

**NEW YORK STATE
COMPONENT RETEST**

**MATHEMATICS A
COMPONENT 5
TASK 2**

THURSDAY, APRIL 25, 2002

**SCORING KEY
AND
RATING GUIDE**

Multiple Choice Key

| | |
|---|---|
| 1 | 4 |
| 2 | 4 |
| 3 | 4 |
| 4 | 1 |
| 5 | 2 |
| 6 | 3 |

COMPONENT 5
MODULE 2

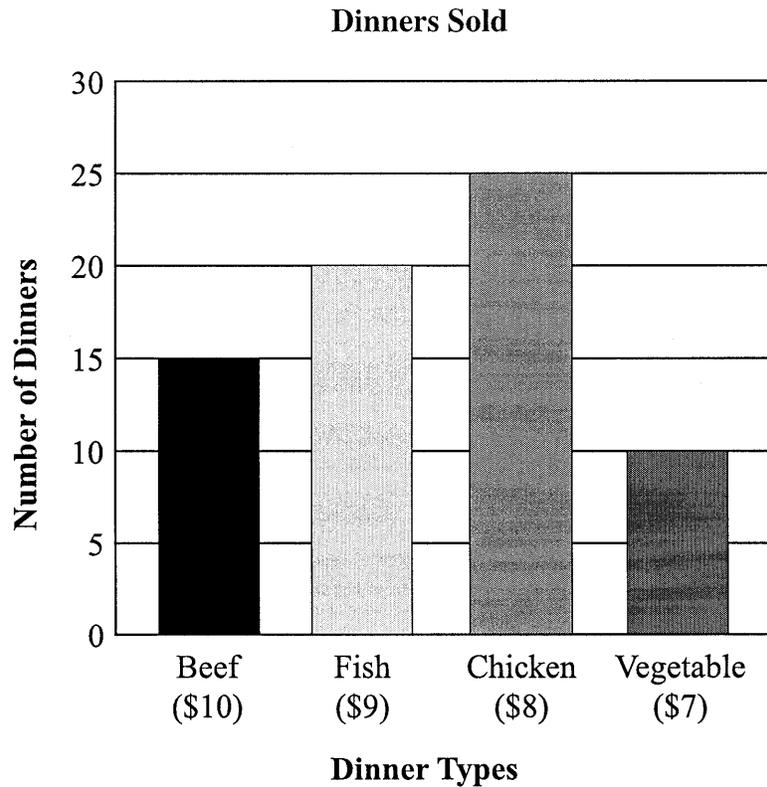
QUESTION 7

TRAINING SET

Part II

Answer all questions in this part. Each correct answer will receive 4 credits. Clearly indicate the necessary steps, including appropriate formula substitutions, diagrams, graphs, charts, etc. For all questions in this part, a correct numerical answer with no work shown will receive only 1 credit. [12]

- 7 The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



Sample Response:

$$\frac{15 \times 10 + 20 \times 9 + 25 \times 8 + 10 \times 7}{15 + 20 + 25 + 10} = \$8.57$$

Rubric:

[4] \$8.57, and appropriate work is shown, such as the method shown in the sample response.

[3] Appropriate work is shown, but one computational error is made.

[2] Appropriate work is shown, but more than one computational error is made.

or

[2] Appropriate work is shown, but one conceptual error is made.

[1] The prices are averaged without considering the number of meals served at each

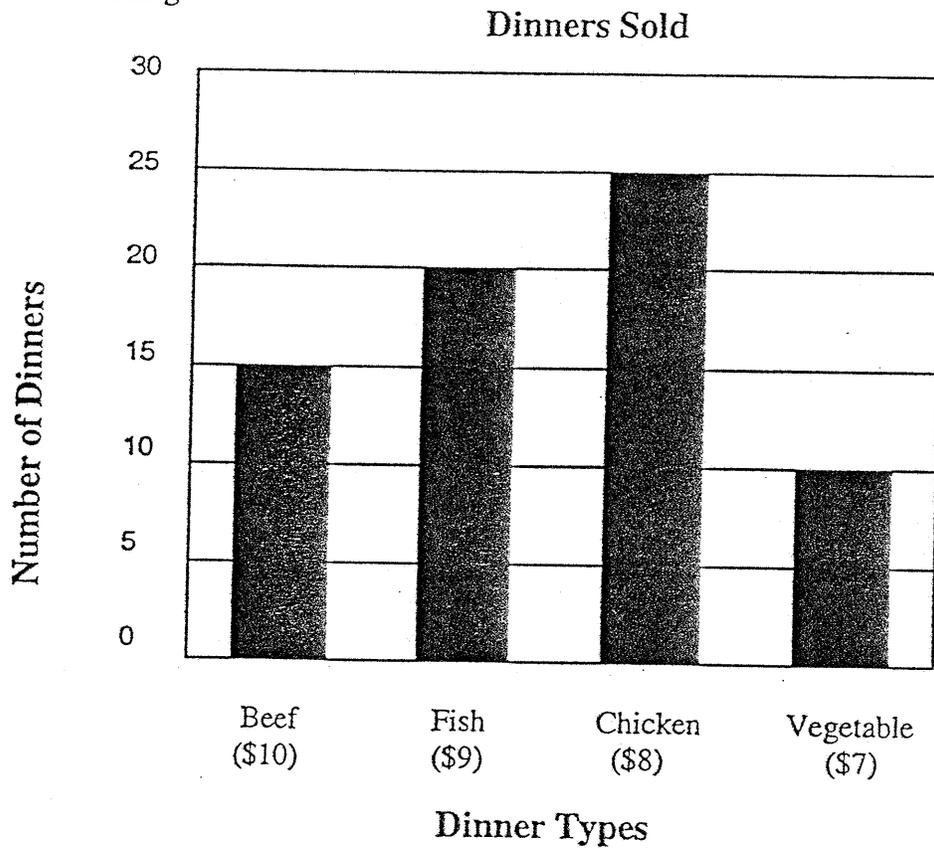
price, such as $\frac{34}{4} = \$8.50$.

or

[1] \$8.57, but no work is shown.

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



$\frac{15 \times 10 + 19 \times 9 + 24 \times 8 + 9 \times 7}{35}$
 35

150 180 200 70

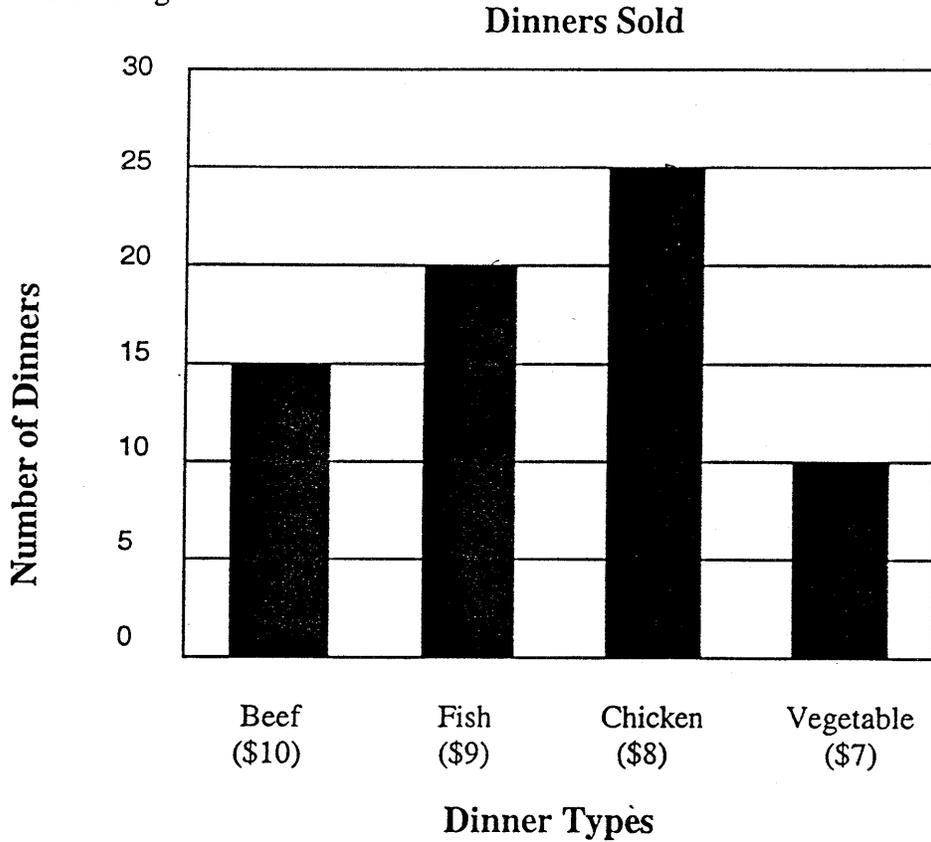
$\$8.57$

$$\begin{array}{r} 270 \\ 330 \\ \hline 600 \end{array}$$

$$70 \overline{) 600}$$

SCORE POINT: 4

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



$$(10)(15) + (9)(20) + (8)(25) + (7)(10)$$

70

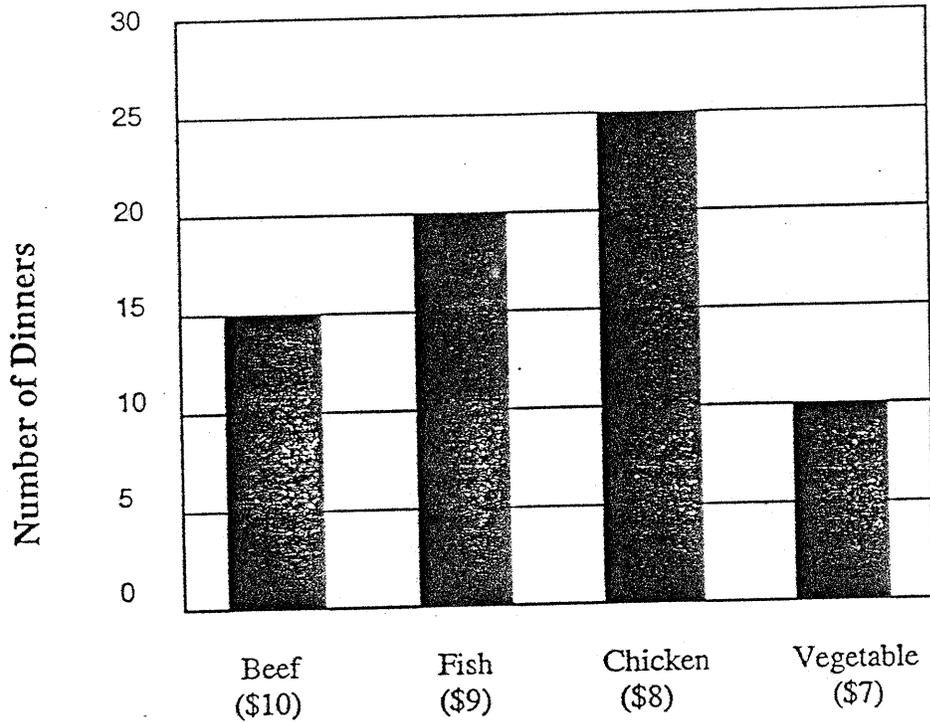
$$\frac{150 + 180 + 200 + 70}{70}$$

average price = \$8.57

SCORE POINT: 4

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?

Dinners Sold



Dinner Types

| | | | | |
|-----------|----------|----------|----------|-----------|
| | | | | 15 |
| | | | | 20 |
| | | | | 25 |
| | | | | 10 |
| | | | | <u>70</u> |
| Beef | Fish | Chicken | veg | |
| 15 (\$10) | 20 (\$9) | 25 (\$8) | 10 (\$7) | |
| \$ 150 | \$ 180 | \$ 200 | \$ 70 | |

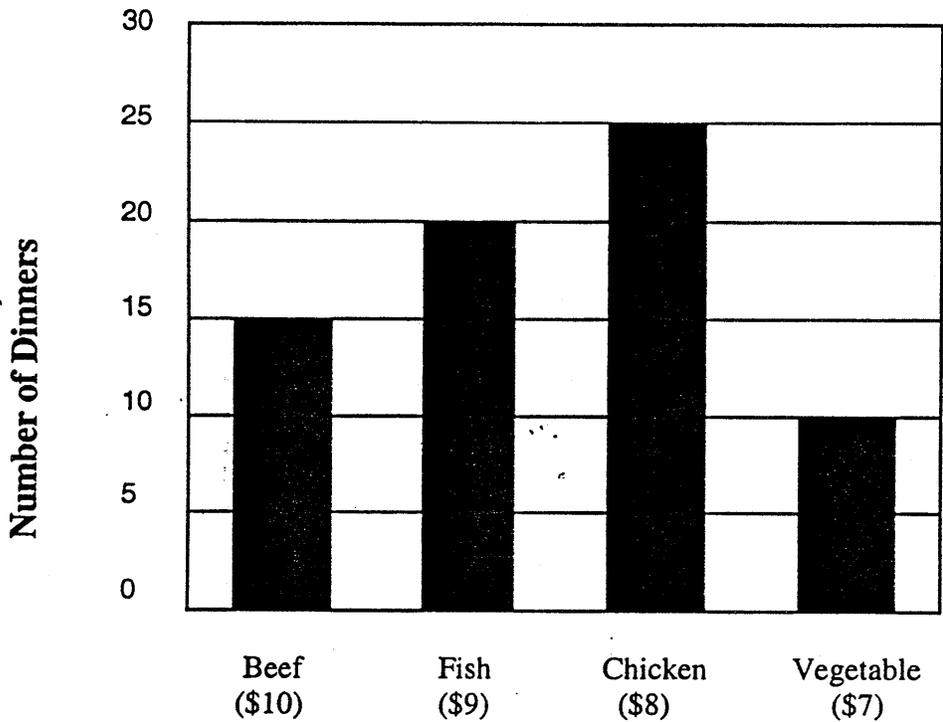
$$70 \overline{) 670} \quad 9.57$$

The mean of the price of all dinners served was about \$9.57.

SCORE POINT: 3

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?

Dinners Sold



Dinner Types

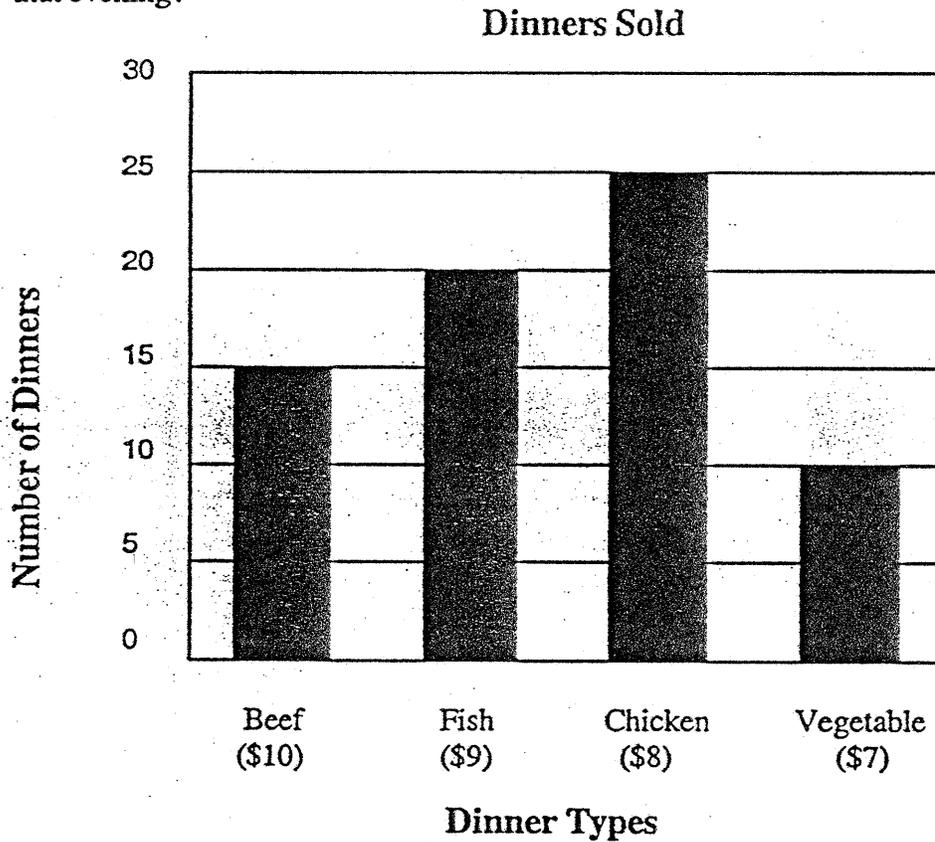
$$\begin{array}{r}
 15 \times 10 = 100 \\
 20 \times 9 = 180 \\
 25 \times 8 = 200 \\
 10 \times 7 = 70 \\
 \hline
 550
 \end{array}$$

$$\begin{array}{r}
 15 \\
 20 \\
 25 \\
 \hline
 10 \\
 \hline
 70
 \end{array}$$

Average price $\frac{550}{70} = \cancel{6.43}$
7.86

SCORE POINT: 3

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



$$\begin{aligned}
 \text{Money spent on beef} &= 10 \times 15 = 150 \\
 \text{" " " fish} &= 9 \times 20 = 180 \\
 \text{" " " chicken} &= 8 \times 25 = 200 \\
 \text{" " " vegetable} &= 7 \times 10 = 70
 \end{aligned}$$

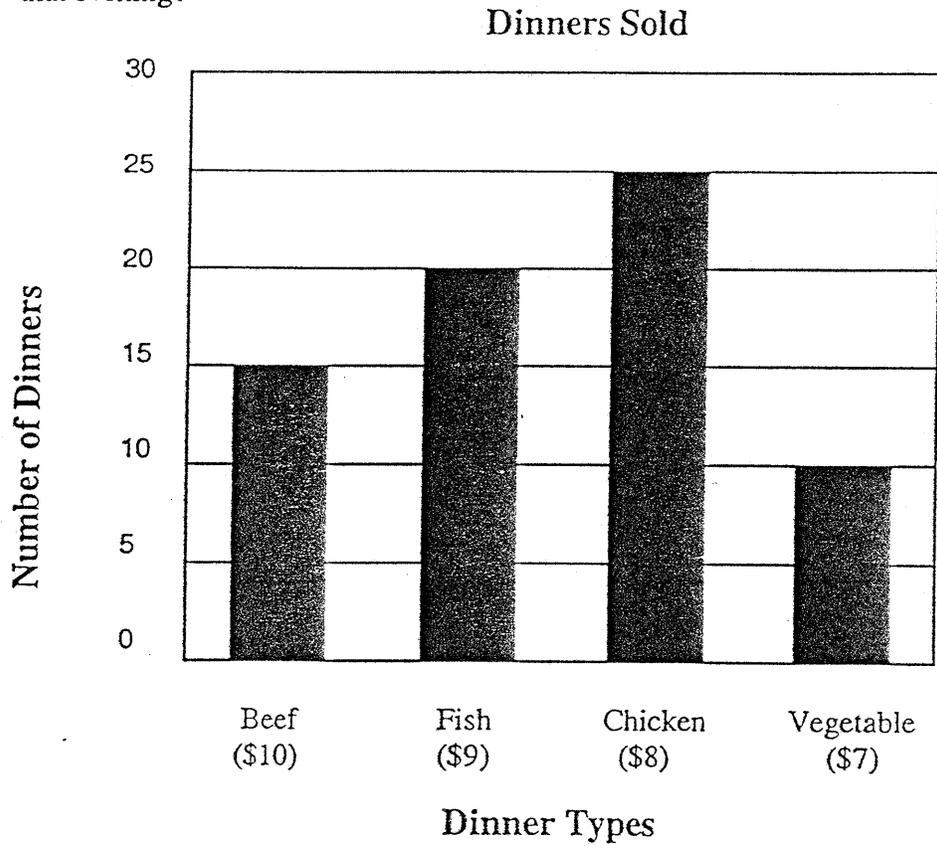
580

$$\text{Number of dinners} = 15 + 20 + 25 + 10 = 70$$

$$\frac{150 + 180 + 200 + 70}{70} = \underline{\underline{7.71}}$$

SCORE POINT: 2

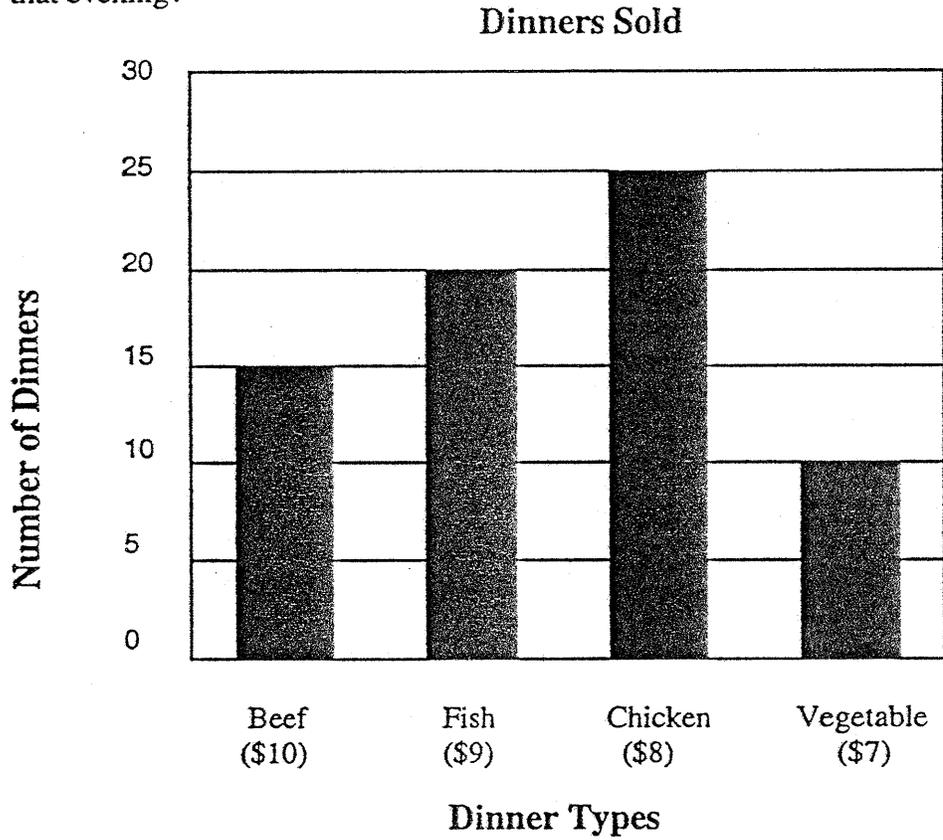
The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



~~8.57~~ \$ 8.57

SCORE POINT: 1

The number and cost of each type of dinner sold one evening at a local restaurant are summarized in the bar graph below. What was the mean (average) price of all the dinners served that evening?



Score Point: 0

Beef
 5, \$10
 Fish 20, \$9
 chicken 25, \$8
 veg. 10, \$7
 60, 34
 30, 17

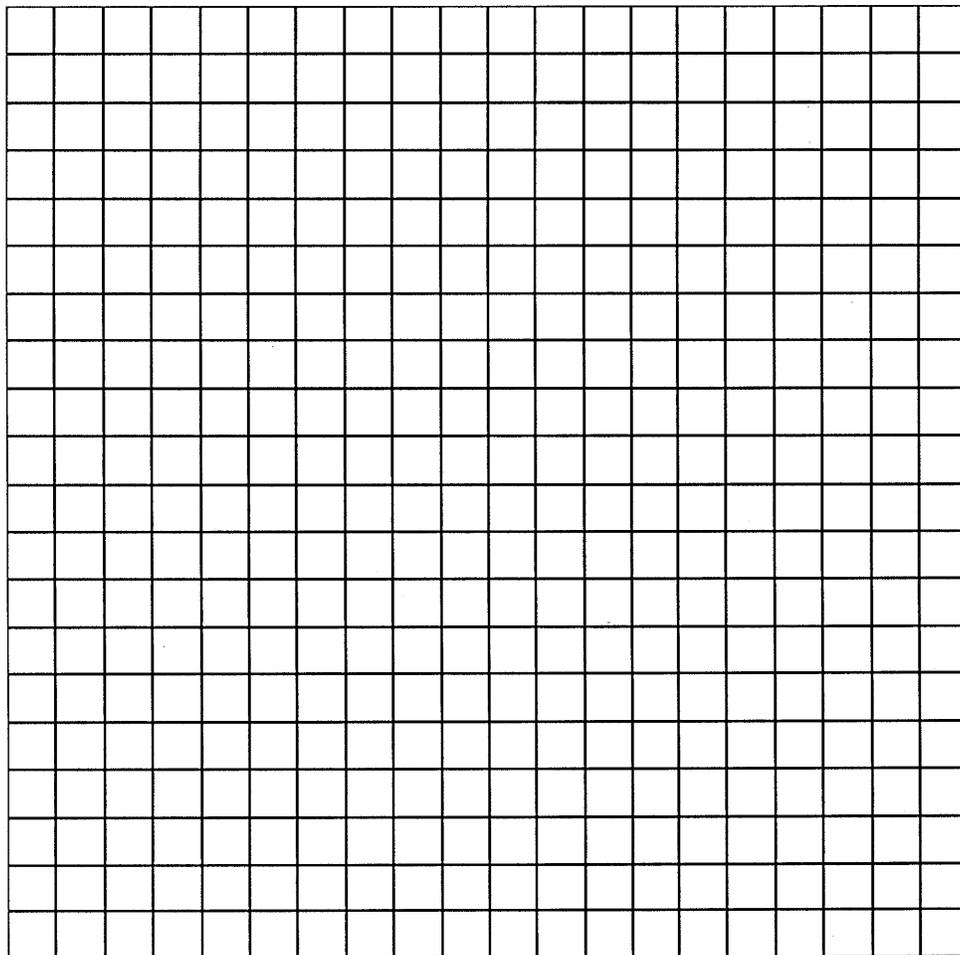
7

COMPONENT 5
MODULE 2

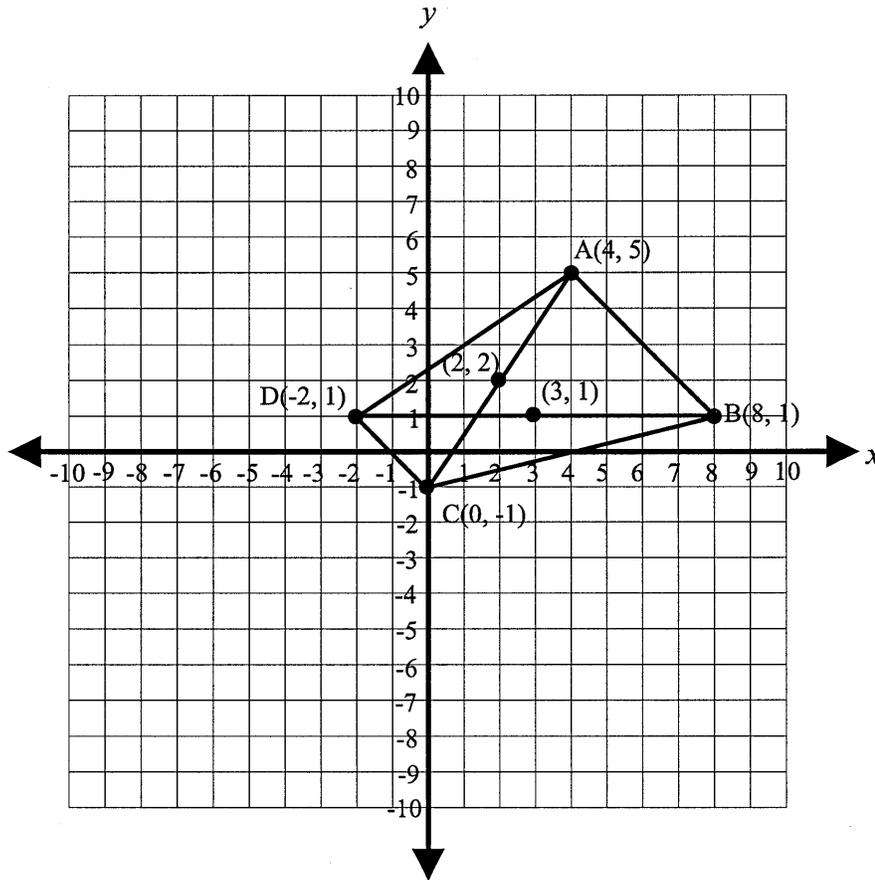
QUESTION 8

TRAINING SET

- 8 Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



Sample Response:



$$\text{Midpoint } \overline{AC} = \left(\frac{4+0}{2}, \frac{5-1}{2} \right) = (2, 2)$$

$$\text{Midpoint } \overline{BD} = \left(\frac{8-2}{2}, \frac{1+1}{2} \right) = (3, 1)$$

Or

Midpoint $\overline{AC} = \left(\frac{4+0}{2}, \frac{5-1}{2} \right) = (2, 2)$, and I graphed the other diagonal, and it does not go through this point.

Rubric

- [4] A correct graph is shown, and an indication that the midpoints are not the same is given, and appropriate work is shown, such as the methods shown in the sample response.
- [3] A correct graph and appropriate work are shown, but one computational error is made.
or
- [3] No graph or an incorrect graph is shown, but both midpoints are determined correctly, and appropriate work is shown, and a correct justification is given.
or
- [3] A correct graph is drawn, and appropriate work is shown, but no justification or an incorrect justification is given.
or
- [3] Appropriate work is shown, but one graphing error is made, yet an appropriate justification is given.
- [2] A correct graph and appropriate work are shown, but more than one computational error is made.
or
- [2] Appropriate work is shown, but more than one graphing error is made, yet an appropriate justification is given.
or
- [2] Appropriate work is shown, but one computational error and one graphing error are made, yet an appropriate justification is given.
or
- [2] A correct graph is drawn, but one conceptual error is made, such as subtracting instead of adding the x 's in the formula, yet an appropriate justification is given.
or
- [2] A correct graph is drawn, but only one midpoint is determined, and an incorrect justification or no justification is given.
or
- [2] A correct graph is drawn, and the midpoints are plotted correctly, a correct justification is given, but no work is shown.
- [1] A correct graph is shown and an indication that the midpoints are not the same is given, but no work is shown.
or
- [1] Only one correct midpoint is determined, but no further work is shown.
or
- [1] A correct graph is drawn, but no further correct work is shown.
- [0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

NOTE: No or yes, but no graph or an incorrect graph is drawn and no justification is given.

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(6, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.

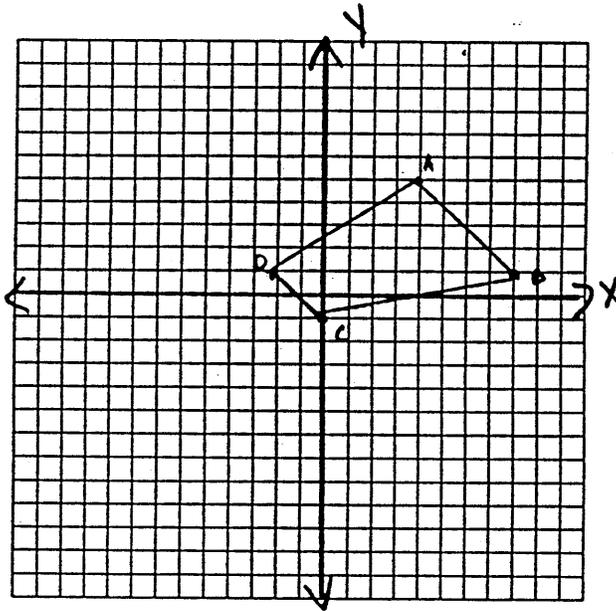
$$m = \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}$$

$$\begin{aligned} m \text{ of } \overline{AC} &= \frac{4+0}{2}, \frac{5+(-1)}{2} \\ &= \frac{4}{2}, \frac{4}{2} \end{aligned}$$

$$m \text{ of } \overline{AC} = (2, 2)$$

$$\begin{aligned} m \text{ of } \overline{BD} &= \frac{6+(-2)}{2}, \frac{1+1}{2} \\ &= \frac{6}{2}, \frac{2}{2} \end{aligned}$$

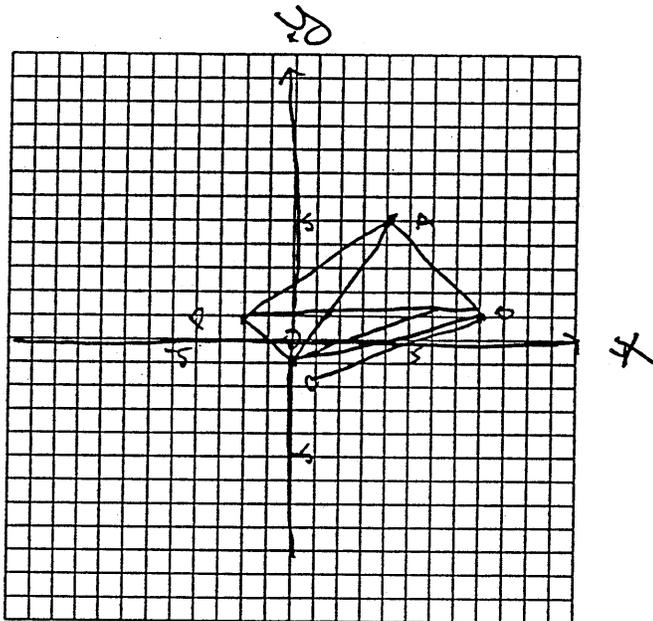
$$m \text{ of } \overline{BD} = (3, 1)$$



Since the midpoint of $\overline{AC} = (2, 2)$ and the midpoint of $\overline{BD} = (3, 1)$, they are not the same.

SCORE POINT: 4

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



$$\begin{array}{r} A(4, 5) \\ C(0, -1) \\ \hline 4, 4 \\ \hline 2 \\ (2, 2) \end{array}$$

$$\begin{array}{r} B(8, 1) \\ D(-2, 1) \\ \hline 6, 2 \\ \hline 3 \\ (4, 1) \end{array}$$

they don't have the same midpoint.

SCORE POINT:3

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.

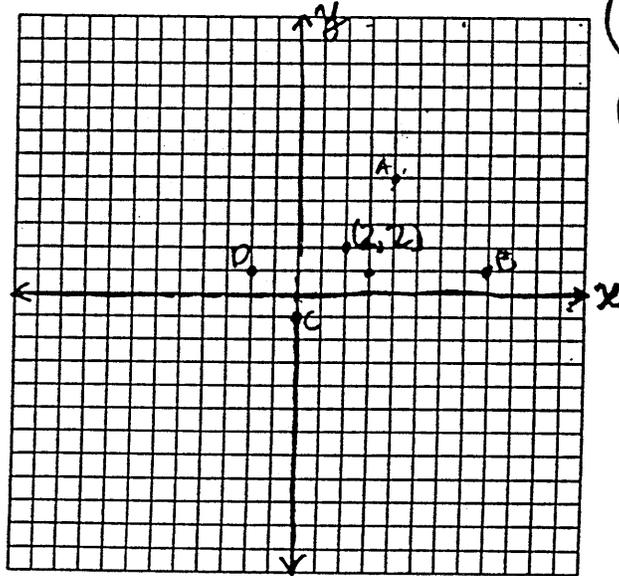
$$DB(-2, 1)(8, 1)$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left(\frac{-2 + 8}{2}, \frac{1 + 1}{2} \right)$$

$$\left(\frac{6}{2}, \frac{2}{2} \right)$$

$$(3, 1)$$



$$\overline{AC}$$

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

$$\left(\frac{4 + 0}{2}, \frac{5 + (-1)}{2} \right)$$

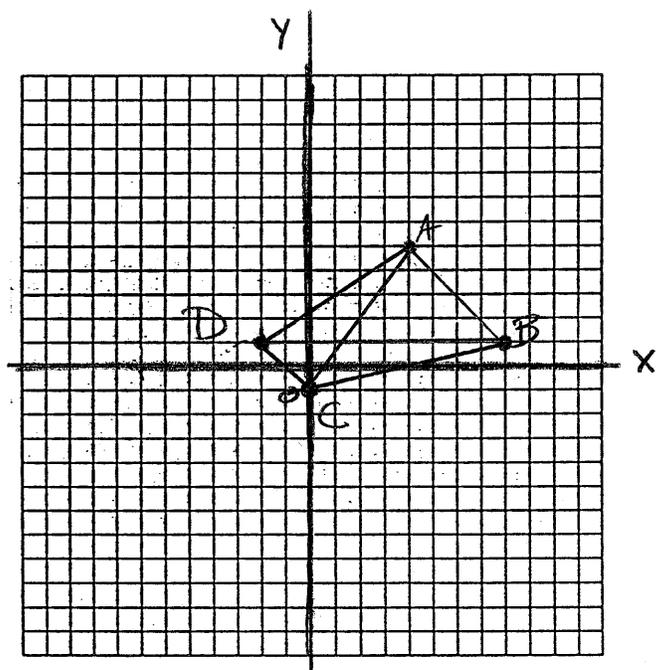
$$\left(\frac{4}{2}, \frac{4}{2} \right)$$

$$(2, 2)$$

\overline{AC} and \overline{BD} do not have the same midpoint because the midpoint of \overline{AC} is $(2, 2)$ and the midpoint of \overline{BD} is $(3, 1)$

SCORE POINT: 3

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



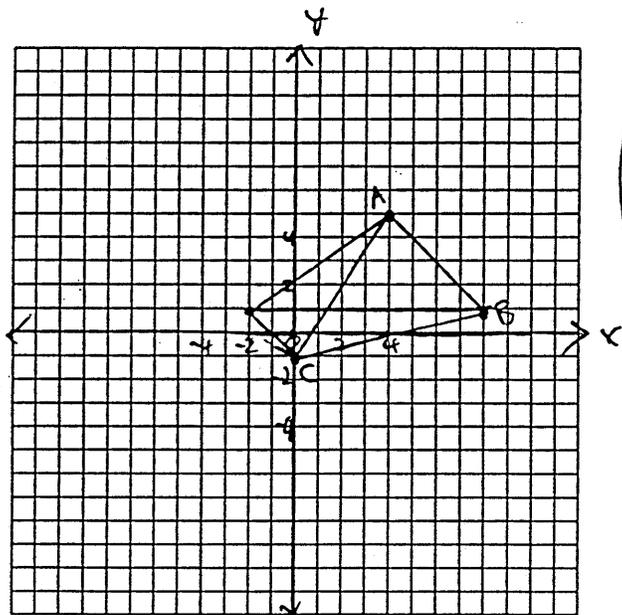
$$\overline{AC} = \left. \begin{array}{l} \frac{4 - 0}{2} = 2 \\ \frac{5 - (-1)}{2} = 3 \end{array} \right\} (2, 3)$$

$$\overline{BD} = \left. \begin{array}{l} \frac{8 - (-2)}{2} = 5 \\ \frac{1 - 1}{2} = 0 \end{array} \right\} (5, 0)$$

NO:
DIAGONALS DO NOT
HAVE THE SAME MIDPOINT
ONE IS (2, 3) AND THE
OTHER IS (5, 0)

SCORE POINT: 2

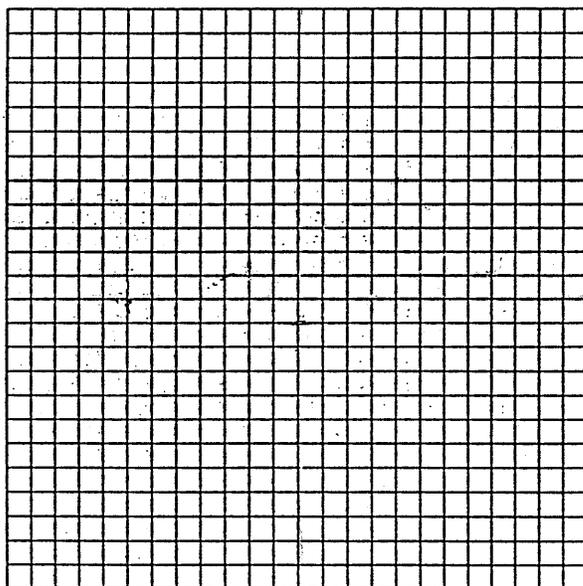
Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



They do not have the same midpoints.

SCORE POINT: 1

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



\overline{AC} midpoint $\frac{4}{2} + \frac{0}{2} \rightarrow \frac{4}{2} \rightarrow 2 \rightarrow (2, 2)$

$\frac{5}{2} + \frac{-1}{2} \rightarrow \frac{4}{2} \rightarrow 2$

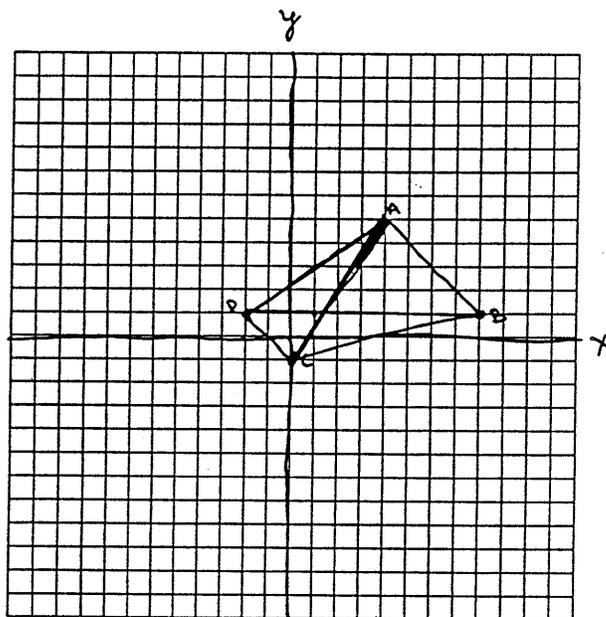
\overline{BD} midpoint

$\frac{8}{2} + \frac{2}{2} \rightarrow \frac{10}{2} \rightarrow 5 \rightarrow (5, 1)$

$\frac{1}{2} + \frac{1}{2} \rightarrow \frac{2}{2} \rightarrow 1$

SCORE POINT: 1

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.

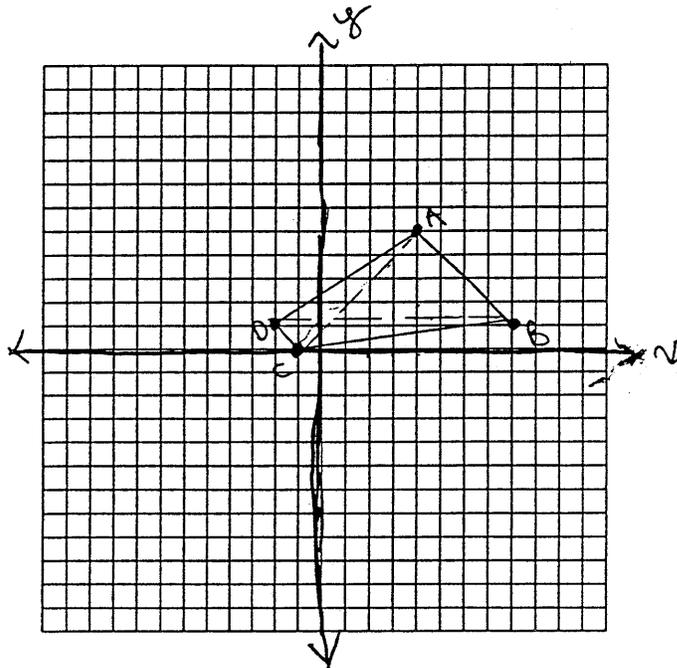


$$\begin{aligned}
 &X_1 - X_2 + Y_1 - Y_2 \\
 &4 - 0 \quad 5 - (-1) \\
 &4 + 6 = 10 \\
 &\boxed{10} \\
 &AC = 10
 \end{aligned}$$

$$\begin{aligned}
 &B, D = (8, 1) \quad 8 - (-2) + 1 - 1 \\
 &(-2, 1) \quad 10 + 0 \\
 &\boxed{10} \\
 &BD = 10
 \end{aligned}$$

SCORE POINT: 1

Quadrilateral $ABCD$ has coordinates $A(4, 5)$, $B(8, 1)$, $C(0, -1)$, and $D(-2, 1)$. On the accompanying grid, draw and label quadrilateral $ABCD$. Determine if the diagonals of quadrilateral $ABCD$, \overline{AC} and \overline{BD} , have the same midpoint. Justify your answer.



yes, ~~AB~~ \overline{AC} and \overline{BD} have the same midpoint. they cross at $(2, 1)$.

Score Point: 0

COMPONENT 5
MODULE 2

QUESTION 9

TRAINING SET

9 The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

9 The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

Sample Response:

$$\frac{6^2\pi + 8^2\pi}{12} \approx 26.18$$

$$\frac{314}{12} = 26.\overline{16}$$

$$\frac{12^2\pi}{14} \approx 32.31$$

$$\frac{452}{14} = 32.2857$$

Special 2

Rubric

- [4] Special 2, and appropriate work is shown, such as the method shown in the sample response, or another appropriate approach such as using cents per square inch or other unit price.
- [3] Appropriate work is shown, but one computational error is made.
- [2] Appropriate work is shown to determine the area relationship, but unit pricing is not used, but an appropriate special is chosen.
- or*
- [2] Appropriate work is shown, but more than one computational error is made.
- or*
- [2] One conceptual error is made, such as using the circumference instead of the area to determine unit price, but an appropriate special is chosen.
- [1] Appropriate work is shown to determine the area relationship, but no further work is shown or the incorrect special is chosen.
- or*
- [1] An appropriate unit price based on an incorrect procedure, but an appropriate special is chosen.

Rubric for #9 continued.

[0] Special 2, but no work is shown.

or

[0] A zero response is completely incorrect, irrelevant, or incoherent or is a correct response that was obtained by an obviously incorrect procedure.

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

Small pizza - 6 in. r

$$A = \pi r^2$$

$$A = 113.0973$$

Medium Pizza - 8 in. r

$$A = 201.0619$$

Large Pizza - 12 in. r

$$A = 452.3893$$

452 sq. in. of pizza for \$14

$$\text{Total pizza} = 314.1592$$

314 square inches of pizza for \$12

About \$.04 per sq. inch of pizza

About \$.03 per sq inch of pizza

Special #2 is the best deal

SCORE POINT: 4

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

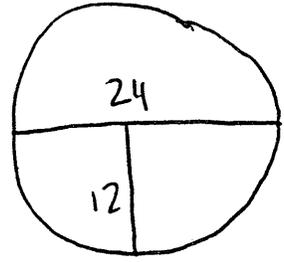
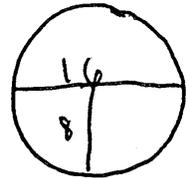
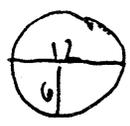
One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

Special 1

Special #2

special 2



$$a = \pi r^2$$

$$a = \pi 6^2$$

$$a = \pi r^2$$

$$a = \pi 8^2$$

$$a = \pi r^2$$

$$a = \pi 12^2$$

$$a = 36\pi +$$

$$64\pi = a$$

$$a = 144\pi$$

$$100\pi$$

$$452.16$$

$$314$$

$$2 \text{ \$}$$

$$138.16$$

②6

③2

SCORE POINT: 4

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

$$\begin{aligned}\text{Special one} &: \pi r_s^2 + \pi r_m^2 \\ &= \pi (6)^2 + \pi (8)^2 \\ &= 36\pi + 64\pi \\ &= 314.16\end{aligned}$$

$$\begin{aligned}\text{Special two} &: \pi r_L^2 \\ &= \pi (14)^2 \\ &= 196\pi \\ &= 615.75\end{aligned}$$

$$\frac{314.16}{12} = 26.18$$

$$\frac{615.75}{14} = 43.98$$

Special two gives more square inches per dollar

SCORE POINT: 3

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

Step 1: $A = \pi r^2$ $A = 3.14 (6)^2 + 3.14 (8)^2$
314 in²

Large pizza
 $A = 3.14 (12)^2$
452.16 in²

Special 2 because more area of pizza

SCORE POINT: 2

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

$$A = \pi r^2$$

$$A = \pi 6^2 = 36\pi$$

$$113.09734$$

$$A = \pi 12^2$$

$$144\pi = 452.3893$$

$$A = \pi 8^2 = 64\pi$$

$$201.06193$$

$$\frac{452.38934}{14.00} = 32.3$$

$$\frac{314.1593}{12.00}$$

26.17994 per ²inch

Special #1 is better because its only approximately 26 cents per in² and Special 2 is 32 cents per in².

SCORE POINT: 2

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

$$\begin{aligned}\text{Special one} &= \pi r_1^2 + \pi r_2^2 \\ &= \pi (6^2) + \pi (8^2) \\ &= 314.16\end{aligned}$$

$$\begin{aligned}\text{Special two} &= \pi r^2 \\ &= \pi (12^2) \\ &= 452.39\end{aligned}$$

SCORE POINT: 1

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

Special 2

Score Point: 0

The local take-out pizza restaurant is offering two specials:

Special 1

One small pizza (6-inch radius) and one medium pizza (8-inch radius) for a total of \$12.00

Special 2

One large pizza (12-inch radius) for \$14.00

For which offer will you get the most pizza per dollar?

$$C = \pi \cdot r$$

Special 1

$$= 6 \cdot \pi \approx 19 \text{ in}$$

$$= \pi \cdot 8 \approx 25 \text{ in}$$

$$44 \text{ in}$$

Special 2

$$= \pi \cdot 12 = 38 \text{ in}$$

Special 1 has the best offer because you get more pizza for the best dollars.

Score Point: 0