one variable which result in linear or quadratic equations
	A.A.27	Understand and apply the multiplication property of zero to solve quadratic equations with integral coefficients and integral roots
	A.A.28	Understand the difference and connection between roots of a quadratic equation and factors of a quadratic expression

Students will recognize, use, and represent algebraically patterns, relations, and functions.

Patterns, Relations, and Functions	A.A.29	Use set-builder notation and/or interval notation to illustrate the elements of a set, given the elements in roster form
	A.A.30	Find the complement of a subset of a given set, within a given universe
	A.A.31	Find the intersection of sets (no more than three sets) and/or union of sets (no more than three sets)
Coordinate Geometry	A.A.32	Explain slope as a rate of change between dependent and independent variables
	A.A.33	Determine the slope of a line, given the coordinates of two points on the line
	A.A.34	Write the equation of a line, given its slope and the coordinates of a point on the line
	A.A.35	Write the equation of a line, given the coordinates of two points on the line
	A.A.36	Write the equation of a line parallel to the x- or y-axis
	A.A.37	Determine the slope of a line, given its equation in any form
	A.A.38	Determine if two lines are parallel, given their equations in any form
	A.A.39	Determine whether a given point is on a line, given the equation of the line
	A.A.40	Determine whether a given point is in the solution set of a system of linear inequalities

	A.A.41	Determine the vertex and axis of symmetry of a parabola, given its equation (See A.G.10)
Trigonometric Functions	A.A.42	Find the sine, cosine, and tangent ratios of an angle of a right triangle, given the lengths of the sides
	A.A.43	Determine the measure of an angle of a right triangle, given the length of any two sides of the triangle
	A.A.44	Find the measure of a side of a right triangle, given an acute angle and the length of another side
	A.A.45	Determine the measure of a third side of a right triangle using the Pythagorean theorem, given the lengths of any two sides

Geometry Strand

Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Shapes	A.G.1	Find the area and/or perimeter of figures composed of polygons and circles or sectors of a circle *Note: Figures may include triangles, rectangles, squares, parallelograms, rhombuses, trapezoids, circles, semi-circles, quarter-circles, and regular polygons (perimeter only).*
	A.G.2	Use formulas to calculate volume and surface area of rectangular solids and cylinders

Students will apply coordinate geometry to analyze problem solving situations.

Coordinate Geometry	A.G.3	Determine when a relation is a function, by examining ordered pairs and inspecting graphs of relations
	A.G.4	Identify and graph linear, quadratic (parabolic), absolute value, and exponential functions
	A.G.5	Investigate and generalize how changing the coefficients of a function affects its graph
	A.G.6	Graph linear inequalities
	A.G.7	Graph and solve systems of linear equations and inequalities with rational coefficients in two variables (See A.A.10)

	A.G.8	Find the roots of a parabolic function graphically *Note: Only quadratic equations with integral solutions.*
	A.G.9	Solve systems of linear and quadratic equations graphically *Note: Only use systems of linear and quadratic equations that lead to solutions whose coordinates are integers.*
	A.G.10	Determine the vertex and axis of symmetry of a parabola, given its graph (See A.A.41) *Note: The vertex will have an ordered pair of integers and the axis of symmetry will have an integral value.*

Measurement Strand

Students will determine what can be measured and how, using appropriate methods and formulas.

Units of Measurement	A.M.1	Calculate rates using appropriate units (e.g., rate of a space ship versus the rate of a snail)
	A.M.2	Solve problems involving conversions within measurement systems, given the relationship between the units

Students will understand that all measurement contains error and be able to determine its significance.

Error and Magnitude	A.M.3	Calculate the relative error in measuring square and cubic units, when there is an error in the linear measure

Statistics and Probability Strand

Students will collect, organize, display, and analyze data.

Organization and Display of Data	A.S.1	Categorize data as qualitative or quantitative
	A.S.2	Determine whether the data to be analyzed is univariate or bivariate
	A.S.3	Determine when collected data or display of data may be biased
	A.S.4	Compare and contrast the appropriateness of different measures of central tendency for a given data set

	A.S.5	Construct a histogram, cumulative frequency histogram, and a box-and-whisker plot, given a set of data
	A.S.6	Understand how the five statistical summary (minimum, maximum, and the three quartiles) is used to construct a box-and-whisker plot
	A.S.7	Create a scatter plot of bivariate data
	A.S.8	Construct manually a reasonable line of best fit for a scatter plot and determine the equation of that line
Analysis of Data	A.S.9	Analyze and interpret a frequency distribution table or histogram, a cumulative frequency distribution table or histogram, or a box-and-whisker plot
	A.S.10	Evaluate published reports and graphs that are based on data by considering: experimental design, appropriateness of the data analysis, and the soundness of the conclusions
	A.S.11	Find the percentile rank of an item in a data set and identify the point values for first, second, and third quartiles
	A.S.12	Identify the relationship between the independent and dependent variables from a scatter plot (positive, negative, or none)
	A.S.13	Understand the difference between correlation and causation
	A.S.14	Identify variables that might have a correlation but not a causal relationship

Students will make predictions that are based upon data analysis.

Predictions from Data	A.S.15	Identify and describe sources of bias and its effect, drawing conclusions from data
	A.S.16	Recognize how linear transformations of one-variable data affect the data's mean, median, mode, and range
	A.S.17	Use a reasonable line of best fit to make a prediction involving interpolation or extrapolation

Students will understand and apply concepts of probability.

Probability

A.S.18 Know the definition of conditional probability and use it to solve for probabilities in finite sample spaces

A.S.19 Determine the number of elements in a sample space and the number of favorable events

A.S.20 Calculate the probability of an event and its complement

A.S.21 Determine empirical probabilities based on specific sample data

A.S.22 Determine, based on calculated probability of a set of events, if:

- some or all are equally likely to occur
- one is more likely to occur than another
- whether or not an event is certain to happen or not to happen

A.S.23 Calculate the probability of:

- a series of independent events
- a series of dependent events
- two mutually exclusive events
- two events that are not mutually exclusive

Geometry

In implementing the Geometry process and content performance indicators, it is expected that students will identify and justify geometric relationships, formally and informally. For example, students will begin with a definition of a figure and from that definition students will be expected to develop a list of conjectured properties of the figure and to justify each conjecture informally or with formal proof. Students will also be expected to list the assumptions that are needed in order to justify each conjectured property and present their findings in an organized manner.

The intent of both the process and content performance indicators is to provide a variety of ways for students to acquire and demonstrate mathematical reasoning ability when solving problems. The variety of approaches to verification and proof is what gives curriculum developers and

teachers the flexibility to adapt strategies to address these performance indicators in a manner that meets the diverse needs of our students. Local curriculum and local/state assessments must support and allow students to use any mathematically correct method when solving a problem.

Throughout this document the performance indicators use the words *investigate, explore, discover, conjecture, reasoning, argument, justify, explain, proof,* and *apply*. Each of these terms is an important component in developing a student's mathematical reasoning ability. It is therefore important that a clear and common definition of these terms be understood. The order of these terms reflects different stages of the reasoning process.

Investigate/Explore - Students will be given situations in which they will be asked to look for patterns or relationships between elements within the setting.

Discover - Students will make note of possible relationships of perpendicularity, parallelism, congruence, and/or similarity after investigation/exploration.

Conjecture - Students will make an overall statement, thought to be true, about the new discovery.

Reasoning - Students will engage in a process that leads to knowing something to be true or false.

Argument - Students will communicate, in verbal or written form, the reasoning process that leads to a conclusion. A valid argument is the end result of the conjecture/reasoning process.

Justify/Explain - Students will provide an argument for a mathematical conjecture. It may be an intuitive argument or a set of examples that support the conjecture. The argument may include, but is not limited to, a written paragraph, measurement using appropriate tools, the use of dynamic software, or a written proof.

Proof - Students will present a valid argument, expressed in written form, justified by axioms, definitions, and theorems using properties of perpendicularity, parallelism, congruence, and similarity with polygons and circles.

Apply - Students will use a theorem or concept to solve a geometric problem.

Problem Solving Strand

Students willbuild new mathematical knowledge through problem solving.

G.PS.1 Use a variety of problem solving strategies to understand new mathematical content

Students will solve problems that arise in mathematics and in other contexts.

G.PS.2 Observe and explain patterns to formulate generalizations and conjectures

G.PS.3 Use multiple representations to represent and explain problem situations (e.g., spatial, geometric, verbal, numeric, algebraic, and graphical representations)

Students will apply and adapt a variety of appropriate strategies to solve problems.

G.PS.4 Construct various types of reasoning, arguments, justifications and methods of proof for problems

G.PS.5 Choose an effective approach to solve a problem from a variety of strategies (numeric, graphic, algebraic)

G.PS.6 Use a variety of strategies to extend solution methods to other problems

G.PS.7 Work in collaboration with others to propose, critique, evaluate, and value alternative approaches to problem solving

Students will monitor and reflect on the process of mathematical problem solving.

G.PS.8 Determine information required to solve a problem, choose methods for obtaining the information, and define parameters for acceptable solutions

G.PS.9 Interpret solutions within the given constraints of a problem

G.PS.10 Evaluate the relative efficiency of different representations and solution methods of a problem

Reasoning and Proof Strand

Students will recognize reasoning and proof as fundamental aspects of mathematics.

G.RP.1 Recognize that mathematical ideas can be supported by a variety of strategies

G.RP.2 Recognize and verify, where appropriate, geometric relationships of perpendicularity, parallelism, congruence, and similarity, using algebraic strategies

Students will make and investigate mathematical conjectures.

G.RP.3 Investigate and evaluate conjectures in mathematical terms, using mathematical strategies to reach a conclusion

Students will develop and evaluate mathematical arguments and proofs.

G.RP.4 Provide correct mathematical arguments in response to other students' conjectures, reasoning, and arguments

G.RP.5 Present correct mathematical arguments in a variety of forms

G.RP.6 Evaluate written arguments for validity

Students will select and use various types of reasoning and methods of proof.

G.RP.7 Construct a proof using a variety of methods (e.g., deductive, analytic, transformational)

G.RP.8 Devise ways to verify results or use counterexamples to refute incorrect statements

G.RP.9 Apply inductive reasoning in making and supporting mathematical conjectures

Communication Strand

Students will organize and consolidate their mathematical thinking through communication.

G.CM.1 Communicate verbally and in writing a correct, complete, coherent, and clear design (outline) and explanation for the steps used in solving a problem

G.CM.2 Use mathematical representations to communicate with appropriate accuracy, including numerical tables, formulas, functions, equations, charts, graphs, and diagrams

Students will communicate their mathematical thinking coherently and clearly to peers, teachers, and others.

G.CM.3 Present organized mathematical ideas with the use of appropriate standard notations, including the use of symbols and other representations when sharing an idea in verbal and written form

G.CM.4 Explain relationships among different representations of a problem

G.CM.5 Communicate logical arguments clearly, showing why a result makes sense and why the reasoning is valid

G.CM.6 Support or reject arguments or questions raised by others about the correctness of mathematical work

Students will analyze and evaluate the mathematical thinking and strategies of others.

G.CM.7 Read and listen for logical understanding of mathematical thinking shared by other students

G.CM.8 Reflect on strategies of others in relation to one's own strategy

G.CM.9 Formulate mathematical questions that elicit, extend, or challenge strategies, solutions, and/or conjectures of others

Students will use the language of mathematics to express mathematical ideas precisely.

G.CM.10 Use correct mathematical language in developing mathematical questions that elicit, extend, or challenge other students' conjectures

G.CM.11 Understand and use appropriate language, representations, and terminology when describing objects, relationships, mathematical solutions, and geometric diagrams

G.CM.12 Draw conclusions about mathematical ideas through decoding, comprehension, and interpretation of mathematical visuals, symbols, and technical writing

Connections Strand

Students will recognize and use connections among mathematical ideas.

G.CN.1 Understand and make connections among multiple representations of the same mathematical idea

G.CN.2 Understand the corresponding procedures for similar problems or mathematical concepts

Students will understand how mathematical ideas interconnect and build on one another to produce a coherent whole.

G.CN.3 Model situations mathematically, using representations to draw conclusions and formulate new situations

G.CN.4 Understand how concepts, procedures, and mathematical results in one area of mathematics can be used to solve problems in other areas of mathematics

G.CN.5 Understand how quantitative models connect to various physical models and representations

Students will recognize and apply mathematics in contexts outside of mathematics.

G.CN.6 Recognize and apply mathematics to situations in the outside world

G.CN.7 Recognize and apply mathematical ideas to problem situations that develop outside of mathematics

G.CN.8 Develop an appreciation for the historical development of mathematics

Representation Strand

Students will create and use representations to organize, record, and communicate mathematical ideas.

G.R.1 Use physical objects, diagrams, charts, tables, graphs, symbols, equations, or objects created using technology as representations of mathematical concepts

G.R.2 Recognize, compare, and use an array of representational forms

G.R.3 Use representation as a tool for exploring and understanding mathematical ideas

Students will select, apply, and translate among mathematical representations to solve problems.

G.R.4 Select appropriate representations to solve problem situations

G.R.5 Investigate relationships between different representations and their impact on a given problem

Students will use representations to model and interpret physical, social, and mathematical phenomena.

G.R.6 Use mathematics to show and understand physical phenomena (e.g., determine the number of gallons of water in a fish tank)

G.R.7 Use mathematics to show and understand social phenomena (e.g., determine if conclusions from another person's argument have a logical foundation)

G.R.8 Use mathematics to show and understand mathematical phenomena (e.g., use investigation, discovery, conjecture, reasoning, arguments, justification and proofs to validate that the two base angles of an isosceles triangle are congruent)

Algebra Strand

Note: The algebraic skills and concepts within the Algebra process and content performance indicators must be maintained and applied as students are asked to investigate, make conjectures, give rationale, and justify or prove geometric concepts.

Geometry Strand

Students will use visualization and spatial reasoning to analyze characteristics and properties of geometric shapes.

Geometric Relationships

Note: Two-dimensional geometric relationships are addressed in the Informal and Formal Proofs band.

G.G.1 Know and apply that if a line is perpendicular to each of two intersecting lines at their point of intersection, then the line is perpendicular to the plane determined by them

G.G.2 Know and apply that through a given point there passes one and only one plane perpendicular to a given line

G.G.3 Know and apply that through a given point there passes one and only one line perpendicular to a given plane

G.G.4 Know and apply that two lines perpendicular to the same plane are coplanar

G.G.5 Know and apply that two planes are perpendicular to each other if and only if one plane contains a line perpendicular to the second plane

G.G.6 Know and apply that if a line is perpendicular to a plane, then any line perpendicular to the given line at its point of intersection with the given plane is in the given plane

G.G.7 Know and apply that if a line is perpendicular to a plane, then every plane containing the line is perpendicular to the given plane

G.G.8 Know and apply that if a plane intersects two parallel planes, then the intersection is two parallel lines

G.G.9 Know and apply that if two planes are perpendicular to the same line, they are parallel

G.G.10 Know and apply that the lateral edges of a prism are congruent and parallel

G.G.11 Know and apply that two prisms have equal volumes if their bases have equal areas and their altitudes are equal

G.G.12 Know and apply that the volume of a prism is the product of the area of the base and the altitude

G.G.13 Apply the properties of a regular pyramid, including:
- lateral edges are congruent
- lateral faces are congruent isosceles triangles
- volume of a pyramid equals one-third the product of the area of the base and the altitude

G.G.14 Apply the properties of a cylinder, including:
- bases are congruent
- volume equals the product of the area of the base and the altitude
- lateral area of a right circular cylinder equals the product of an altitude and the circumference of the base

G.G.15 Apply the properties of a right circular cone, including:
- lateral area equals one-half the product of the slant height and the circumference of its base
- volume is one-third the product of the area of its base and its altitude

G.G.16 Apply the properties of a sphere, including:
- the intersection of a plane and a sphere is a circle

- a great circle is the largest circle that can be drawn on a sphere
- two planes equidistant from the center of the sphere and intersecting the sphere do so in congruent circles
- surface area is $4\pi r^2$
- volume is $\frac{4}{3}\pi r^3$

Constructions	G.G.17	Construct a bisector of a given angle, using a straightedge and compass, and justify the construction
	G.G.18	Construct the perpendicular bisector of a given segment, using a straightedge and compass, and justify the construction
	G.G.19	Construct lines parallel (or perpendicular) to a given line through a given point, using a straightedge and compass, and justify the construction
	G.G.20	Construct an equilateral triangle, using a straightedge and compass, and justify the construction
Locus	G.G.21	Investigate and apply the concurrence of medians, altitudes, angle bisectors, and perpendicular bisectors of triangles
	G.G.22	Solve problems using compound loci
	G.G.23	Graph and solve compound loci in the coordinate plane

Students will identify and justify geometric relationships formally and informally.

Informal and Formal Proofs	G.G.24	Determine the negation of a statement and establish its truth value
	G.G.25	Know and apply the conditions under which a compound statement (conjunction, disjunction, conditional, biconditional) is true
	G.G.26	Identify and write the inverse, converse, and contrapositive of a given conditional statement and note the logical equivalences
	G.G.27	Write a proof arguing from a given hypothesis to a given conclusion
	G.G.28	Determine the congruence of two triangles by using one of the five congruence techniques (SSS, SAS, ASA, AAS, HL), given

sufficient information about the sides and/or angles of two congruent triangles

G.G.29 Identify corresponding parts of congruent triangles

G.G.30 Investigate, justify, and apply theorems about the sum of the measures of the angles of a triangle

G.G.31 Investigate, justify, and apply the isosceles triangle theorem and its converse

G.G.32 Investigate, justify, and apply theorems about geometric inequalities, using the exterior angle theorem

G.G.33 Investigate, justify, and apply the triangle inequality theorem

G.G.34 Determine either the longest side of a triangle given the three angle measures or the largest angle given the lengths of three sides of a triangle

G.G.35 Determine if two lines cut by a transversal are parallel, based on the measure of given pairs of angles formed by the transversal and the lines

G.G.36 Investigate, justify, and apply theorems about the sum of the measures of the interior and exterior angles of polygons

G.G.37 Investigate, justify, and apply theorems about each interior and exterior angle measure of regular polygons

G.G.38 Investigate, justify, and apply theorems about parallelograms involving their angles, sides, and diagonals

G.G.39 Investigate, justify, and apply theorems about special parallelograms (rectangles, rhombuses, squares) involving their angles, sides, and diagonals

G.G.40 Investigate, justify, and apply theorems about trapezoids (including isosceles trapezoids) involving their angles, sides, medians, and diagonals

G.G.41 Justify that some quadrilaterals are parallelograms, rhombuses, rectangles, squares, or trapezoids

G.G.42 Investigate, justify, and apply theorems about geometric relationships, based on the properties of the line segment joining the midpoints of two sides of the triangle

G.G.43 Investigate, justify, and apply theorems about the centroid of a triangle, dividing each median into segments whose lengths are in the ratio 2:1

G.G.44 Establish similarity of triangles, using the following theorems: AA, SAS, and SSS

G.G.45 Investigate, justify, and apply theorems about similar triangles

G.G.46 Investigate, justify, and apply theorems about proportional relationships among the segments of the sides of the triangle, given one or more lines parallel to one side of a triangle and intersecting the other two sides of the triangle

G.G.47 Investigate, justify, and apply theorems about mean proportionality:

- the altitude to the hypotenuse of a right triangle is the mean proportional between the two segments along the hypotenuse
- the altitude to the hypotenuse of a right triangle divides the hypotenuse so that either leg of the right triangle is the mean proportional between the hypotenuse and segment of the hypotenuse adjacent to that leg

G.G.48 Investigate, justify, and apply the Pythagorean theorem and its converse

G.G.49 Investigate, justify, and apply theorems regarding chords of a circle:

- perpendicular bisectors of chords
- the relative lengths of chords as compared to their distance from the center of the circle

G.G.50 Investigate, justify, and apply theorems about tangent lines to a circle:

- a perpendicular to the tangent at the point of tangency
- two tangents to a circle from the same external point
- common tangents of two non-intersecting or tangent circles

G.G.51 Investigate, justify, and apply theorems about the arcs determined by the rays of angles formed by two lines intersecting a circle when the vertex is:
- inside the circle (two chords)
- on the circle (tangent and chord)
- outside the circle (two tangents, two secants, or tangent and secant)

G.G.52 Investigate, justify, and apply theorems about arcs of a circle cut by two parallel lines

G.G.53 Investigate, justify, and apply theorems regarding segments intersected by a circle:
- along two tangents from the same external point
- along two secants from the same external point
- along a tangent and a secant from the same external point
- along two intersecting chords of a given circle

Students will apply transformations and symmetry to analyze problem solving situations.

Transformational Geometry

G.G.54 Define, investigate, justify, and apply isometries in the plane (rotations, reflections, translations, glide reflections) *Note: Use proper function notation.*

G.G.55 Investigate, justify, and apply the properties that remain invariant under translations, rotations, reflections, and glide reflections

G.G.56 Identify specific isometries by observing orientation, numbers of invariant points, and/or parallelism

G.G.57 Justify geometric relationships (perpendicularity, parallelism, congruence) using transformational techniques (translations, rotations, reflections)

G.G.58 Define, investigate, justify, and apply similarities (dilations and the composition of dilations and isometries)

G.G.59 Investigate, justify, and apply the properties that remain invariant under similarities

G.G.60 Identify specific similarities by observing orientation, numbers of invariant points, and/or parallelism

G.G.61 Investigate, justify, and apply the analytical representations for translations, rotations about the origin of 90º and 180º, reflections over the lines $x = 0$, $y = 0$, and $y = x$, and dilations centered at the origin

Students will apply coordinate geometry to analyze problem solving situations.

Coordinate Geometry

G.G.62 Find the slope of a perpendicular line, given the equation of a line

G.G.63 Determine whether two lines are parallel, perpendicular, or neither, given their equations

G.G.64 Find the equation of a line, given a point on the line and the equation of a line perpendicular to the given line

G.G.65 Find the equation of a line, given a point on the line and the equation of a line parallel to the desired line

G.G.66 Find the midpoint of a line segment, given its endpoints

G.G.67 Find the length of a line segment, given its endpoints

G.G.68 Find the equation of a line that is the perpendicular bisector of a line segment, given the endpoints of the line segment

G.G.69 Investigate, justify, and apply the properties of triangles and quadrilaterals in the coordinate plane, using the distance, midpoint, and slope formulas

G.G.70 Solve systems of equations involving one linear equation and one quadratic equation graphically

G.G.71 Write the equation of a circle, given its center and radius or given the endpoints of a diameter

G.G.72 Write the equation of a circle, given its graph
Note: The center is an ordered pair of integers and the radius is an integer.

G.G.73 Find the center and radius of a circle, given the equation of the circle in center-radius form

G.G.74 Graph circles of the form $(x - h)^2 + (y - k)^2 = r^2$