The Core Curriculum Companion
For The
New York State Mathematics Resource Guide

INTRODUCTION

The **Core Curriculum Companion** is the result of a cooperative effort between the New York City members of the New York State Mathematics Mentors and the New York City Urban Systemic Initiative. This document was designed to support the implementation of the New York City Performance Standards in Mathematics and the key ideas identified in the New York State Mathematics Resource Guide/Core Curriculum.

Each section of this book focuses on a specific grade level from Prekindergarten through Grade 8. Within each grade level, there are investigations to enable students to explore important concepts in mathematics in each of the seven key ideas.

To facilitate the explorations, we have included vocabulary, a list of materials needed, the related New York City Performance Standards, and all the performance indicators. To further enhance student learning, each investigation includes ideas for implementation, possible solutions, and extensions of the original activity. We have also included model assessment questions in both multiple choice and extended response formats to support the mathematics testing program in New York City and New York State.

It is our expectation that the **Core Curriculum Companion** will assist teachers in providing their students with rich experiences in mathematics. In addition, we believe this document will support the high standards that have been established for our students.

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PREKINDERGARTEN

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Use models, facts and relationships to draw conclusions about mathematics and explain their reasoning.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Explain their answers and solution processes.

INVESTIGATION

"I'VE GOT IT"

Students will make observations to determine eligibility criteria of a sorted group. Once criteria is established students will be able to add objects to the group.

WHAT'S THE MATHEMATICS?

- identifying similarities and differences
- creating sets
- problem solving

RELATED New York CITY PERFORMANCE STANDARDS

M5b Implementation
M5c Conclusion
M7c Explains solutions to problems clearly and logically, and supports solutions with evidence, in both oral and written work.

VOCABULARY:	sort	same	alike
	different	group	set

SUGGESTED TIME: 15 minutes/day 4x/week whole group

MATERIALS: children's footwear

DOING THE INVESTIGATION

1. Have children place their footwear (ask for only one) in the middle of the circle.
2. With children watching, start forming a sub-group by placing several of the tie sneakers in a separate spot.
3. Tell them that all of the items in the new group are the same in a special way. If you can tell me how they are the same shout "I've Got It!"
4. When students think they have identified the sorting criteria, have them place a sneaker in the pile.
5. When about half of the sneakers are sorted ask if anyone can give a name to the newly formed group. Encourage students to be specific about the label: a group of sneakers you can tie. Explain how you figured it out.
6. Repeat the label and show the students that all the members of this sub-group are the same.
7. Continue sorting sneakers until completion.
8. Work on sorting throughout the week, setting up a different group each day, i.e., Velcro sneakers, shoes, boots.
9. Can the entire group sort a set without revealing the criteria?
10. At the end of the week try to sort several sub-groups at the same time.

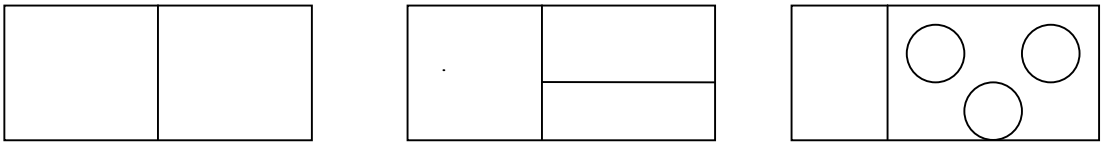
POSSIBLE SOLUTIONS

Two groups of footwear: One group of mixed footwear
One group of tie sneakers only

Several smaller groups of footwear: One group of Velcro sneakers
One group of shoes
One group of boots, etc.

EXTENDING THE ACTIVITY

1. Set up a sorting activity for individual or small group use.
   Prepare a variety of colored items: small cubes, Lego blocks, pattern blocks and pegs. Children can sort by color (different items) or by item. It might be helpful to put out a sorting mat. Use a 12x18 piece of construction paper. Draw lines as shown.



2. Challenge critical thinking skills by asking children to resort a sub-group, i.e., from the tie sneaker group sort white tie sneakers.

3. Sort seasonal objects:
   Apples or leaves for Fall
   Gloves, mittens or hats for Winter
   Seeds or flowers for Spring
   Sandals, caps or fruit for Summer

ASSESSMENT 1

Teacher Observation, Listening and Recording:

- Can the student sort items consistently?
- Is the student able to identify the sorting criteria?
- Can the student sort groups and sub-groups without a given criteria?

ASSESSMENT 2

Sort a group of girls into two or more sets.

For additional information and activities, see pages 10 and 18
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA #2NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Use whole numbers to determine number positions and quantify groups of objects.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions.
2C. Relate counting to grouping and place value.
2D. Recognize the order of whole numbers.

INVESTIGATION

DISTRIBUTION

Students will match one notice with one child for all children in their group. They will determine the correct number of notices needed by for the children in their group.

WHAT'S THE MATHEMATICS?

- one to one correspondence
- counting
- equivalent sets

RELATED New York CITY PERFORMANCE STANDARDS

M1a Arithmetic and Number Concepts (one to one correspondence and counting)
M7a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

VOCABULARY: same amount enough as many as

SUGGESTED TIME: 10 – 15 minutes/3x week

MATERIALS: notices, correspondence or letters to go home, home folder

DOING THE INVESTIGATION

1. Children should be seated in small groups at the tables with; their home folder or book bag ready to accept notices.
2. Give one child at the table the appropriate number of notices and ask him/her to give one to each child at the table. Observe to be sure that all children are receiving papers.
3. You might want to suggest some type of sequence, i.e., walk around the table while distributing. Continue this process until all children have had at least two turns at distributing (about 2 weeks).
4. This time when you approach the table count out loud the number of children seated. Then count out loud the number of notices needed and hand them to a child for distribution. Point out that the distributor has as many papers as there are children. Do this for a few days.
5. Start questioning the children in each group by asking, How many papers do you think you need at this table today? It is important to ask the children how they arrived at their answer.
6. When the children are secure with one to one correspondence directly at the table, you can move all notices to another place. Have the distributor come and take enough notices for all the people at their table.

POSSIBLE SOLUTIONS

Each child should have a paper to go home.
Students should be able to count the number of children in their group to determine the amount of papers needed.
Students might be able to recognize quantity without counting. “I just knew there were four.”
Students should be able to determine the correct amount, get it and distribute to the group.

EXTENDING THE ACTIVITY

1. When the teacher is distributing notices at the table, count out too many or too few and ask, "Are _____ enough for this group?"
   "Will we need more or less for everyone to get one?"

2. Set up a center activity with pegboards and pegs. Students are required to fill two rows with same color pegs.

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Is the student counting correctly?
- Can the student explain his/her method for solving the problem?

ASSESSMENT 2

During snack or a party can the child take enough cups for the children at his/her table?

For additional information and activities, see pages 11 and 19
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA #3OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Develop strategies for selecting the appropriate computational and operational methods in problem solving.
3B. Develop readiness for single-digit addition and subtraction facts.
3C. Understand the commutative and associative properties.

INVESTIGATION

THE BAKERY SHOP

Students will use manipulatives to establish quantity. They will determine which strategy to use to find out how many cookies are left in the bakery shop. The song can be sung or chanted.

WHAT'S THE MATHEMATICS?

- one to one correspondence
- counting, forward and backward
- subtraction
- problem solving

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Arithmetic and Number Concepts

VOCABULARY:	How many?	How many taken?	How many left?
	count	forward	backward

SUGGESTED TIME: 5 minutes 3x/week

MATERIALS: Hands and song:

Five little cookies in the bakery shop
Sitting on the counter with some sugar on the top.
Along came _____ with a penny to pay.
He grabbed a little cookie and he ran a-way.

DOING THE INVESTIGATION

1. In a large or small group have the children warm up their fingers and mathematical minds with a finger play. Try:

   Five little soldiers standing in a row.
   Three stood straight and two stood so (bent).
   Along came the captain (one finger moves in front) and
   what do you think?
   They all stood straight as quick as a wink!

2. Ask the students to hold up one hand and count out loud the number of fingers. Tell them to pretend we are in a bakery shop and there are five cookies on the counter. "Let's make believe your fingers are cookies."
3. Start the cookie song. Students should have five fingers displayed. When the cookie is purchased, physically bend and hide a finger on your hand. Ask the students if they can tell you how many cookies are left in the shop? With responses, be sure to ask how the student came to his/her answer.
4. It is important that the students use their fingers to represent quantity as they are counting.
5. Repeat the song, hiding fingers one at a time, until all are gone.
6. This song can begin at any number up to ten.
7. Modeling and participation from the teacher will encourage active participation from the students.

POSSIBLE SOLUTIONS

Students should be able to count forward on their tall fingers to determine the number of cookies left.
Students might recognize quantity visually.
Students might be able to count backward using bent fingers to determine the amount of cookies left.

EXTENDING THE ACTIVITY

1. Sing three other take away songs: Five Green and Speckled Frogs – RAFFI
   Five Little Monkeys Swinging from a Tree
   Ten in the Bed
2. Using the Bakery Shop song take away two or three cookies at a time. The teacher could also vary take away amounts in the song.

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Students are able to manipulate fingers to show quantity.
- Student counting forward and backward is accurate.
- Quantity said and shown match, one to one correspondence is demonstrated.

ASSESSMENT 2

Repeat songs using a different manipulative to see if students can translate from one material to another.

For additional information and activities, see pages 12 and 20
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA #4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Use concrete materials to model spatial relationships.
4B. Construct charts and graphs to display and analyze real-world data.
4C. Use multiple representations (manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.
4D. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

CAN YOU BUILD THIS?

Students will explore spatial relationships by building with color cubes. They will identify positions, i.e., top, bottom, under, etc., by replicating a model and coloring it.

WHAT'S THE MATHEMATICS?

- observation
- sorting
- one to one correspondence
- counting
- spatial relationships

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Gives and responds to directions about location, e.g., by using words such as " in front of," "right," and "above."
M6e Refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle, square, rectangle, side, edge, face, cube, point, line, perimeter, area and circle; and refers with assistance to rhombus, parallelogram, quadrilateral, polygon, polyhedron, angle, vertex, volume, diameter, circumference, sphere, prism, and pyramid.
M7a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

VOCABULARY:	bottom	middle	on top	under
	underneath	next to	over	higher
	lower	tall	short	in front of

SUGGESTED TIME: 5 - 20 minutes 3-4x per small group

MATERIALS: colored cubes, paper, markers or crayons

DOING THE INVESTIGATION

1. Work in a small group, dividing students in pairs. Children should work facing each other.
2. Prepare a basket of colored cubes for each pair.
3. Give the students a few minutes of free exploration with the cubes.
4. Using 6-8 cubes, build a structure between each pair. As you are building use color words and positions to describe where you are placing each cube.
5. Ask the students to replicate the structure in front of them.
6. Repeat the process but this time, describe one spatial feature of the building. Have the children describe the rest of the structure before they build it. Encourage language by questioning:

Tell me about the structure.
How did you make it so tall?
What else do you notice?
Why did you put that cube there?

7. Now ask one child in each pair to create a structure and the second child to replicate it. Have both describe it.
8. Draw a quick two-dimensional sketch of the structure. Have the children outline the squares with the appropriate colors and then color with crayons.

POSSIBLE SOLUTIONS

Accept all structures.
Language should include some position words and several color words.
Accept all pictures.

EXTENDING THE ACTIVITY

1. Require a specified number of cubes be used in the structure
2. Change the building material to pattern blocks or Lego.
3. Working in pairs, have one child use language only to tell his partner where to place the cubes.
4. Have the children draw their own sketches.
5. Use sketches to make tasks cards available with cubes for individual use.

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Does child use model appropriately? Are cubes placed correctly?
- Are color words used?
- Does child use any position words?
- Can s/he use both to correctly describe placement?

ASSESSMENT 2

Play a Simple Simon game using position words, body parts and a chair. Simple Simon says put your arm over the chair. Simple Simon says put your foot under the chair. Simple Simon says stand next to the chair. Sit on the chair, etc.

For additional information and activities, see pages 13 and 21
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Select appropriate standard and nonstandard measurement tools in measurement activities.
5B. Understand the attributes of length, capacity, weight, time, money, and temperature.
5C. Estimate measures such as length and volume, using both standard and nonstandard units.
5D. Collect and display data.
5E. Use statistical methods such as graphs and charts to interpret data.

INVESTIGATION

PEG RACE

Students will explore time by comparing the duration of an egg timer with how many pegs they can put in a pegboard.

WHAT'S THE MATHEMATICS?

- one to one correspondence
- counting
- measuring time
- comparing
- estimation

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1 Arithmetic and Number Concepts (one-to-one correspondence, counting)
M2i Selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as weight, length, area, volume, and time.

VOCABULARY: time minute(s) how many? estimate
few a lot a little

SUGGESTED TIME: 15 minutes per small group

MATERIALS: pegs, pegboards, egg timer

DOING THE INVESTIGATION

1. Introduce the egg timer to the whole group. What do you think this is? How do you use it? What do you use it for?
2. Try a timed experiment with the egg timer: How many times can you jump up and down?
3. Compare the results.
4. Prepare pegs and pegboards for small group work, about four students.
5. Ask students to predict (guess) how many pegs they can put in the pegboard before the timer is up. If counting is not yet established you can ask the child to show you how many. Put the pegs in a pile near the child. Ready.. Set.. GO!
6. Have the student's compare what is in their pegboard to what is in their pile. Was your guess correct? Did you have any left over? A lot? A little?
7. Count the pegs that are in each board and compare them. Who put in the most? The least? The same? Emphasize that everyone had the same amount of time.
8. Try the experiment again. Now can you guess how many pegs you will need? Did students adjust their amounts?
9. Focus on time by asking, "What would happen if you flipped the timer over twice?" Or stopped it halfway? Try it.

POSSIBLE SOLUTIONS

Students need many chances to experiment in order to become familiar with the amount of time.
As the duration of a specified time is internalized, students' estimated amounts should become more accurate.

EXTENDING THE ACTIVITY

1. Try stacking colored cubes. How tall a tower can you build in this amount of time?
2. Try stringing beads. How long a necklace can you make in this amount of time?
3. Vary the measuring tool. Use a bell timer or the second hand on a clock.
4. Use the egg timer to measure other times during the school day.
   It should take this much time to line up. Let's see if we can do it.
   Can you wash up in this amount of time?
   Can you clean up the block area before the timer is finished?

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Did the child make an adjustment in his/her predicted amount? Was it appropriate?
- When the time was lengthened, was the peg amount increased? When shortened, was it decreased?

ASSESSMENT 2

Teacher Questioning

"Tell me one thing it takes a long time to do." "Tell me one thing it takes a short time to do."

For additional information and activities, see pages 14, 15 and 22
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Recognize situations in which only an estimate is required.
6B. Develop a variety of estimation skills and strategies.
6C. Predict experimental probabilities.

INVESTIGATION

RED OR YELLOW?

Students will explore probability by tossing a color die then examining the results. A game of chance will be played by tossing two-color counters.

WHAT'S THE MATHEMATICS?

- making predictions
- probability
- one to one correspondence
- counting

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1 Arithmetic and Number Concepts (one to one correspondence and counting)
M4e Predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely.

VOCABULARY: more less same more likely

SUGGESTED TIME: 2 lessons: one whole group, one small group

MATERIALS: Several dice: four sides with one red dot and two sides with one yellow dot, counters and two-color counters

DOING THE INVESTIGATION

1. Arrange the students in groups of four with a die and 10 counters for each group.
2. Tell the students they will work in teams within their groups, one yellow team and one red team. Each team will have a turn to roll the die. If your color is thrown your team will take a counter. The team with the most counters after 10 rolls wins.
3. As the students begin, listen for comments about the game. Ask which team has more? Which team has less? Why?
4. Have the students play several rounds. Ask if they notice anything about the game. Elicit responses that focus on the red team winning more than the yellow team and how unfair the game is. Why do you think this is happening? Have the students examine the dice. Who is more likely to win the game?
5. Try a few more rounds to confirm the hypothesis.
6. How do you think we can change the game to make it a fair one?
7. The next day review information discovered the day before. Tell the students you want to play the same game but this time you will toss a two-color counter instead of a die. The color names for each team should be the same as the counter.
8. Pass counters around for observation. What do you think will happen in today's game?
9. After playing a few rounds discuss the results. Was the game fair or unfair? Why do you think this?

POSSIBLE SOLUTIONS

1st Activity: The chances of the red team winning are far greater than the yellow team. The students should conclude that the red team is winning more because there are more red dots on the die than yellow ones. To make it fair the die would have to have the same number of yellow dots as red ones.

2nd Activity: In this game each team has an even chance of winning. The game is fair because there is one red side and one yellow side.

EXTENDING THE ACTIVITY

1. Use the adjusted dot die (3 red dots and 3 yellow dots) to play the game. Who is most likely to get a lot of counters?
2. Play Heads or Tails with a coin. Who is most likely to win?
3. Start the two color counter game with a stack of ten blocks. Each time your team tosses it's color, take a block off. Who is most likely to win?

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Are students attending to and comparing the amounts collected?
- Is there discussion on winning or not winning?
- Are they keeping count?

ASSESSMENT 2

Given a die with five sides blue and one side red can a pair of students answer the following questions?
In five tries which color team do you think will win more?
In five tries which color team do you think will win less?

For additional information and activities, see pages 16 and 23
in the New York State Core Curriculum/Mathematics Resource Guide.

PREKINDERGARTEN

KEY IDEA #7PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Use a variety of manipulative materials to explore patterns.
7D. Interpret graphs.
7E. Explore and develop relationships among two and three-dimensional geometric shapes.
7F. Discover patterns in nature, art, music, and literature.

INVESTIGATION

Using names the students will explore, rhythm and patterns. They will use this information to create a visual representation of the pattern with blocks.

WHAT'S THE MATHEMATICS?

- one to one correspondence
- counting
- sequence

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3 Function and Algebra Concepts (linear patterns).
M7b Shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

VOCABULARY: rhythm beat repetition sequence

SUGGESTED TIME: 10 - 15 minutes 3 - 4 x/week for 2 weeks

DOING THE INVESTIGATION

1. In a whole group start by chanting two children's names in a repetitive sequence, i.e., Ka - ren Jon – a – than, Ka – ren Jon – a- than, Ka - ren, Jon-a-than. Ask the students to join in the chant when they are ready.
2. Tell them you have created a pattern using their names. Can the students continue the pattern without you?
3. Keep the rhythm, change the names and continue chanting, i.e., Su –san An–ge-la, Su-san An-ge-la.
4. Add to the rhythm by clapping on the beat of the names. Students listen then join in.
5. Stop chanting and listen to the pattern. Can the students extend this clapping pattern?
6. Continue this game for a week, changing names and resulting rhythms. You can also alternate your method of starting either clapping or chanting.
7. The following week, introduce a visual pattern with blocks. Begin with two names and blocks to represent each beat (same shape and size for each name).
8. Start the pattern and say the name as you display them. Ask the students to extend this pattern and share knowledge about how they knew what came next.
9. Repeat several times during the week alternating between claps and chants as you point to the blocks.

POSSIBLE SOLUTIONS

There will be numerous patterns resulting from this activity.
The complexity of the patterns will depend on the names chosen and the order in which they are used. Try to keep it simple.

EXTENDING THE ACTIVITY

1. Add three names to the pattern.
2. Use rhythm sticks to accompany or create a pattern instead of clapping.
3. Ask students to create patterns and have the class extend them.
4. Have the students create patterns in the block area. Draw a picture of their work.

ASSESSMENT 1

Teacher Observation, Listening and Recording

- Students can identify sequence in a simple (AB type) pattern.
- Students can extend simple patterns (AB type).
- Students can share their knowledge for extending the pattern.

ASSESSMENT 2

Working with a partner can you create a pattern on a felt board using shapes? Make another and have your partner extend it.

For additional information and activities, see pages 17 and 24
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Use models, facts and relationships to draw conclusions about mathematics and explain their reasoning.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Explain their answers and solution processes.

INVESTIGATION

Students will determine possible solutions for the following nontraditional word problem:

There are some animals behind a fence. You can see four heads sticking up from the top of the fence and twelve legs sticking out under the fence. What animals could they be?

WHAT'S THE MATHEMATICS?

- number and numeration, including one to one correspondence, counting, comparing and joining
- problem solving

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Add subtracts, multiply, and divide whole numbers.
M5a Formulation: Given the basic statement of a problem situation, the student:
1. Makes the important decisions about the approach, materials, and strategies to use.
2. Uses previously learned strategies, skills, knowledge, and concepts to make decisions.
3. Uses strategies such as using manipulatives or drawing sketches, to model problems.

M5b Implementation: The student makes the basic choices involved in planning and carrying out a solution; that is the student:

1. Makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate;
2. Makes connections among concepts in order to solve problems;
3. Solves problems in ways that make sense and explains why these make sense, e.g., defends the reasoning, explains the solution.

M6a Adds, subtracts, multiplies, and divides whole numbers correctly.
M7c Explain solutions to problems clearly and logically.

VOCABULARY: more less join

SUGGESTED TIME: 25 - 35 minutes

MATERIALS: Unifix cubes or other counters, paper, crayons, markers, pencils, animal resource pictures/books

DOING THE INVESTIGATION

This investigation will be most successful if it is preceded by a series of lessons identifying attributes of a variety of animals including mammals, reptiles and birds. Students may be required to sort animal photographs based on their coverings, where they live and/or the number of legs they have. After students have been tuned into the number of legs various animals have, they will be able to use this prior knowledge to help them solve the problem.

1. Have materials readily available for students working in the math center.
2. Pose the problem to the students working in the math center.
3. Ask children to write their solutions on a piece of paper and explain how they figured it out to the other students.

POSSIBLE SOLUTIONS

Children may formulate and implement a variety of strategies to solve this problem. They may use counters to represent the heads/legs; they may draw circles and tally marks to represent heads/legs; they may draw actual animals. Solutions include:

- Two four-legged animals and two two-legged animals
- Three four-legged animals and one animal with no legs
- Two six-legged animals and two animals with no legs
- One eight-legged animal, one four-legged animal, two animals with no legs
- One eight-legged animal, two two-legged animals, one animal with no legs
- A four- headed, twelve-legged creature
- Four four-legged animals where two are standing on their hind legs

Students may arrive at some very creative solutions. If their reasoning is valid, then the solution should be accepted as correct.

EXTENDING THE ACTIVITY

Present the same problem to the students, but change the numbers. If there are 3 heads and 14 legs, this will lend itself to students using knowledge of insects and spiders; insects have six legs and spiders have eight legs.

ASSESSMENT 1

Note the student's solution strategy.

ASSESSMENT 2

Interview the student with similar problems progressively making the numbers more difficult.

For additional information and activities, see pages 10 and 18
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA #2NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Use whole numbers to determine number positions and quantify groups of objects.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions.
2C. Relate counting to grouping and place value.
2D. Recognize the order of whole numbers

INVESTIGATION

A student will be assigned the responsibility of taking attendance.

WHAT'S THE MATHEMATICS?

- one to one correspondence
- counting
- sorting
- combining
- symbolic representation of numbers
- sorting

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Add, subtract, multiply, and divide whole numbers.
M4c Makes statements and draws simple conclusions based on data.

VOCABULARY: more less total number words

SUGGESTED TIME: 5 minutes each day

MATERIALS: class set of student name cards, pocket chart, word cards “Absent”, “Boys”, “Girls”, “Total Present”

DOING THE INVESTIGATION

1. Assign a student to be the attendance monitor for a week.
2. At the start of the school day direct the student to “take attendance” by sorting student name cards by who is absent/present.
3. Direct the student to put absent name cards in the pocket chart next to the word “Absent”.
4. The student then is to sort the cards into groups of boys/girls and put those cards in the pocket chart next to the appropriate word cards.
5. The student then writes the number of present boys next to the pocket chart on the chalkboard.
6. Do the same for girls.
7. Student writes the total number of boys + girls next to “Total Present”.
8. Ask the student to explain to the class how he/she arrived at the answer.

POSSIBLE SOLUTIONS

The student will most likely make adjustments to the data from the day before by moving student word cards onto the appropriate lines. For example, if Li is absent, his card will be moved from the “Boys” line to the “Absent” line.
After adjusting cards to properly reflect the attendance data of the day, a student may:

- Count all the cards in a group to come up with a total for each group, then count the boys group and the girls groups together by starting with the number one.
- See the total number of boys from the day before and change the number accordingly without counting cards. For example, if 12 boys were present yesterday, one boy is absent today, the student will know to change the number 12 to 11 because it is one less.

EXTENDING THE ACTIVITY

The student who is in charge of attendance may be asked to distribute materials such as drawing paper to the class. Direct the student to use the data from the attendance chart to figure out how many pieces of paper to get from the closet. Ask the child for another way he/she could figure out how many pieces of paper to get without using the attendance chart. (For example, counting the students, or seeing one seat unoccupied indicating one student absent, which would mean 26 pieces of paper instead of 27.)

ASSESSMENT 1

Note the student's solution strategy.

ASSESSMENT 2

Ask the student to use student attendance data to distribute materials, for example, drawing paper. Observe student's strategy.

For additional information and activities, see pages 11 and 19
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA #3OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Develop strategies for selecting the appropriate computational and operational methods in problem solving.
3B. Develop readiness for single-digit addition and subtraction facts.
3C. Understand the commutative and associative properties.

INVESTIGATION

A student is assigned to distribute snacks to a table seated with 3 to 5 children for one week. Each day fewer snack items than needed are placed in a basket by the teacher. The student will be required to figure out how many more items will be needed so that each child at the table will get his/her share and explain his/her strategy for arriving at the solution.

WHAT'S THE MATHEMATICS?

- finding the missing addend
- joining/separating whole numbers

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Add, subtracts, multiply, and divide whole numbers.
M5a Formulation – Given the basic statement of a problem situation, the student:
- Makes the important decisions about the approach, materials, and strategies to use…
- Uses previously learned strategies, skills, knowledge, and concepts to make decisions.
- Uses strategies such as using manipulatives or drawing sketches, to model problems.

M7c Explain solutions to problems clearly and logically.

VOCABULARY: add subtract more
join separate less

SUGGESTED TIME: 10 minutes each day

MATERIALS: Baskets, snack items for each day, i.e., graham crackers, saltines, pretzel rods, raisins, etc.

DOING THE INVESTIGATION

1. Prepare baskets with snack items, one per child.
2. If there are five children at the table, only put in three items.
3. Say, "There are three graham crackers in the basket. There are five children at the table. How many more graham crackers do you need to add to the basket so that there will be enough graham crackers for each child to get one?"
4. Give the basket to the student.
5. Ask the student to explain how he/she got the answer.
6. The next day vary the number of snack items in the basket.
7. On subsequent days, provide snacks where each child will receive two snack items. For example, "There are seven graham crackers in the basket. You need ten graham crackers so that each child at the table will get two. How many more graham crackers do you need to add to the basket?"

POSSIBLE SOLUTIONS

A student will use his/her fingers and use a direct modeling strategy by counting on. He/she may report to the class that "I saw three crackers in the basket and then I counted two more… four, five to get my answer."

Another student may use an estimating strategy by placing a random number of crackers in the basket and then counting all the crackers beginning with the number one. He/she will then make appropriate adjustments by adding or taking crackers to/from the basket to get five.

EXTENDING THE ACTIVITY

Many students have a basic foundation of division and multiplication in kindergarten. Students can be asked to prepare snack baskets (or book baskets for each table) where they are given the problem: "There are four children at the table. Each child will get two crackers. How many crackers do you need to put in the basket?" or "There are 10 books in the baskets. There are five children at the table. How many books will each child get?"

ASSESSMENT 1

Note the student's solution strategy.

ASSESSMENT 2

Interview the student with similar problems progressively making the numbers more difficult.

For additional information and activities, see pages 12 and 20
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA #4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Use concrete materials to model spatial relationships.
4B. Construct charts and graphs to display and analyze real-world data.
4C. Use multiple representations (manipulative materials, pictures, diagrams) as tools to explain the operation of everyday procedures.
4D. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

Students will create a train using attribute blocks based on predetermined criteria developed by the teacher.

WHAT'S THE MATHEMATICS?

- identifying attributes of shapes – triangle, circle, rectangle, square
- developing mathematics vocabulary
- matching same and similar shapes

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Uses many types of figures (angles, triangles, squares, rectangles, rhombi, parallelograms, quadrilaterals, polygons, prisms, pyramids, cubes, circles, and spheres) and identifies the figures by their properties, e.g., symmetry, number of faces, two- or three-dimensionality, no right angles.
M2e Solves problems by showing the relationship between and among figures, e.g., using congruence and similarity, and using transformations including flips, slides, and rotations.
M2f Extends and creates geometric patterns using concrete and pictorial models.
M7a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.

VOCABULARY:	circle	square	triangle	rectangle
	shape	same	angle	side
	corner	big	small	different

SUGGESTED TIME: 20 -30 minutes

MATERIALS: Attribute blocks – small and large circles, triangles, rectangles, squares, large work mat (optional), music (optional)

DOING THE INVESTIGATION

1. Review the names of the shapes by holding up each one.
2. Place the shapes on the work mat so that all children can see them.
3. Review attributes by posing the following queries:
   - Find a small circle (Select a student to pick up a small circle.)
   - Find a large rectangle (Select a student to pick up a large rectangle.)
   - Find a small square (Select a student to pick up a small square.)
   - Find a large triangle (Select a student to pick up a large rectangle.)
   - Find a shape with four sides (Follow established procedure.)
   - Find a different shape with four sides.
   - Find a shape with three sides.
   - Find a shape with no sides.

   (Also ask students to identify shapes with a particular number of corners. Do this until you are confident that all students can identify the attributes of the shapes.)
4. Instruct students that now we are going to make a train.
5. Select one child to start the train. Instruct the child to select a shape from the work mat.
6. Establish one constant "rule" for this train.

   EXAMPLE OF RULES:
   The next person to join the train has to pick a shape that:
   - Has the same number of sides from the previous shape.
   - Has a different number of sides from the previous shape.
   - Has the same number of corners from the previous shape.
   - Is the same size as the previous shape.

7. Select a student to become part of the train Instruct the student to select a shape that will fit the rule. Ask the student to explain why he/she selected the shape. The child then joins the train with the shape.
8. After a select number of students have had a turn to "board the train" put on some music and have them "chug-chug" around the room.

POSSIBLE SOLUTIONS

A student will select an appropriate shape and use mathematical vocabulary words to explain why the shape supports the rule.

EXTENDING THE ACTIVITY

1. Conduct the same activity. However, use two criteria to establish the train rule. For example, in order to board the train, you need to select a shape that is a different size and a different shape than the previous one.
2. Allow a student to establish the rule and determine if others may board the train.

ASSESSMENT 1

Observe student's shape selection and explanation.

ASSESSMENT 2

Students can develop a two-dimensional train on paper using prepared shapes based on the teacher's rule.

For additional information and activities, see pages 13 and 21
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Select appropriate standard and nonstandard measurement tools in measurement activities.
5B. Understand the attributes of length, capacity, weight, time, money, and temperature.
5C. Estimate measures such as length and volume, using both standard and nonstandard units.
5D. Collect and display data.
5E. Use statistical methods such as graphs and charts to interpret data

INVESTIGATION

Students will contribute and organize data for a class birthday graph.

WHAT'S THE MATHEMATICS?

- collecting data
- organizing data
- representing data

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Collects and organizes data to answer a question or test a hypothesis by comparing sets of data.
M4b Displays data in line plots, graphs, tables and charts.
M7d Considers purpose and audience when communicating about mathematics.

VOCABULARY: data collect organize group sort

SUGGESTED TIME: 30 minutes

MATERIALS: 3 X 5 post-it notes, crayons/pencils/markers, poster board, oak tag

DOING THE INVESTIGATION

1. Have a discussion with the students about birthdays. Ask, "Who has a birthday? How do you celebrate a birthday?" Students will inevitably react in excitement by stating their ages and their birth dates.
2. Ask the students if they would like to know everyone's birthday in the class. (They will say yes.) Explain that we can make a display to be hung up in the class.
3. Show students the post-it notes and the writing implements. Explain that these papers could be used for the display.
4. Ask the students how the post-its could be used. (Each child in the class can get one.) Ask the students to determine what data needs t be put onto each post-it. (Name, birth date.)
5. After it is determined what each student will write on his/her paper, direct students to complete their data.
6. After all students have their papers completed, ask them how the papers could be organized on the poster board so that we can easily read the data.
7. Take suggestions. Allow students to manipulate the post-its to try some of the possibilities. Leading questions include:
   - If we organize the data this way, can we see how many children have a birthday in January?
   - If we organize the data this way is it easy to quickly see which month has the most birthdays?
8. When the graph is organized to your liking (horizontal or vertical bar graph, sorted appropriately), ask the students to determine a name for the graph. Make a sign and add it to the graph.

POSSIBLE SOLUTIONS

Students tend to be very enthusiastic about birthday data. Some responses you may get when eliciting how to organize the data are:

- Put all the same birthdays together.
- Put all the same months together.

While these suggestions are on the right track, the data isn't lined up. It is only grouped/sorted.

- Put all the same birthday months next to each other.
- Make a line with the papers.

Appropriate Titles:

"Our Birthdays" "Happy Birthday to Us!"
"Birthdays" "See our Birthdays"

EXTENDING THE ACTIVITY

Ask small groups of children if they would like to find something else about the students in the class. If a group is interested, have them decide what data they would like to collect and how they will go about collecting and organizing the data. Some topics they come up with may be: Languages spoken at home, Number of sisters/brothers, Who has a pet?

ASSESSMENT 1

Observe the student's collection and organization of data.

ASSESSMENT 2

Ask the student to organize data in a comprehensive way that you provide for him/her.

For additional information and activities, see pages 14, 15 and 22
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Recognize situations in which only an estimate is required.
6B. Develop a variety of estimation skills and strategies.
6C. Predict experimental probabilities.

INVESTIGATION

Students will estimate the distance of students' broad jumps.

WHAT'S THE MATHEMATICS?

- estimation of length
- non-standard measurement

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1c Estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers, as appropriate, in calculations.
M2g Uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area
M2I Selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as weight, length, area, volume, and time.
M2j Carries out simple conversions, such as between cm, and m, and between hours and minutes.
M5a Problem Solving and Reasoning – Formulation – Given the basic statement of a problem situation, the student:
- Makes the important decisions about the approach, materials, and strategies to use, i.e., does not merely fill in a chart, use a pre-specified manipulative, or go through a predetermined set of steps;

- Uses previously learned strategies, skills, knowledge, and concepts to make decisions, uses strategies, such as using manipulatives or drawing sketches, to model problems.

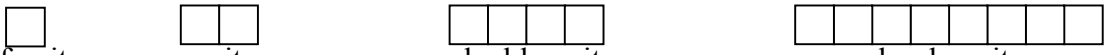
M5b Implementation – The student makes the basic choices involved in planing and carrying out a solution; that is, the student:

- Makes up and uses a variety of strategies and approaches to solving problems and uses or learns approaches that other people use, as appropriate;
- Makes connections among concepts in order to solve problems;
- Solves problems in ways that make sense and explains why these ways make sense, e.g., defends the reasoning, explains the solution.

VOCABULARY: estimate length measure distance predict

SUGGESTED TIME: 15 - 30 minutes

MATERIALS: blocks (unit, double unit, half unit, quadruple unit), masking tape or artist tape (easily peels off) or small post-it notes.



DOING THE INVESTIGATION

This activity can take place in the block center with a small group of students. However, you may choose to introduce the investigation to the whole group at first.

1. Explain to the students that they are going to jump from a starting point (marked with the masking tape), keeping both feet together.
2. Ask the students how we can determine how far each person can jump by asking" "Can anything in the block center help us to measure how far different children jump?" Children will arrive at using the blocks to measure. Ask which size blocks they would like to use to measure.
3. Discuss the results that may occur if smaller blocks are used to measure compared to longer blocks, eliciting thoughts and responses from the students.
4. Suggest that students test their ideas by having one child jump, mark off the spot he/she landed, and measure using two different sizes.
5. Ask the children to agree upon a unit of measurement.

6. Each student can then take one or two turns jumping. The group will measure each jump after it is executed. Discuss with the group if they think it would be beneficial to record the data. Encourage students to collect the data to refer back to when they begin making estimates.
7. After students have measured some test jumps, ask them to predict and estimate each subsequent jump without measuring. For each jump and estimate, they should check their estimate(s) by measuring.

POSSIBLE SOLUTIONS

1. When determining which size block to use, students may decide to use different size blocks for one measurement. If this should arise, discuss why this would be an inaccurate measurement by comparing two students of extremely different heights who both measure the same amount of blocks (one using more longer blocks and one using more shorter blocks).
2. In selecting what size block to use, students may say half unit, unit, double unit or quadruple unit.
3. When measuring, distances may not be exact. Ask the students how they want to deal with that situation.
4. Based on prior knowledge of measuring distances, student estimations should be close. If they are not, have them refer back to the data they collected.

EXTENDING THE ACTIVITY

Students can convert the units of measurement. For example, if a student uses unit blocks to measure, have him/her determine what the distance would be using half unit blocks. (If a child jumps eight unit blocks, it would be the same as 16 half unit blocks.)

ASSESSMENT 1

Observe the student's estimations while doing the activity. Note problem solving strategies.

ASSESSMENT 2

Pose another problem where the student will be required to estimate distance using the same units of measurement. Observe accuracy of estimate.

For additional information and activities, see pages 16 and 23
in the New York State Core Curriculum/Mathematics Resource Guide.

KINDERGARTEN

KEY IDEA #7PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Use a variety of manipulative materials to explore patterns.
7D. Interpret graphs.
7E. Explore and develop relationships among two- and three- dimensional geometric shapes.
7F. Discover patterns in nature, art, music, and literature.

INVESTIGATION

Students will interpret data from a bar graph.

WHAT'S THE MATHEMATICS?

- interpreting graphs
- comparing and analyzing data

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4c Makes statements and draws simple conclusions based on data, that is:
- Reads data in line plots, graphs, tables, and charts
- Compares data in order to make true statements

VOCABULARY:	data	interpret	more	less
	count	same	most	least

SUGGESTED TIME: 5 - 10 minutes ongoing each month

MATERIALS: bar graph reflecting students' birthday data

DOING THE INVESTIGATION

While it is important to have graphs around the classroom for students to read, one of the most authentically purposeful graphs is a birthday graph. Children tend to be self initiated at reading the data each month. The purpose of a birthday graph in a kindergarten classroom can be to determine how many cards/books will need to be generated over the course of the month for each of the children celebrating a birthday.

1. Display a bar graph reflecting birthday data about the students in the class.
2. Tell the students that each month they will make cards or a class birthday book for each child celebrating a birthday that month.
3. Ask the students:
   - How many books will need to be made in October?
   - Which month(s) will we be celebrating three?
   - Which month will we be celebrating the most birthdays?
   - Which month will we be making the least books for the least birthdays?
4. Every month revisit the graph to interpret the number of books that will need to be made.

POSSIBLE SOLUTIONS

Student Responses to the queries asking for a specific number:

- 3 birthdays. I know because I counted from the bottom up, 1, 2, and 3.
- 3 birthdays. I know because I read the number three on the side and the bar goes up to the three.

Student responses to queries specifying particular months:

- October has the most birthdays. The bar is the tallest.
- January and May have the same number of birthdays. The bars are the same.

EXTENDING THE ACTIVITY

Display other graphs around the room that are of interest to the students; display graphs with helpful information for students. For example, a group of students may want to collect data about who can tie shoes. After this data is organized and displayed, a student can retrieve information about who can tie his/her shoe if he/she needs assistance.

ASSESSMENT 1

Observe the students' responses including the answer and an explanation.

ASSESSMENT 2

Ask student for another way they might know the answer by reading the graph.

For additional information and activities, see pages 17 and 24
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 1

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their reasoning.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Justify their answers and solution processes.
1D. Use logical reasoning to reach simple conclusions.

INVESTIGATION

SO WHAT'S THE DIFFERENCE?

Students will group and explore attribute blocks. They will determine ways that the blocks are alike and ways that they are different. After grouping the blocks in various ways, the students will focus on blocks that are different in only one way.

WHAT'S THE MATHEMATICS?

- recognizing sameness and difference
- recognizing shapes, colors, and sizes
- expressing more than one solution to a problem

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Uses many types of figures and identifies relationships between and among the figures by their properties.
M2e Solves problems by showing relationships between and among figures.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY:	same	alike	different	small
	large	color	sort	group
	little	big		

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: attribute blocks, paper, pencils, crayons

DOING THE INVESTIGATION

1. Children should work in pairs or groups.
2. Each group or pair should receive a set of attribute blocks.
3. Allow the children time to explore the blocks and then ask them to put the blocks into groups. Accept all the reasonable different types of groupings that the students may create. Ask them to describe their groupings.
4. Discuss the various sizes, colors and shapes of the attribute blocks.
5. How many different shapes are there? What are the shapes?
6. How many different sizes are there? What are the sizes?
7. How many different colors are there? What are the colors?
8. Is your grouping like your neighbor's or is it different?
9. NOW focus on one particular block. (e.g., the large red circle).
10. Ask the students to find a block that is different in only one way.
11. How many blocks did they find?
12. Have the students record their findings with words, pictures, or diagrams.
13. Students may need to be encouraged to search through the blocks and to accept more than one solution.
14. Continue this activity by then focusing on a different attribute block.
15. Let the children now select an attribute block and let their partner respond.

POSSIBLE SOLUTIONS

1st Activity: Students should be able to conclude that for the large, red circle the following blocks differ in only one way regarding color, size, or shape.

Color:	large green circle	large blue circle	large yellow circle
Size:	small red circle		
Shape:	large red square, large red hexagon	large red triangle	large red rectangle

2nd Activity: The student responses will differ according to the attribute block that you focus on.
The three main differences will again focus on color, size and shape.

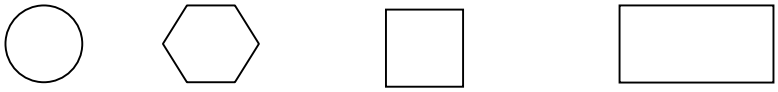
EXTENDING THE ACTIVITY

1. Allow the students to further explore with the attribute blocks.
2. Now have the students focus on two different attributes.
3. Select an attribute block such as the little green triangle.
4. Have the students find all of the blocks that have two differences.
   Possible responses:
   - Color: Students would select only the green shapes and the other attributes of size and shape and would be the two differences.
   - Shape: Students would only select triangles and the other attributes of color and size would be the two differences.
   - Size: Students would only select the little shapes and the other attributes of shape and color would be the two differences.

ASSESSMENT 1

Look at this shape. 

Mark the one that is different in only one way.



a) b) c) d)

Ans. c) Only the small square is different in one way.

ASSESSMENT 2

Use other manipulatives and have them group them so that only one difference is evident in each group. Teddy Bear Counters, beans, coins, or other manipulatives can be used.

Ans. Answers will vary according to the manipulative that you may choose.

For additional information and activities, see pages 25 and 33
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 1

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions including decimal fractions.
2C. Relate counting to grouping and place value.
2D. Recognize the order of whole numbers and commonly used fractions.
2E. Demonstrate the concept of ratio through problems related to actual situations.

INVESTIGATION

TWO FOR TWO

Children will generate 2 different numbers from two number cards.
They will make models of the two numbers and recognize how the two numbers look different when the one digit is in the ones place value or in the tens place value.

WHAT'S THE MATHEMATICS?

- recognize tens and ones place value
- make models of the numbers
- compare greater and less than numerals
- order numerals
- use manipulatives to model numeration

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1b Demonstrate understanding of the base ten place value and uses this knowledge to solve arithmetic tasks.
M5a Formulation
M5b Implementation
M5c Conclusion

M7a Uses appropriate mathematical terms, vocabulary, and language, based on prior conceptual work.
M7b Shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

VOCABULARY:	place value	tens	ones	groups
	digit	numeral	order	

SUGGESTED TIME: 2 - 3 lessons

MATERIALS: connecting cubes, numeral cards 0 - 5 (2 sets), macaroni, construction paper, glue, markers

DOING THE INVESTIGATION

1. Children shuffle the two sets of cards and put them face down in a pile.
2. One child turns over two cards. If the numbers on the cards are the same, draw again.
3. The two children work together and show how they can build two different numbers using the same two cards.
4. The children represent the numbers using connecting cubes. The children build towers of ten to represent the tens place value numeral.
5. Taking turns, the children pick two cards and repeat the process.
6. The children should compare the model of the one numeral vs. the model of the digits reversed (e.g., 21 and 12).
7. The children should recognize that when the larger numeral is in the tens place it will result in a higher number.
8. After the children have made several representations they will then make representations using macaroni.
9. The student will turn over two cards. They will label ½ of the construction paper as the tens place value and the other side as the ones place value. On the tens side the student should draw the appropriate number of circles depending on the card drawn. (e.g., If the numeral is 3 for the tens place, then three circles should be drawn on the tens side.) Inside each circle the student should glue 10 macaroni pieces. On the ones side, the student should use single macaroni pieces to represent the numeral drawn.
10. The student writes the number at the bottom of the paper.
11. Finally, the students will arrange and hang their papers in consecutive order.

POSSIBLE SOLUTIONS

1st Activity: The solutions will vary according to the cards that are drawn. The numbers will range from 1-50.

2nd Activity: As the macaroni representations are consecutively hung, it will help to develop number sense.

EXTENDING THE ACTIVITY

1. Allow the students to continue making representations of numbers from the number cards.
2. As the children continue to work allow the children to use two cards with the same number on each. The picture looks the same.
3. This happens when the same numeral is drawn for both the tens place and the ones place. (e.g., 1,1 2,2 3,3 4,4 etc.)
4. Introduce the word "palindrome".

ASSESSMENT 1

If the numerals 4 and 5 are drawn, which is the largest number that can be made?

a) 24 b) 54 c) 45 d) 42

Ans. b) 54

ASSESSMENT 2

Using the following numerals:

3, 5, 2, 1

Make the largest two digit number that you can.

Ans. 53

For additional information and activities, see pages 26 and 34, and 35
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 1

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add and subtract whole numbers.
3B. Develop strategies for selecting the appropriate computational and operational method in problem solving.
3C. Know single-digit addition and subtraction facts and develop readiness for multiplication and division facts.
3D. Understand the commutative and associative properties.

INVESTIGATION

TOWERS OF TEN

The students will build towers of ten cubes to represent possible combinations of sums to ten. They will only use two different colors of cubes to discover the various possibilities that exist. The students will explore equality.

WHAT'S THE MATHEMATICS?

- explore equations with sums of ten
- addition skills
- commutative and associative property of addition
- using manipulatives to increase addition skills
- making models of addition equations

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Adds, subtracts, multiplies, and divides whole numbers, with and without calculators.
M5a Formulation
M5b Implementation
M5c Conclusion
M6a Adds, subtracts, multiplies, and divides whole numbers correctly.

M7b Shows mathematical ideas in a variety of ways, including words, numbers, symbols, pictures, charts, graphs, tables, diagrams, and models.

VOCABULARY:	addition	sum	equal	count
	total	plus	in all	group
	all together	the same as		

SUGGESTED TIME: 2 - 3 lessons

MATERIALS: snap cubes, paper, pencil

DOING THE INVESTIGATION

1. Children should work in pairs.
2. Each pair of students should be given two different colors of cubes.
3. The students should create a tower of ten cubes.
4. Using the two colors of cubes, the students should build towers that are the same height, but look different.
5. All the cubes of the same color should stay together so there are only two addends.
   (e.g., 4 + 6 = 10 4 red cubes together and 6 blue cubes together, equal a tower of ten)
6. Have the students compare and tell how the towers are different. Have the students relate the mathematical equation.
7. As the students make various towers of ten, let them use the towers to compare to another tower instead of the single color tower of ten. (e.g., 6 + 4 = 7 + 3)
8. How many towers are possible?
   - Allow a 6 green and 4 red tower to equal a 4 red and 6 green tower to show the commutative property.
   - Explore all the possibilities.
9. Let the pairs of the students work together to record the equations.
10. Let the students discuss how the change in one color affects the other color in the equation.
11. If one more of one color is used, then it will decrease the amount of the other color by one.
12. These relationships become enriching as the students make discoveries.

POSSIBLE SOLUTIONS

If the students use red and blue cubes the following solutions are possible:

9 red and 1 blue
8 red and 2 blue
7 red and 3 blue
6 red and 4 blue
5 red and 5 blue
4 red and 6 blue
3 red and 7 blue
2 red and 8 blue
1 red and 9 blue

EXTENDING THE ACTIVITY

1. Instead of using two colors of cubes, now use three different colors of cubes to make sums of tens.
2. This may seem a bit advanced for their level but as the students explore addition in this context they become aware of even more equations.
3. The possible solutions increase with three addends.
4. It becomes even richer when the students compare 2 color towers with three color towers. (e.g., 1 red + 4 blue + 5 yellow = 6 red + 4 yellow)

ASSESSMENT 1

Which number sentence equals 10?

a) 7 + 2 b) 6 + 2 c) 3 + 8 d) 7 + 3

Ans. d) Only number sentence 7 + 3 equals 10.

ASSESSMENT 2

If I have four red cubes, how many green cubes would I need to make a tower of 10?

a) 5 b) 4 c) 6 d) 7

Ans. c) 6

ASSESSMENT 3

Create three different towers that equal a tower of eight. Use green and yellow cubes.

Ans. 1 green + 7 yellow
2 green + 6 yellow
3 green + 5 yellow
4 green + 4 yellow
5 green + 3 yellow
6 green + 2 yellow
7 green +1 yellow

For additional information and activities, see pages 27 and 36
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 1

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representations to provide means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Use concrete materials to model spatial relationships.
4B. Construct charts and graphs to display and analyze real-world data.
4C. Use multiple representations (manipulative materials, pictures, diagrams) as tools to explain the operation of everyday procedures.
4D. Use variables such as height, weight, and hand size to predict changes over time.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

ROCKET RIOT

The students will construct rocket ships using pattern blocks. After they have made the rocket ships, they will count and tally the various pattern blocks that they used. They will record the pattern blocks that they used on a graph.

☆The object that you have the student create with pattern blocks can be determined by the thematic unit you are using in your classroom.

WHAT'S THE MATHEMATICS?

- making geometric pictures
- exploration of shapes
- recording data
- comparing data – less than, more than

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Uses many types of figures (angles, triangles, squares, rectangles, rhombi, parallelograms, quadrilaterals, polygons, prisms, pyramids, cubes, circles, and spheres) and identifies the figures by their properties.
M2f Extends and creates geometric patterns using concrete and pictorial models.

M4a Collects and organizes data to answer a question or test a hypothesis by comparing sets of data.
M4b Displays data in line plots, graphs, tables, and charts.
M4c Makes statements and draws simple conclusions based on data.
M5a Formulation
M5b Implementation
M5c Conclusion
M6e Refers to geometric shapes and terms correctly with concrete objects or drawings, including triangle, square, rectangle, side, edge, face, cube, point, line, perimeter, area, and circle; and refers with assistance to rhombus, parallelogram, angle, vertex, volume, diameter, circumference, sphere, prism, and pyramid.
M6g Reads creates and represents data on line plots, charts, tables, diagrams, bar graphs, simple circle graphs, and coordinate graphs.

VOCABULARY:	triangle	rhombus	trapezoid
	hexagon	square	parallelogram
	graph	pattern blocks	tally
	record	bar graph	

SUGGESTED TIME: 2 - 3 lessons

MATERIALS: pattern blocks, crayons, paper, pencil

DOING THE INVESTIGATION

1. The students will use pattern blocks to create a rocket ship to go with a theme unit on Space.
2. They should trace and color the pattern blocks to record their rocket ship.
3. Accept all reasonable likeness to rocket ships.
4. The students should count and record the total number of pattern blocks used to make the rocket ship.
5. Next, the students should create a pictograph representing the number of each pattern block used.
6. The students then compare the different numbers of blocks used.
7. The students then analyze which blocks were used the most and the least.

POSSIBLE SOLUTIONS

1st Activity: The students' rocket ships will all be different looking but total number of blocks may be the same. This allows for individual responses but a comparison of number of blocks.

2nd Activity: The number of blocks will affect the outcome of the individual pictographs.

EXTENDING THE ACTIVITY

1. When the students have explored with the pattern blocks, they will see the equalities of the various shapes.
   e.g., 2 triangles = 1 rhombus
   3 triangles = 1 trapezoid
   6 triangles = 1 hexagon
2. The following number value can be assigned to the pattern blocks:
   triangle = 1
   rhombus = 2
   trapezoid = 3
   hexagon = 6
3. Using these values the students can use their rocket ship pictures and figure out what their rocket is worth.

Solutions- Values will vary according to the rocket ship picture.

ASSESSMENT 1

Which shape has six sides?

a) triangle b) square c) trapezoid d) hexagon

Ans. d) hexagon – Only the hexagon has 6 sides.

ASSESSMENT 2

Create a picture using ten pattern blocks.
Record on a graph the number of each block used.

Ans. Graphs and pictures will vary. Pictographs should coincide with the pattern block pictures created.

For additional information and activities, see pages 28 and 37
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 1

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Understand that measurement is approximate, never exact.
5B. Select appropriate standard and non-standard measurement tools in measurement activities.
5C. Understand the attributes of area, length, capacity, volume, weight, time, temperature, and money.
5D. Estimate measures such as length, perimeter, area, and volume, using both standard and non-standard units.
5E. Collect and display data.
5F. Use statistical methods such as graphs, tables, and charts to interpret data.

INVESTIGATION

SHAPE BALANCE

The students will explore weight comparison using pattern blocks. They will discover the weight relevance with the size and combination.

WHAT'S THE MATHEMATICS?

- comparison of size and weight
- exploring the comparison and combination of shapes
- using non-standard objects for units of measurement
- using a balance scale to compare weights
- understanding the relevance of size to weight

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2g Uses basic ways of estimating and measuring the size of figures and objects in the real world, including length, width, perimeter, and area.

M2i Selects and uses units, both formal and informal as appropriate, for estimating and measuring quantities such as weight, length, area, volume, and time.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: weight heavy balance comparison light

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: pattern blocks, balance scale

DOING THE INVESTIGATION

1. The students should work in pairs.
2. Each pair of students should have pattern blocks and a balance scale.
3. The only pattern blocks used should be the hexagon, rhombus, trapezoid, and the triangle.
4. By looking at the various pattern blocks let the students make predictions on the weights.
5. Have the students arrange the blocks from lightest to heaviest.
6. Now have the students test their predictions using balance scales
7. As the students further explore the weights of the pattern blocks prior knowledge of the pattern blocks can be enhanced.
8. As the students discover that two triangles cover one rhombus, let the students experiment by putting two triangles on one side of the balance scale and a rhombus on the other side.
9. Let the students further explore various ways to create hexagons with the pattern blocks.
10. Compare the weights of the created hexagons with the yellow hexagon.
11. Students will discover that the pattern blocks are not only area related, but are weight related also.
12. Encourage the children to record their findings using drawings.

POSSIBLE SOLUTIONS

1st Activity: Using only the triangle, rhombus, trapezoid and hexagon, they range from lightest to heaviest in the following way:
triangle, rhombus, trapezoid, hexagon

2nd Activity: The students will discover some of the various weight relationships:
2 triangles balance 1 rhombus
3 triangles balance 1 trapezoid
6 triangles balance 1 hexagon
1 rhombus and 1 triangle balance 1 trapezoid
1 rhombus, 1 triangle and 1 trapezoid balance 1 hexagon

2 rhombi and 2 triangles balance 1 hexagon

EXTENDING THE ACTIVITY

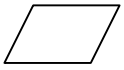
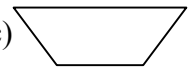
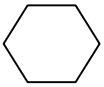
1. As the students continue to explore the weight relationships of the pattern blocks they will come up with further creative equalities.
2. Using the equalities, assign the various shapes the following number values:
   triangle = 1
   rhombus = 2
   trapezoid = 3
   hexagon = 6
3. Lead the students to discover the number relationship.
   1 triangle + 1 triangle balance 1 rhombus
   1 + 1 = 2
4. Have the students look at their balance recordings from the previous activity.
5. Have the students write the number sentences that go with their balance pictures.

   Following are a few possible solutions:

   1 rhombus + 1 triangle balance 1 trapezoid
   2 + 1 = 3
   1 rhombus + 1 rhombus balance 4 triangles
   2 + 2 = 4
   1 triangle + 1 rhombus + 1 trapezoid balance 1 hexagon
   1 + 2 + 3 = 6

ASSESSMENT 1

Solve the following balance scale picture:

a)  b)  c)  d) 

Ans. c) Trapezoid - Only the trapezoid equals the same weight as the 3 triangles.

ASSESSMENT 2

What ways could you balance the scale if two trapezoids are put on one side?

Ans. Answers will vary but may include:
1 hexagon
3 rhombi
6 triangles
1 triangle, 1 trapezoid, 1 rhombus
2 rhombi, 2 triangles
4 triangles, and 1 rhombus

For additional information and activities, see pages 29, 30, and 38
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 1

KEY IDEA # 6 UNCERTAINTY

Students use ideas to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Make estimates to compare to actual results of both formal and informal measurement.
6B. Make estimates to compare to the actual results of computations.
6C. Recognize situations in which only an estimate is required.
6D. Develop a wide variety of estimation skills and strategies.
6E. Determine the reasonableness of results.
6F. Predict experimental probabilities.
6G. Make predictions, using unbiased random samples.
6H. Determine probabilities of simple events.

INVESTIGATION

BLAST OFF !

Students will create rocket ships with snap cubes based on spinners. The first rocket to reach 10 cubes can blast off! Three different color rocket ships will race for blast off. The students will make predictions on the rocket ship to blast off first based on the spinners used.

WHAT'S THE MATHEMATICS?

- make predictions on the likelihood of events
- conduct and predict outcomes
- determine probabilities of simple events
- explore reasonableness
- record the outcome of events

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Collects and organizes data to answer a question or test a hypothesis by comparing sets of data.
M4b Displays data in line plots, graphs, tables, and charts.
M4c Makes statements and draws simple conclusions based on data.
M4e Predicts results, analyzes data, and finds out why some results are more likely, less likely, or equally likely.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY:	prediction	more likely	possible
	outcome	less likely	predictable
	likelihood	equally likely	

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: snap cubes, red, yellow and blue 4" squares of construction paper, spinners

DOING THE INVESTIGATION

1. Display a variety of spinners.
2. Let the students work in pairs
3. Each pair of students should create 3 "launch pads" of the 4" squares of construction paper in 3 different colors- 1 red, 1 yellow and, 1 blue.
4. Show a spinner that is ½ red. ¼ yellow, and ¼ blue.
5. Explain that the colored squares are launch pads for the rocket ship they are going to build.
6. The students will spin the spinner and whichever color the spinner lands on will give one cube for that color rocket ship.
   (e.g., If the spinner lands on blue, then one cube will be added to the blue rocket ship.)
7. Ten cubes are needed to complete the rocket ship for BLAST OFF!
8. The children will look at the spinner and predict which rocket ship will be completed first.
9. Let the students run the experiment. Each pair will stop when one of the rocket ships is complete with 10 cubes.
10. When the rocket ship is complete, the students will record on a classroom chart which color rocket ship is ready for blast off.
11. Based on the results of the data, let the students compare and explain the outcome of the sample spinner event.
12. Show the students another spinner where ½ is blue and ¼ is yellow, ¼ red.
13. Ask the students to predict which rocket ship would "BLAST OFF!" first. Students should explain their responses.
14. Let the students conduct the experiment and compare the results with their predictions.

POSSIBLE SOLUTIONS

1st Activity: The students should conclude that the red rocket ship will probably blast off first. Conclusions should be based on the classroom chart.
The red rocket will have a 1/2 probability.
The yellow rocket will have a 1/4 probability.
The blue rocket will have a 1/4 probability.

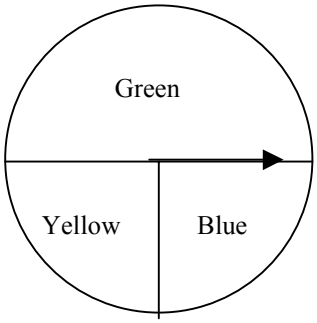
2nd Activity: The students should conclude that the blue rocket ship will now blast off first. Their conclusion should be based on the previous sample and the similarity to the previous spinner.

EXTENDING THE ACTIVITY

1. Prepare a spinner with an equally likely result (1/3 yellow, 1/3 red, 1/3 blue).
2. Ask the children to predict which rocket ship will blast off first. The student responses may vary and a discussion may develop that they are unsure of the results.
3. Let the students conduct the experiment and record their results on a classroom chart. The responses will likely show an equally likely result of rocket ships being constructed.

ASSESSMENT 1

Look at the spinner to the right:



Which color is most likely to occur?

a) Yellow b) Green c) Blue

Ans. b) Green The green covers 1/2 of the spinner where as the yellow and blue each only cover 1/4 of the spinner.

ASSESSMENT 2

Design a spinner using the four colors: red, blue, yellow, and green, that shows an equally likely chance to occur.

Ans. Each color should cover 1/4 of the spinner.

For additional information and activities, see pages 31 and 39, 40
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 1

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Explore and express relationships, using variables and open sentences.
7D. Solve for an unknown, using manipulative materials.
7E. Use a variety of manipulative materials and technologies to explore patterns.
7F. Interpret graphs.
7G. Explore and develop relationships among two- and three-dimensional geometric shapes.
7H. Discover patterns in nature, art, music, and literature.

INVESTIGATION

CRAZY CATERPILLARS

The students will explore creating simple patterns with pattern blocks. The students will create crazy caterpillars on adding machine tape. For a simple pattern use only two different pattern blocks.

WHAT'S THE MATHEMATICS?

- to recognize and continue a simple pattern
- to make a pattern
- to record and explain a pattern

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3a Uses linear patterns to solve problems.
M5a Formulation
M6a Implementation
M5c Conclusion
M3d Uses letters, boxes, or other symbols to stand for any number, measured quantity, or object in simple situations with concrete materials.

VOCABULARY: pattern shape repeat sequence

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: pattern blocks, adding machine tape, crayons

DOING THE INVESTIGATION

1. Give each child an 18 inch strip of adding machine tape.
2. Draw a little caterpillar face at one end of the strip of paper.
3. Using two different pattern blocks, make a simple pattern of the body of the caterpillar.
4. Attach a card where student explains and records the pattern.
5. Have the children compare their caterpillar to the other students' caterpillars.
6. Have the students find other caterpillars that have the same pattern sequence.
7. Group the caterpillars by the same pattern sequence.

POSSIBLE SOLUTIONS

1st Activity: Possible simple pattern solutions include:
ABABAB
AABAAB
ABBABB
ABBBABBB
AABBAABB

2nd Activity: Children will group their caterpillars by patterns that are the same.

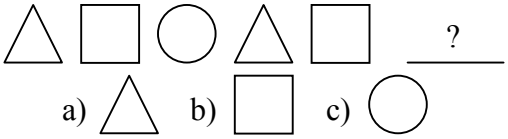
EXTENDING THE ACTIVITY

1. The children will create more Crazy Caterpillars.
2. The students will use three different pattern blocks.
3. As they explore the possible pattern sequence, they should see that there are more possibilities for Crazy Caterpillars.

Solutions will include some of the possible sequences:
ABCABC
AABBCCAABBCC
ABBCABBC
AABCAABC
ABCCABCC
ABABCABABC
ABBBCABBBC

ASSESSMENT 1

Complete the following pattern:



Ans. c)  A circle will complete the simple pattern.

ASSESSMENT 2

Give each child two different colors of snap cubes. They should build three different towers of a simple pattern. Then they should record their towers on squared paper.

Ans. Some of the possible sequence towers include:

ABABAB
AABBAABB
ABBABB
AABAAB

For additional information and activities, see pages 32 and 41, 42
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 2

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence and construct an argument.

PERFORMANCE INDICATORS

1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their reasoning.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Justify their answers and solution processes.
1D. Use logical reasoning to reach simple conclusions.

INVESTIGATION

DRESSING THE BABY

Every morning it was Sarah's job to dress her baby brother. One morning, she saw that most of his clothes were still being washed. She was only able to find 3 different shirts and 3 different pairs of pants.

- How many different ways could Sarah dress the baby using only the clothes she found?
- Draw a picture to show each outfit.
- Organize the information. Make sure to include all the possible combinations of shirts and pants.
- Tell how you figured it out.

WHAT'S THE MATHEMATICS?

- Draw pictures and use manipulatives to represent problems.
- Use concrete objects, pictorial representation, and tables to represent and solve problems.
- Brainstorm possible solutions before starting the problem.
- Introduce concept of arrangements and combinations.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4f Find all possible combinations.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: combinations arrangements strategy
organized list table

SUGGESTED TIME: 1 – 2 lessons

MATERIALS: colored cubes, tiles or squares of colored paper, crayons or markers

DOING THE INVESTIGATION

- Bring 3 different pairs of pants and 3 different shirts (pictures or cutouts are fine) to class and ask a student to choose one from each bag. Ask another student to do the same.
- How many different outfits can you make with these shirts and pants? What will they be?
- Elicit the need to identify the clothes, either by color or pattern.
- Identify the need for some way to organize the information.
- Read the investigation aloud and then give each pair of students a bag of colored cubes or cutouts to find a solution.

POSSIBLE SOLUTIONS

If there are 3 shirts and 3 pairs of pants, 9 different combinations of outfits are possible.

Student work needs to reflect the following:

1. Each of the items is identified with a color or a description.
2. There is a model or a picture of each outfit.
3. The information is organized in a table, a list or shown in pictures.
4. A description of the solution process.

EXTENDING THE ACTIVITY

4. Sarah was able to find kind 4 different shirts and only 3 pairs of pants. How many different outfits could she make? Show or tell what each outfit would be.
5. Sarah was able to find 2 different shirts, 3 different pants and 2 different vests. Show or tell what each outfit would look like.

ASSESSMENT 1

Sam has a pair of red socks, a pair of green socks, and a pair of yellow socks. He has a pair of white sneakers and a pair of brown sneakers. Draw a picture of all the different combinations Sam can make with these socks and sneakers.

Ans. Accept any reasonable drawings which show all the combinations.

ASSESSMENT 2

Make an organized list to show the different combinations Sam can make with the socks and sneakers.

Ans. Accept any reasonable lists.

For additional information and activities, see pages 25 and 33
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 2

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real word, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS

2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions including decimal fractions
2C. Relate counting to grouping and place value.
2D. Recognize the order of whole numbers and commonly used fractions.
2E. Demonstrate the concept of ratio through problems related to actual situations.

INVESTIGATION

WHAT DOES 1,000 LOOK LIKE?

Students will collect and build a model of 1,000 items, counting by tens, then by hundreds.

WHAT'S THE MATHEMATICS?

- Represent 2 and 3 digit numbers up to 999 using concrete models.
- Recognize the meaning of zero in the place value system.
- Understand the meaning of digits in 3 digit numbers.
- Develop a variety of skip counting strategies.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1b Demonstrates understanding of the base ten place value system and uses that knowledge to solve arithmetic tasks.
M1c Estimate, approximate, round off, use landmark numbers or use exact numbers in calculation.
M1f Describe and compare quantities by using whole numbers up to 10,000.

VOCABULARY: ones tens hundreds thousand
tally digits landmarks skip counting

SUGGESTED TIME: This investigation begins with 1 or 2 lessons, but the actual collecting and counting can take several weeks.

MATERIALS Any small, inexpensive and easy to handle items can be used to make the collections. Pasting tiny stickers on index cards is suggested.

DOING THE INVESTIGATION

- Begin by asking the students what they think 1,000 looks like. Display a box of pins, a bag of M&M's, a jar filled with pasta or pennies and perhaps a box of cookies. Ask which holds about 1,000. Record their guesses.
- Ask students if they think they can make a collection of 1,000 for the classroom. About how long would it take? How much space would it take up? How would you count it? Could we bring 1,000 elephants? 1,000 refrigerators? Elicit the need for small items.
- Tiny stickers on index cards are the easy to manage and fun to use. This can be a whole class activity or each group can make their own collections of 1,000.
- To begin the collections, students need to discover a way to count accurately. For most second graders, this is a challenge. Encourage them to skip count by 2's or by 5's. Counting what has been collected needs to be ongoing and relatively accurate. Ask students to check each others work.
- Find landmark numbers in the collections. Do you have 10? Do you have 100? 100 tiny stickers fit nicely on an index card and they are easily displayed.
- There should be daily record keeping and discussion of the collection. How many do we have? About how many more do we need?

POSSIBLE SOLUTIONS

5. To make counting easier, landmark numbers and counting strategies need to be established. First, establish 10 as a landmark number. As the collection grows to the first hundred, elicit from the students that it is easier to count by hundreds.
6. Students should be able to tell how many hundreds are in the collection.
7. Writing about the collection is an important part of the activity. Begin with a whole class writing experience and when the students get the idea, move to group or individual writing.

EXTENDING THE ACTIVITY

6. Bring a Guessing Jar to school every week. Begin with a small jar filled with large items and progress to a large jar filled with small items. Tape the lid shut. Students put one guess in the guessing box. Every week the box is emptied and the student with the most accurate guess keeps the contents of the jar.
7. Provide opportunities for the students to add the parts of their collections together. Sam has 83 stickers and Bob has 54 stickers. How many do they have together? Mary has 202. Do they have as many as she does? How can you tell?
8. How does the size of the items in the collection change how the collection will look? Suppose we collected 1000 shoes. About how much space would the collection take up in the room?

ASSESSMENT 1

Every student needs to make part of the collection and count it accurately. Students should be able to write as well as tell about the collection explaining how the collection began, and how it was completed. Student work needs to show an understanding of skip counting and landmark numbers.

ASSESSMENT 2

What number comes next?

15, 20, 25, 30, __

Ans. 35

For additional information and activities, see pages 26 and 34-35
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 2

KEY IDEA #3OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS

3A. Add and subtract whole numbers.
3B. Developing strategies for selecting the appropriate computational and operational method in problem solving.
3C. Know single-digit addition and subtraction facts and develop readiness for multiplication and division facts.
3D. Understand the commutative and associative properties.

INVESTIGATION

WHAT'S FAIR?

By deciding what is fair, students will share to make equal groups and begin to explore the remainder.

WHAT'S THE MATHEMATICS?

- Develop readiness for multiplication and division facts.
- Write a variety of division algorithms.
- Demonstrate an understanding of equality.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Adds, subtracts multiples and divides whole numbers.
M1c Estimates, approximates, rounds off, uses landmark numbers, or uses exact numbers in calculations.
M6a Adds, subtracts, multiplies and divides whole numbers correctly.

VOCABULARY: fair equal divide
multiply remainder

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: counters, cups, “The Doorbell Rang,” by Pat Hutchins

DOING THE INVESTIGATION

1. Begin by reading “The Doorbell Rang." Did the students in the story share the cookies fairly? How do you know?
2. Discuss and demonstrate what is meant by “fair." Elicit that fair can sometimes mean equal.
3. Give each group of students a cup of less than 20 counters (make sure the amount in each cup is different).
4. Ask the students to share the counters in the cup with their group. Wait until they have completed the task and then ask each group to show and tell what they did.
5. Set up a chart to record the information. How can we tell if it is fair?
   - How many counters were in the cup at the beginning?
   - How many students were in your group?
   - How many did each student get?
   - Are there any counters leftover?
   - What should we do with the leftovers?

POSSIBLE SOLUTIONS

1. Students should be able to show equal groups at every table. There should be some discussion of the “fairest” way to deal with the remainder.
2. Students should be able to fill in the chart on the board or on a worksheet at every table.

EXTENDING THE ACTIVITY

1. There are 5 children in Sam's family. Whenever Sam gets something just for himself, his mother makes him share it equally with his brothers and sisters.

a. Sam got a bag of 13 cookies. Show or explain how he can share them fairly with his brothers and sisters.

b. Are there any cookies left over? What should Sam do with them?

2. Show or explain with a picture or a story:
   - What happens to your share when there are more cookies to divide?
   - What happens to your share when there are fewer cookies to divide?
   - What happens to your share when there are more students at the table?
   - What happens to your share when there are fewer students at the table?

ASSESSMENT 1

Six friends want to share a box of cookies. There are 19 cookies in the box. How many will each friend get if they share the cookies fairly? Will there be any cookies leftover? What do you think they should do with the leftover cookies?

ASSESSMENT 2

Wendy had 17 cookies to share equally with her 3 friends. How many cookies will Wendy get? How many cookies will each friend get? Will there be any leftovers? What do you think she should do with the leftovers? Why?

For additional information and activities, see page 27 and 36
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 2

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS

4A. Use concrete materials to model spatial relationships.
4B. Construct charts and graphs to display and analyze real-world data.
4C. Use multiple representations (manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.
4D. Use variables such as height, weight, and hand size to predict changes over time.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

CAN YOU GUESS MY FAVORITE COLOR?

I'm using these pattern block tiles to cover my kitchen floor, but I don't like all the colors. Can you help me make new tiles that are the same shape and size, but are different colors?

WHAT'S THE MATHEMATICS?

- Compare attributes of pattern block shapes.
- Solve problems by showing relationships between and among pattern blocks.
- Establish congruency as a basic principle of geometry.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Use many types of figures.
M2e Solve problems by showing relationships between and among figures.

VOCABULARY:	triangle	square	hexagon	trapezoid
	rhombus	area	equal	congruent
	symmetry			

SUGGESTED TIME: 1 – 2 lessons

MATERIALS: a set of pattern blocks, crayons and markers for every pair of students.

DOING THE INVESTIGATION

- Give each pair of students a bag of pattern blocks. Ask them to sort the blocks by shape.
- Establish why the blocks are the same. Discuss attributes; size, color, shape.
- Are the blocks the same for any other reason? Establish that the blocks are congruent by placing one over the other.
- Congruent blocks match each other exactly in size and shape. You can put one on top of the other to check if they are congruent.
- Display a blue rhombus, a red trapezoid, and a yellow hexagon.
- Give out paper, crayons or markers. Identify the shapes, pointing out that the size of the tile must remain the same. Read the problem aloud.
- Can you make a new rhombus?
- Can you make a new trapezoid?
- Can you make a new hexagon?
- Draw (or trace) and color the new figures.
- Show or tell how you solved the problem.

POSSIBLE SOLUTIONS

9. After exploring with the blocks and finding that blocks of the same color and shape are congruent, students will use the blocks in combinations to make other congruent shapes.
10. Collect and display student work.
    - 2 triangles make a new green rhombus
    - 3 triangles make a new green trapezoid
    - 6 triangles make a new green hexagon
    - 2 red trapezoids make a new red hexagon
11. Students might discover multicolor combinations that will fit the conditions of the problem.

EXTENDING THE ACTIVITY

1. Students complete pattern block equations, first with models on the overhead then with tracings or pictures and finally with numbers.
2. What is the value of each block? Establish that if the value of the green triangle is 10¢, then find the value of the other blocks. Can you find the value of all the blocks? Why or why not?
    - Make a design that is worth exactly $1.50 using the fewest blocks.
    - Make a design that is worth exactly $1.50 using the most blocks.

ASSESSMENT 1

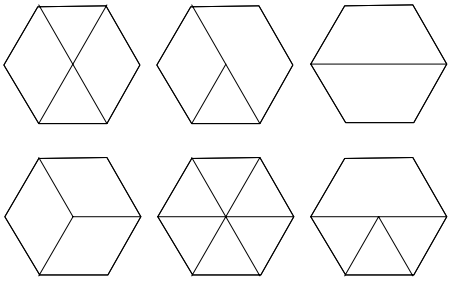
How many triangles will I need to make a new rhombus tile?
How many triangles will I need to make a new trapezoid tile?
How many triangles will I need to make a new hexagon tile?
Are there any other ways I can make new tiles?

Ans. You will need 2 triangles to make a rhombus, 3 triangles to make a trapezoid, 6 triangles to make a hexagon. Some other ways to make new tiles are a triangle and a rhombus to make a trapezoid, 3 rhombi to make a hexagon and 2 trapezoids to make a hexagon.

ASSESSMENT 2

Show three different ways to make a hexagon.

Ans. Six samples are shown below; other solutions are possible.



For additional information and activities, see pages 28 and 37
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 2

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS

5A. Understand that measurement is approximate, never exact.
5B. Select appropriate standard and non-standard measurement tools in measurement activities.
5C. Understand the attributes of area, length, capacity, volume, weight, time, temperature, and money.
5D. Estimate measures such as length, perimeter, area, and volume, using both standard and non-standard measures.
5E. Collect and display data.
5F. Use statistical methods such as graphs, tables and charts to interpret data.

INVESTIGATION

THE CASE OF THE TWISTED PRETZEL

The best data collection investigations come from students' own experiences. Students should formulate their own questions to investigate so that there is a need to collect and organize data. The following is provided as a model.

The students in class 2-234 love to eat pretzels. Every day, their teacher gives each child a little bag of pretzels to eat at snack time. The students are complaining that every day they seem to be getting fewer pretzels in each bag.

- Is this true?
- How can it be proved?
- What can the students do about it?

WHAT'S THE MATHEMATICS?

- Collecting and displaying data in tables and charts.
- Making bar graphs with stacks of cubes and graph paper.
- Comparing data in terms of number, equality, similarities and differences.
- Understanding that simple graphs can help one to understand observations.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Collects and organizes data to answer a question or test a hypothesis.
M4c Makes statements and draws simple conclusions based on data.

VOCABULARY:	data collection	bar graph	conclusion
	more than	less than	one more than
	one less than	same size	equal to

SUGGESTED TIME: 3 - 4 lessons

MATERIALS: Individual bags of pretzels, linking cubes, tiny post-its or 1-inch graph paper cut in strips, centimeter graph paper for recording.

DOING THE INVESTIGATION

8. Day 1 – Give each pair of students an individual sized bag of pretzels and some counting cubes. Students make a stack of cubes to represent the number of pretzels in the bag. If linking cubes are not available, use post-its or strips of 1-inch graph paper. There should be some discussion of how to represent pretzel fragments.
9. Establish the need to record the data. How can we record this data? Identify the variables, and ask each pair to place their stack or strip on the chart on the board.
10. Transfer these numbers to a chart so that the cubes can be reused. Have we collected enough information to answer the question?
11. Day 2 and Day 3 – Collect and display more data. Have we collected enough information to answer the question?

POSSIBLE SOLUTIONS

Comparing the data will provide the students with a visual representation of the answer to the question. Which bag had the most? Which bag had the least? Which bags were equal? Does this data prove that we are getting fewer pretzels in each bag?

EXTENDING THE ACTIVITY

1. The teacher is complaining that individual packages of pretzels are very expensive. Is there a cheaper way to buy pretzels?
2. Students investigate the cost of buying pretzels in a big bag.
3. Which brand is the least expensive?
4. How will the big bag of pretzels be divided up?
5. Who will divide them?
6. What are the real costs involved in this plan? (napkins, baggies)
7. After all factors are considered, what is the best plan for the class?
8. A taste test might be needed before a decision is made.

ASSESSMENT 1

Write a description of the data collection investigation. Make sure to include the question, how the data was collected, and how the conclusion was reached.

ASSESSMENT 2

Write a letter to the company that manufactures the pretzels. Include copies of the data you collected and how you reached your conclusion.

For additional information and activities, see pages 29-30 and 38
in the Core New York State Curriculum/Mathematics Resource Guide.

GRADE 2

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS

6A. Make estimates to compare to actual results of both formal and informal measurement.
6B. Make estimates to compare to the actual results of computations.
6C. Recognize situations in which only an estimate is required.
6D. Develop a wide variety of estimation skills and strategies.
6E. Determine the reasonableness of results.
6F. Predict experimental probabilities.
6G. Make predictions, using unbiased random samples.
6H. Determine probabilities of simple events.

INVESTIGATION

WHAT ARE MY CHANCES?

Students will determine probabilities of simple events by performing experiments with three or more likely outcomes. Without looking, they will pick a color tile from a bag, replacing it after every pick. After tallying the results, they will investigate the outcomes.

WHAT'S THE MATHEMATICS?

- classify outcomes
- perform experiments with three or more likely outcomes
- tally results
- begin to explore experimental probability

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4e Predict results and find out why some results are more likely, less likely or equally likely.

VOCABULARY	certain	possible	impossible
	predict	tally	likely
	unlikely	chance	less likely

SUGGESTED TIME: 1 - 2 lessons.

MATERIALS: Paper bags with an equal number of red, yellow, green and blue square tiles in each bag, paper folded in 4 parts and labeled for tallies.

DOING THE INVESTIGATION

1. Begin by filling a bag with green tiles. Ask several students to pick a yellow tile without looking from the bag. Establish the idea that it is impossible to pick a yellow from a bag that contains all green tiles.
2. Using the same bag of green tiles, ask another child to pick a green. Establish the idea that picking a green tile from a bag of green tiles is certain.
3. What do you think will happen when you don't know what is in the bag?
   Which color will you pick the most?
   Which color will you pick the least?
   Which color will you never pick?
4. Record student predictions on the board.
5. Give each pair of students a bag containing an equal number of red, blue, green and yellow tiles and a recording sheet. Ask them to take turns picking and recording on the tally sheet. Remind them that the tiles they pick must be put back in the bag before they pick again.
6. After about 10 minutes, the students add up their totals. Enter the totals for each pair of students on a chart or on the blackboard.
7. Go back and compare the results with student predictions. Did the results of the experiment match the predictions? Why or why not?

POSSIBLE SOLUTIONS

1. The results of a small number of trials is not necessarily going to match the one out of four that we would expect to happen, but if the tally marks were added correctly, the results should be reasonably close.
2. If the outcomes are equally likely, the chances of picking each color are equally likely.

EXTENDING THE ACTIVITY

12. Students design an experiment where the outcome is impossible.
13. Students design an experiment where the outcome is certain.
14. Put twice as many green tiles in the bag as yellow. Ask the students to predict and classify the outcome. Repeat the experiment described above and compare the results.
15. Can you guess what is in the bag? Place 10 yellow tiles and 1 red tile in a bag. Ask students to pick a tile. Can you guess what's in the bag? How many tiles need to be picked before you can guess?

ASSESSMENT 1

Students should be able to classify everyday events.

certain -------- likely --------- less likely -------- impossible

- The sun will rise tomorrow.
- It will snow on Sunday.
- The school telephone will ring today.
- If we buy a lottery ticket, we will win.
- It will be dark outside tonight.

ASSESSMENT 2

Show or tell about the experiment we did in class. Make sure to include the results of the experiment.

For additional information and activities, see pages 31 and 39-40
in the New York State Core Curriculum/Mathematics Resource Guide.

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

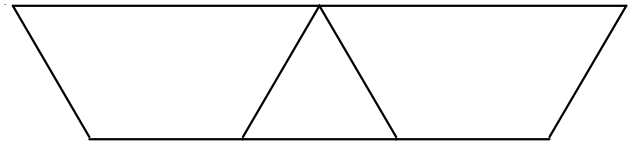
PERFORMANCE INDICATORS

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Explore and express relationships, using variables and open sentences.
7D. Solve for an unknown, using manipulative materials.
7E. Use a variety of manipulative materials and technologies to explore patterns.
7F. Interpret graphs.
7G. Explore and develop relationships among two- and three- dimensional geometric shapes.
7H. Discover patterns in nature, art, music, and literature.

INVESTIGATION

TWENTY GREEN GATES *

Sam wants to build a fence around his toy city. He wants the fence to have 20 green gates for the cars and people to go in and out.
If Sam continues building by adding 1 triangle and 1 trapezoid at a time, how many blocks will he need to make a fence with 20 gates?



After seeing only 1 or 2 green triangles in the pattern, students will need to replicate the pattern. Because the number of triangles and trapezoids is limited, students will have to think of alternative ways to extend the pattern.

*Adapted from: Constructing Ideas About Patterns Grades 1-3, by Sandra Ward, Creative Publications.

WHAT'S THE MATHEMATICS?

- Recognize, describe, extend and create patterns with geometric shapes.
- Do skip counting with manipulatives.
- Use manipulative materials to explore linear patterns.
- Solve real world problems involving addition of whole numbers.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2e – Solve problems by showing relationships between and among figures.
M2f – Extend and create geometric patterns using concrete and pictorial figures.
M3a – Use linear patterns to solve problems.

VOCABULARY triangle trapezoid pattern

SUGGESTED TIME: 1 – 2 lessons

MATERIALS: Bags of pattern blocks, paper, crayons or markers, worksheets

DOING THE INVESTIGATION

- Read the problem, then model the pattern on the overhead projector or with pattern blocks taped on the board. How many blocks do you think Sam will need to make a fence with 20 green gates?
- Set up a chart (or a worksheet) so the students can keep track as they work:
  How many green triangles?
  How many red trapezoids?
  How many blocks in all?
- Allow the students to work in pairs. At some point, they will run out of blocks and need to think of an alternative method.

POSSIBLE SOLUTIONS

12. Students may use actual blocks to build the fence with 20 gates, or they may draw the fence with 20 gates, or use any combination of the above to arrive at the solution – Sam will need 20 green triangles and 21 red trapezoids, a total of 41 blocks.
13. Some students will be able to extend the pattern using the chart or worksheet.
14. Some students might need to fill in every line; others might be able to extend the pattern at some point along the way.

How many green triangles?	How many red trapezoids?	How many blocks in all?
1	2	3
2	3	5
3	4	7
4	5	9
5	6	11
6	7	13
7	8	15
8	9	17
9	10	19
10	11	21
20	21	41

EXTENDING THE ACTIVITY

3. If the fence has 50 gates, how many blocks will you need in all?
4. Use the pattern blocks to design your own fence. Show your pattern, tell how many gates, tell how many blocks you used in all.

ASSESSMENT 1

Show and tell about how you found the number of blocks you needed to build the fence with 20 green gates. Could it have been done any other way? Show and tell how you could have done it using your pattern.

ASSESSMENT 2

If your fence used 31 blocks in all, how many gates would you have?

Ans. 15

For additional information and activities, see pages 32 and 41-42
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence and construct an argument.

PERFORMANCE INDICATORS:

1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their thinking.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Justify answers and solution processes.
1D. Use logical reasoning to reach simple conclusions.

INVESTIGATION

The students will determine the outcome of each of the following situations and justify their findings:

- even number + even number = ?
- even number + odd number = ?
- odd number + odd number = ?
- odd number + even number = ?

WHAT'S THE MATHEMATICS?

- addition and subtraction in number patterns
- verify an answer to a problem
- use estimation, number relationships, and mathematical checks to justify answers

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Adds, subtracts, multiplies and divides whole numbers with and without calculators.
M6a Adds, subtracts, multiplies and divides whole numbers correctly.

VOCABULARY:	even	odd	plus	minus
	subtracted from	addition	subtraction	addend
	sum	total		

SUGGESTED TIME: 1 - 2 days

MATERIALS: any type of counters

DOING THE INVESTIGATION

Discuss odd and even numbers.

- Use a simple addition example as a model and discuss whether the addends and sum are odd or even, i.e., 3+2=5.
- Record findings (odd + even = odd) and look for patterns.
- Allow students to work in pairs as they investigate other examples and record their findings.
- Encourage students to choose many sets of numbers to determine if results will always be the same.
- Students will explain their answers.

POSSIBLE SOLUTIONS

even number + even number = even number

- you are always adding groups of two

even number + odd number = odd number

- you always add groups of two plus one extra

odd number + odd number = even number

- you always add groups of two plus two extra

odd number + even number = odd number

- you are always adding groups of two plus one extra

EXTENDING THE ACTIVITY

Students will conduct an investigation to determine the outcome of each of the following situations and justify their findings:

even number – even number = ?
even number – odd number = ?
odd number – odd number = ?
odd number – even number = ?

ASSESSMENT 1

The sum of an odd number and an even number is always

a) even b) zero c) equal d) odd

Ans. d) odd

ASSESSMENT 2

If you add two even numbers your answer is always another even number.
Is this statement true or false?

Explain your answer.

Ans. True.
Accept all reasonable explanations.

For additional information and activities see pages 43 and 52
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.
2B. Use concrete materials to model numbers and number relationships for the whole numbers.
2C. Relate counting to grouping and place value.
2D. Recognize order of whole numbers and commonly used fractions and decimals.
2E. Demonstrate the concept of ratio and percent through problems related to actual situations.

INVESTIGATION

The students will use concrete materials and pictorial models to represent a 3 digit counting number.

WHAT'S THE MATHEMATICS?

- use manipulative materials to represent place value in whole numbers i.e., base 10 blocks
- represent numbers in various ways i.e., expanded notation

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1b Demonstrate understanding of the base ten value system.
M1f Describe and compare quantities by using whole numbers.
M7a Use appropriate mathematical terms, vocabulary and language.

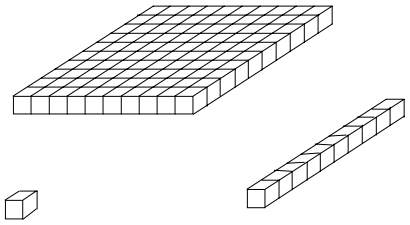
VOCABULARY:	units	ones	place value
	longs	tens	expanded notation
	flats	hundreds	digit

SUGGESTED TIME: 3 - 4 days

MATERIALS: base 10 blocks, graph paper, squared materials

DOING THE INVESTIGATION

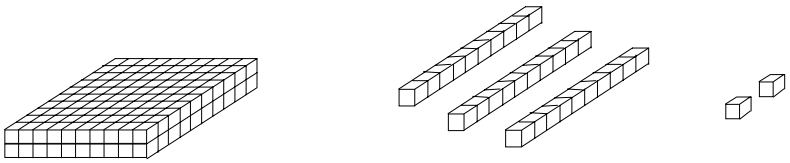
- Display, with base 10 blocks or squared materials, the representation of a two digit number.
- Elicit from the children the number that you are representing.
- Have the children use concrete materials as they work in pairs to represent various two and three digit numbers.



- Represent the same number using expanded notation.
  (1 x 100) + (1 x 10) + (1 x 1) = 111
- Students will represent several three digit numbers in the same manner.

POSSIBLE SOLUTIONS

232



100 100 10 10 10 2

(2 x 100) + (3 x 10) + (2 x 1) = 232

EXTENDING THE ACTIVITY

Use expanded notation to express a four digit number.

ASSESSMENT 1

(3 x 100) + (5 x 10) + (6 x 1) can be written as ____

a. 3005061
b. 356
c. 100 + 10 + 1
d. 350

Ans. b)

ASSESSMENT 2

Represent the number 428 using the least number of base 10 blocks. Explain in words why your answer is correct.

Ans. Students should use 4 hundreds, 2 tens, and 8 ones. Accept all reasonable verbal explanations.

For additional information and activities, see pages 44 and 53-54
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply and divide whole numbers.
3B. Develop strategies for selecting the appropriate computational and operational method in problem-solving situations.
3C. Know single digit addition, subtraction, multiplication, and division.
3D. Understand the commutative and associative properties.

INVESTIGATION

Jim says that to add 2 digit numbers you must:

- first add the ones column
- regroup if necessary
- then add the tens column

Susan says that you must:

- first add the tens
- then add the ones
- then add both results

Who is right? Explain your answer and show an example.

WHAT'S THE MATHEMATICS?

- addition and subtraction of whole numbers
- use commutative, associative, distributive and inverse properties

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Add, subtract, multiply, divide whole numbers
M1b Demonstrate understanding of base 10 value system
M7b Show mathematical ideas in a variety of ways

VOCABULARY: ones tens hundreds column
regroup numeral digit

SUGGESTED TIME: 1 - 3 days

MATERIALS: base-10 blocks

DOING THE INVESTIGATION

- Familiarize students with using base 10 blocks for addition.
- Demonstrate and model the process for recording work.
- Students will demonstrate their ability to add two digit numbers.
- Students will record their work.
- Pairs of students work together as they solve several problems using both methods.
- Allow pairs to share findings with the class.
- If necessary, elicit that when you regroup, the result is the same as adding the ones to the tens.

POSSIBLE SOLUTIONS

They are both right. When you regroup 12 ones, the result is one ten and two ones. It doesn't matter if you do it before or after adding the tens. You will always get the same answer.

Ex.

```
  10
  53
+ 28
  81
```

You regroup before adding the tens.

53 + 28 =

50 + 20 = 70
3 + 8 = 11
70 + 11 = 81

You add the ones after adding the tens.

EXTENDING THE ACTIVITY

Students will conduct the same type of investigation when adding 3 digit numbers.

ASSESSMENT 1

Which digit is missing?

$$\begin{array}{r} 556 \\ +\ 78 \\ \hline 6\_4 \end{array}$$

a) 2 b) 3 c) 12 d) 13

Ans. b)

ASSESSMENT 2

What numeral is missing? _____

Show or explain your thinking.

$$\begin{array}{r} 1\_8 \\ +\ 75 \\ \hline 243 \end{array}$$

Ans. 6
Accept all reasonable explanations.

For additional information and activities, see pages 45 and 55
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating and connecting mathematical information and relationships

PERFORMANCE INDICATORS:

4A. Use concrete materials to model spatial relationships.
4B. Construct tables, charts, and graphs to display and analyze real-world data.
4C. Use multiple representations as tools to explain the operations of every day proofs.
4D. Use variables such as height, weight, temperature, and hand size to predict changes over time.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

Students will work in pairs using the green, blue, red and yellow pattern blocks. They will each construct a shape using only 8 pattern blocks. Then they will work together to determine the area of each shape and which shape has the larger area.

WHAT'S THE MATHEMATICS?

- design patterns using geometric figures
- find the area by counting units

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Use many types of figures.
M2e Solve problems by showing relationships between and among figures.
M2g Use basic ways to estimate and measure the size of figures.
M2h Use models to reason relationships between area of rectangles.
M6e Refer to geometric shapes and terms correctly.
M7b Show mathematical ideas in a variety of ways.

VOCABULARY: equilateral triangle, hexagon, rhombus, area, trapezoid

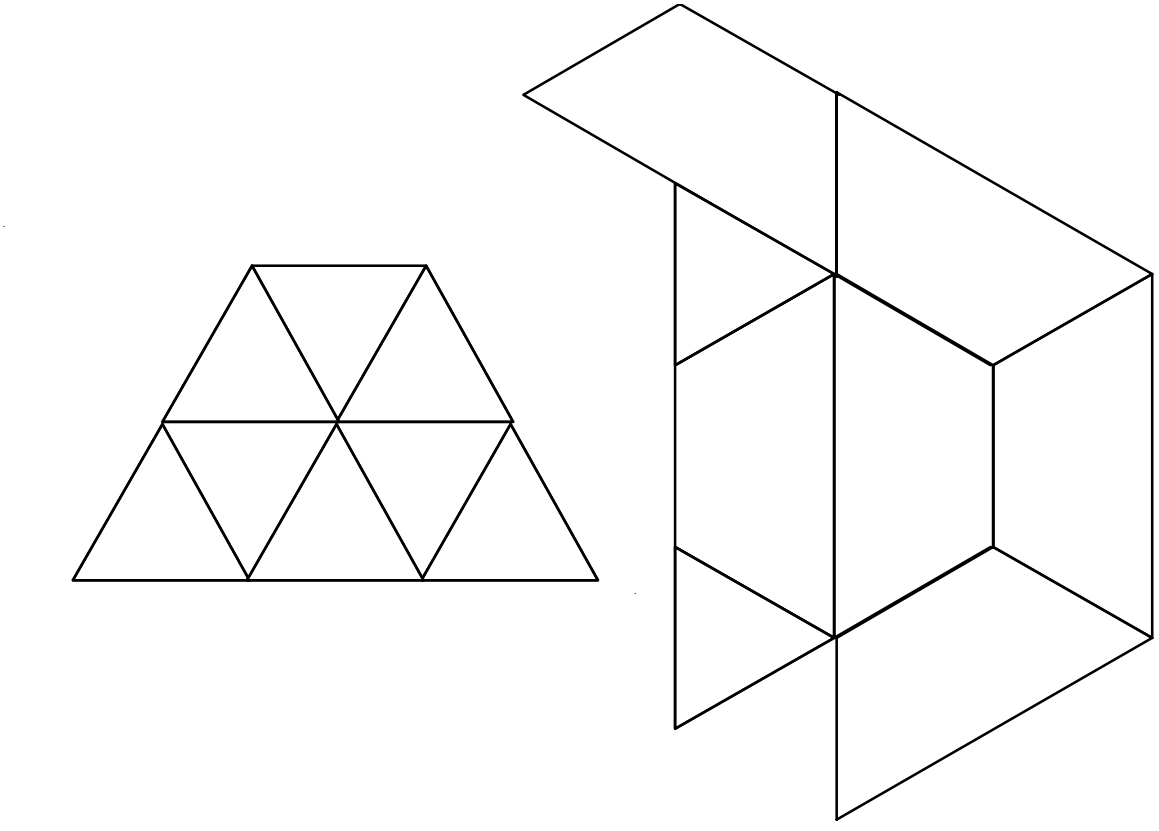
SUGGESTED TIME: 1 - 2 days

MATERIALS: pattern blocks, triangle paper

DOING THE INVESTIGATION

- The teacher displays two pattern block shapes.
- The class determines which shape has the largest area.
- The class arrives at the conclusion that interchanging pieces will help to determine the area (3 green=1 red) of a figure.
- The class then works in small groups, using triangle paper, to make shapes and determine the area of each.
- Students order their shapes by area from smallest to largest.

POSSIBLE SOLUTIONS

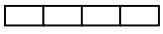
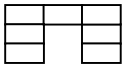


EXTENDING THE ACTIVITY

Students will work in pairs to create shapes that enclose a given area. (Teacher will assign different areas to each pair of students.)

ASSESSMENT 1

Circle the figure that has the greatest area.

a)  b)  c)  d) 

Ans. d)

ASSESSMENT 2

Draw two figures and determine which has the greatest area.
Use pictures or words to prove your answer.

Ans. Accept any reasonable verbal or nonverbal explanation.

For additional information and activities, see pages 46 and 56
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Understand that measurement is approximate, never exact.
5B. Select appropriate standard and nonstandard measurement tools in measurement activities.
5C. Understand the attributes of area, length, capacity, weight, volume, time, temperature, and angles.
5D. Estimate and find measures such as length, perimeter, area, and volume, using both nonstandard and standard units.
5E Collect and display data.
5F Use statistical methods such as graphs, tables, and charts to interpret data.

INVESTIGATION

Students will conduct a data study to find out the number of letters in each other's last name. The data will then be appropriately displayed and analyzed.

WHAT'S THE MATHEMATICS?

- organize data with graphs, models, pictures, lists.
- use concrete materials to develop the concept of average or arithmetic mean.
- find the range and the mean for a collection of organized data.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Collect and organize data to answer a question.
M4b Display data in line plots, graphs, tables, and charts.
M4c Make statements and draw simple conclusions based on data.
M8a Conduct a data study.

VOCABULARY: data mean average bar graph
axis range number scale

SUGGESTED TIME: 4 - 5 days

MATERIALS: cubes or color tiles, graph paper

DOING THE INVESTIGATION

- Read and discuss a book related to graphing (i.e., Lemonade for Sale by Stuart J. Murphy) or names (i.e., Chrysanthemum by Kevin Henkes).
- Make towers of cubes that correspond in length to the number of letters in individual names.
- Model the method of finding the mean by "evening out" towers of cubes so that each tower is the same height.
- Allow students to work in pairs and practice finding the mean of sets of numbers.
- Lead a whole group activity in which students line up according to the number of letters in their first name.
- Represent each line of students with a tower of cubes and then as a bar graph.
- Discuss the use of bar graphs as a means of comparing data.
- Discuss range and parts of graphs such as title, labels, scale etc.
- Have students find mean and range.
- Place students in groups of 4. Instruct them to conduct a poll of the class to find the number of letters in their last names.
- Groups will display the data on bar graphs, find mean, range and make statements based upon their findings.

POSSIBLE SOLUTIONS

16. Appropriate bar graphs
17. Statements such as those saying, "Most students have __ letters in their name." etc.

EXTENDING THE ACTIVITY

Students will conduct a second data study based on topics which they have chosen.
They will organize and display their data in the form of a bar graph with appropriate labels.

ASSESSMENT 1

Computer Games

Child	Number Owned
Shawn	25
Tonya	18
Quan	30
Maria	27
Anna	12

What is the range for the number of games the students own?

a) 30 b) 18 c) 12 d) 5

Ans. b)

ASSESSMENT 2

Use your counters to find the mean of the following set of numbers:
7, 2, 5, 1, 5

Show your work.

Ans. $7 + 2 + 5 + 1 + 5 = 20$, $20 \div 5 = 4$

For additional information and activities, see pages 47 and pages 57-58
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 6 UNCERTAINTY

Ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with every day situations

PERFORMANCE INDICATORS:

6A. Make estimates to compare to actual results of both formal and informal measurement.
6B. Make estimates to compare to actual results of computations.
6C. Recognize situations in which only an estimate is required.
6D. Develop a wide variety of estimation skills and strategies.
6E. Determine the reasonableness of results.
6F. Predict experimental probability.
6G. Make predictions using unbiased random samples.
6H. Determine probabilities of simple events.

INVESTIGATION

Students will set up bags with color cubes to meet the following conditions:

- Bag A – You are certain to choose red.
- Bag B – It is impossible to choose red.
- Bag C – You are equally likely to choose red as any other color.
- Bag D – It is very unlikely that you will choose red.
- Bag E – You are very likely to choose red.

WHAT'S THE MATHEMATICS?

- Conduct and predict outcomes of experiments using equally, unequally and likely outcomes.
- Recognize events that are certain and events that have no chance of occurring.
- Determine the number of ways an event can occur.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4d Gather data by sampling a group.
M4e Predict results and find out why some results are more likely, less likely, or equally likely.

M5b Problem solving/implementation.
M7a Using appropriate math terms, vocabulary, and language.
M7c Explain solutions to problems clearly and logically . . . in both oral and written work.

VOCABULARY:	certain	equally likely	very likely
	majority	impossible	very unlikely
	samples	probability	reliability

SUGGESTED TIME: 3 – 4 lessons

MATERIALS: paper bags, colored cubes

DOING THE INVESTIGATION

- Use paper bags and colored cubes to lead a series of probability activities which lend themselves to various outcomes.
- Familiarize students with appropriate vocabulary.
- Allow students to work in groups as they make predictions and conduct their own experiments.
- Students must provide written and/or oral explanations to validate their results.
- Discuss the reliability of a large number of trials vs. a small number of trials.

POSSIBLE SOLUTIONS

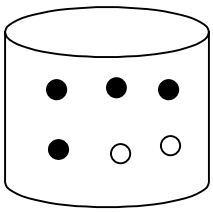
Bag A	All are red.
Bag B	None are red.
Bag C	One half of the cubes are red.
Bag D	A small percent of the cubes are red.
Bag E	A vast majority of the cubes are red.

EXTENDING THE ACTIVITY

Each student will be given a bag of 10 cubes. Students will choose a cube from the bag, note the color and replace the cube. This will be repeated 25 times. Based on the colors drawn the student will make predictions as to the number of red cubes in the bag and explain his/her reasoning.

ASSESSMENT 1

What is the probability of choosing a black marble from the bag without looking?



a) certain b) very likely c) equally likely as white d) very unlikely

Ans. b)

ASSESSMENT 2

Pat and Kim are each holding a bag of marbles. If you choose from Pat's bag you are certain to choose a white marble. If you choose from Kim's bag, you are very likely but not certain to choose a white marble. Draw a picture of what is likely to be in each child's bag.

Ans. Pictures should show Pat's bag with only white marbles and Kim's bag with many more white marbles than any other color.

For additional information and activities, see pages 48-49 and 60
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 3

KEY IDEA # 7 PATTERNS AND FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently

PERFORMANCE INDICATORS:

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Explore and express relationships, using variables and open sentences.
7D. Solve for an unknown, using manipulative material.
7E Use a variety of manipulative materials to explore patterns.
7F. Interpret graphs.
7G. Explore and develop relationships among two- and three-dimensional geometric shapes.
7H. Discover patterns in nature, art, music and literature.

INVESTIGATION

Students construct a symmetrical design using 14 Pattern Blocks. Then they repeat the symmetrical design so that it is symmetrical in shape but not in color.
Children work in groups and discuss the following

- Is there more than one line of symmetry?
- Explain how you are able to recreate original design using greater or fewer blocks than the original design.
- Is there a line of symmetry in an asymmetrical design?

WHAT'S THE MATHEMATICS?

- Determine the interrelationship of different geometric shapes.
- Use manipulatives that allow students to explore geometric shapes and symmetry.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2f Extend and create geometric patterns using concrete and pictorial models.
M7b Showing mathematical ideas in a variety of ways.

VOCABULARY: symmetry horizontal asymmetrical vertical
rhombus parallelogram trapezoid hexagon

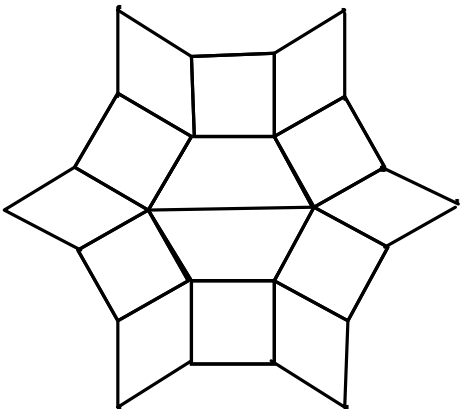
SUGGESTED TIME: 1 - 2 days

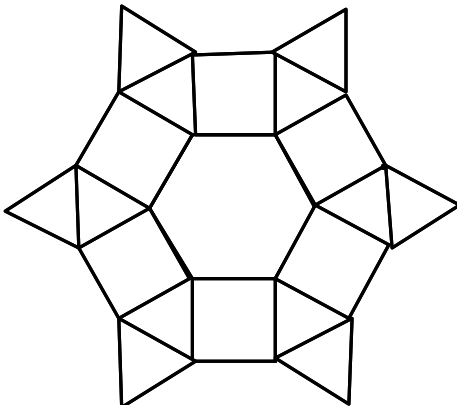
MATERIALS: pattern blocks

DOING THE INVESTIGATION

- Demonstrate for children an example of a symmetrical and an asymmetrical pattern using pattern blocks.
- Define what makes a pattern symmetric and define line symmetry.
- Have children create their own patterns around a line of symmetry which is symmetrical in both shape and color.
- Have the students alter one side of their pattern so that it is still symmetrical in shape but not in color.
- Why it is possible to maintain the same shape while not maintaining the same color?
- Discover shapes with multiple lines of symmetry.

POSSIBLE SOLUTIONS





EXTENDING THE ACTIVITY

Construct a design with your pattern blocks that has both a horizontal and vertical line of symmetry.
Describe your pattern in words using geometric terms.

ASSESSMENT 1

How many lines of symmetry are there in an equilateral triangle?

a) 0 b) 1 c) 2 d) 3

Ans. d)

ASSESSMENT 2

What is a line of symmetry?
Draw a letter of the alphabet that has no line of symmetry.
Draw a letter of the alphabet that has one line of symmetry.

Ans. Accept all reasonable answers.

For additional information and activities, see pages 61-62 and page 50
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures gather evidence and construct an argument.

PERFORMANCE INDICATORS:

1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their thinking.
1B. Use patterns and relationships to analyze mathematical situations.
1C. Justify answers and solution processes.
1D. Use logical reasoning to reach simple conclusions.

INVESTIGATION

You have 11 tables. Each one is shaped like an equilateral triangle. They need to be arranged so that they seat exactly 25 people with one person seated along each edge. Tables may be placed together so that they share a common side or they may stand separately. Draw a picture that shows how the tables should be set up.

WHAT'S THE MATHEMATICS?

- draw pictures and diagrams to represent problems
- clarify problems, using discussions with peers
- use concrete objects to help solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M5b Implementation
M5c Conclusions
M6h Use pencil and paper. . manipulatives and advice from peers to solve problems.

VOCABULARY: triangular arrangements equilateral common side

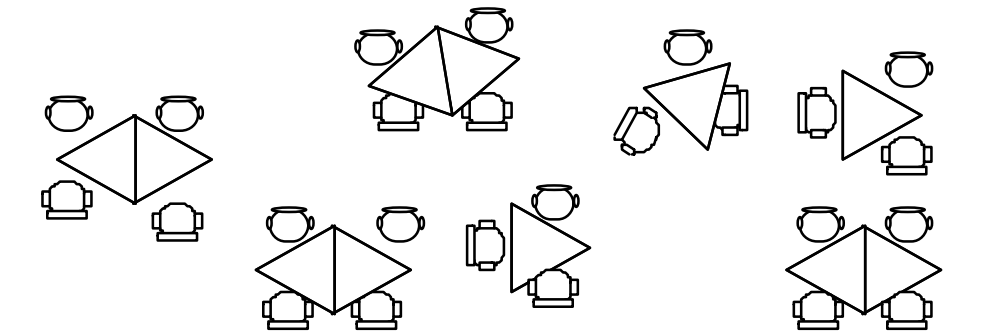
SUGGESTED TIME: 1 – 2 days

MATERIALS: pattern blocks, triangular paper

DOING THE INVESTIGATION

- Use green pattern blocks, which are equilateral triangles, to represent the tables.
- Discuss how 2 or more green triangles can be placed together to share a common side.
- Have students work in groups to explore possible seating arrangements.
- Use manipulatives to solve the investigation.

POSSIBLE SOLUTIONS



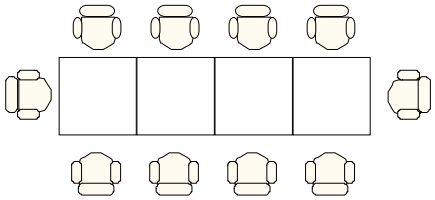
EXTENDING THE ACTIVITY

How would you arrange 11 triangular tables so that they seat the maximum number of people? Show your work and explain your thinking.

ASSESSMENT 1

If 4 square tables were placed together so that they shared at least one common side, what is the maximum number of people that can be seated?

a) 4 b) 6 c) 8 d) 10

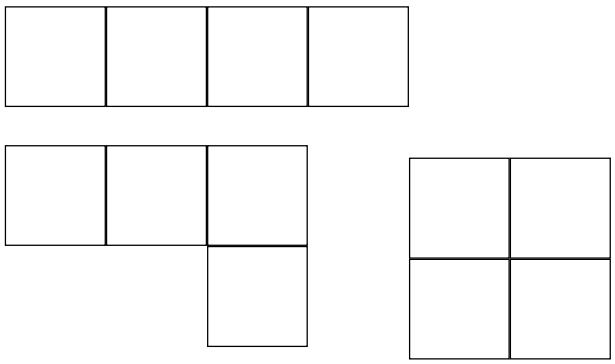


Ans. 10 people. One arrangement is shown at the left. Other arrangements are possible.

ASSESSMENT 2

Show 3 ways to arrange 4 square tables that share at least one common side.

Ans. Three sample arrangements are shown below. Many other arrangements are possible.



For additional information and activities, see pages 43 and 53
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, use of numbers to communicate mathematically, and the use of numbers in the developments of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.
2B. Use concrete materials to model numbers and number relationships for the whole numbers.
2C. Relate counting to grouping and place value.
2D. Recognize order of whole numbers and commonly used fractions and decimals.
2E. Demonstrate the concept of ratio and percent through problems related to actual situations.

INVESTIGATION

The students will use fraction strips to compare and order fractions.

WHAT'S THE MATHEMATICS?

- identify various ways a figure can be divided
- order unit fractions and decimals
- find equivalent fractions
- recognize and order fractions with the denominators 2, 3, 4, 5, 6, 8, 10, 12

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1d Describe and compare quantities by using simple fractions.
M7a Use appropriate mathematical terms, vocabulary and language.
M7c Explain solutions to problems clearly and logically.

VOCABULARY:	fraction	proper fraction	rational number
	equivalent fraction	numerator	denominator

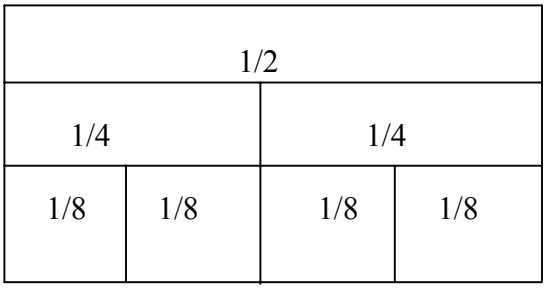
SUGGESTED TIME: 2 - 3 lessons

MATERIALS: fraction strips (made from construction paper) or fraction bars

DOING THE INVESTIGATION

1. Have the class use construction paper to make strips representing fractions with the denominators 2, 3, 4, 5, 6, 8, 10 and 12 and 1 whole. Write the proper fraction on each strip.
2. Students order various unit fractions by comparing the size of the corresponding fraction strips.
3. Students work in pairs to represent various other fractions (i.e., 2/3 , 3/4, 6/12). By comparing the size of the strips, they will:
   - determine which fractions are greater.
   - record their findings.
   - represent fractions which are equivalent to 1/2, 1/3, 1/4.
   - provide written explanations of their work.
4. Students will discuss their findings.

POSSIBLE SOLUTIONS



1/2 = 2/4 = 4/8

1/4 <3/8<1/2

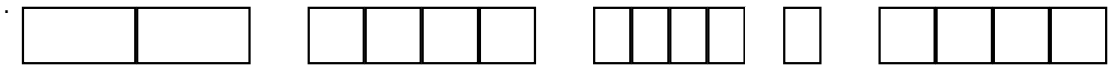
EXTENDING THE ACTIVITY

Students will develop a rule for finding equivalent fractions by looking for patterns.

i.e., 1/2 = 2/4, 3/6, 4/8, 5/10, 6/12
1/3 = 2/6, 4/12
1/4 = 2/8, 3/12

ASSESSMENT 1

One half of Sam's scarf is striped. Which scarf is Sam's?



A B C D

Ans. A, B has 3/4 striped; C has 2/6 or 1/3 striped; D has 1/4 striped.

ASSESSMENT 2

Write 2 fractions that are equivalent to 1/5.
Explain how you arrived at your answer.

Ans. 2/10, 3/15, 4/20, . . . 100/500, . . .

Equivalent fractions can be found by multiplying both the numerator and the denominator of the fraction by the same number.

For additional information and activities, see pages 44 and 53-54
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply and divide whole numbers.
3B. Develop strategies for selecting the appropriate computational and operational method in problem-solving situations.
3C. Know single digit addition, subtraction, multiplication, and division.
3D. Understand the commutative and associative properties.

INVESTIGATION

A candy bar was divided into 12 equal parts. Show how you would divide 10 candy bars equally among 24 children

WHAT'S THE MATHEMATICS?

- division of 3-digit divisors by one and two digit divisors
- breaking the problem into parts

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Add, subtract, multiply and divide whole numbers.
M6a Add, subtract, multiply and divide whole numbers correctly.

VOCABULARY: equally divide

SUGGESTED TIME: 1 day

MATERIALS: color tiles

DOING THE INVESTIGATION

- Do a simpler problem. There are 2 six slice pizzas and 4 children will receive an equal number of slices.
- Show how you will combine and then divide to get equal shares.
- Encourage children to draw illustrations to solve the problem.

POSSIBLE SOLUTIONS

10 candy bars
x 12 pieces in each bar
120 pieces

Then divide 120 pieces among 24 children

```
      5     Each child gets this amount
24 ) 120
     120
       0
```

EXTENDING THE ACTIVITY

If there are 35 children in a class and you wanted to give each child 3 slices of pizza, how many 8 slice pizza pies must you order?

ASSESSMENT 1

What number belongs in the box?

$$\square \div 9 = 21$$

a) 30 b) 12 c) 2 d) 189

Ans. d) 189; 9 x 21 = 189.

ASSESSMENT 2

How would you divide 180 marbles equally among 15 children?
Show your work.

Ans.

```
    12
15) 180
    15
     30
     30
      0
```

Each child would get 12 marbles.

For additional information and activities, see pages 45 and 55
in the New York State Core Curriculum/Mathematics Resource Guide.

Grade 4

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Use concrete materials to model spatial relationships.
4B. Construct tables, charts, and graphs to display and analyze real-world data.
4C. Use multiple representations as tools to explain the operations of every day proofs.
4D. Use variables such as height, weight, temperature, and hand size to predict changes over time.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

INVESTIGATION

Students will use colored cubes to develop a rule for finding the volume of rectangular solids.

WHAT'S THE MATHEMATICS?

- Investigate properties of solid figures.
- Fill space with standard-sized objects to build a foundation for the volume formula.
- Find volume by counting units.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2e Solve problems by showing relationships between and among figures.
M5b Implementation
M5c Conclusion
M6b Estimate numerically and spatially.

VOCABULARY:	length	rectangular	solid	cube
	width	volume	height	depth

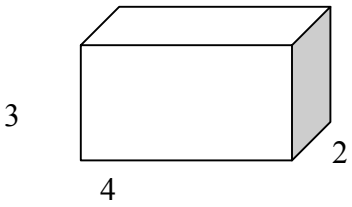
SUGGESTED TIME: 1 – 2 days

MATERIALS: colored cubes

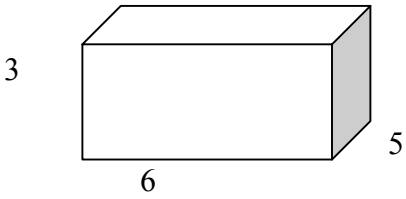
DOING THE INVESTIGATION

- Model filling a rectangular shaped container with cubes. Record the number of cubes needed to occupy the length, width and height of the container and the total number of cubes used.
- Students work in groups as they use cubes to build various rectangular solids. They record the number of cubes used as modeled by the teacher.
- As groups work, circulate and encourage students to look for a relationship between the dimensions and the total number of cubes used (volume).
- Allow groups to share their findings with class. Elicit formula V=L x W x H.

POSSIBLE SOLUTIONS



V = 4 x 2 x 3
V = 24 cubes



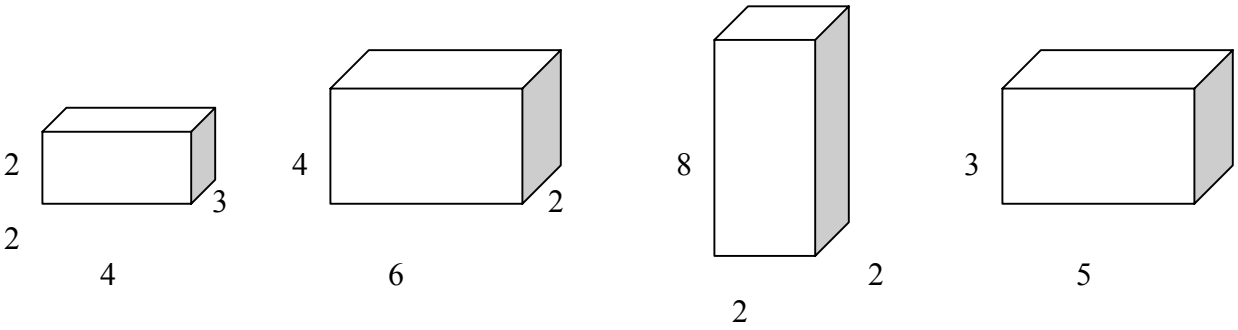
V = 6 x 5 x 3
V = 90 cubes

EXTENDING THE ACTIVITY

Students will use cubes to make solid figures that have the same volume, but have different dimensions.

ASSESSMENT 1

Circle the figure that has the greatest volume.



A B C D

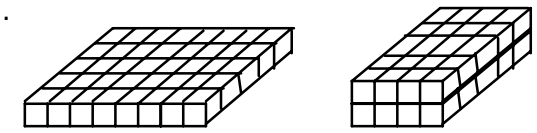
Ans. Figure B

The volumes are:
A 2 x 3 x 4 = 24
B 4 x 6 x 2 = 48
C 8 x 2 x 2 = 32
D 3 x 5 x 2 = 30

ASSESSMENT 2

Anna has 40 cubes. Show 2 different rectangular solids that she can make. She must use all of the cubes for each solid. Give the length, width and height for each solid.

Ans. Two possible solids are shown below. They are an 8 by 5 by 1 solid and a 4 by 5 by 2 solid. Any rectangular solid for which the product of the length, width and height is 40 will be a correct answer.



For additional information and activities, see pages 46 and 56
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Understand that measurement is approximate, never exact.
5B. Select appropriate standard and nonstandard measurement tools in measurement activities.
5C. Understand the attributes of area, length, capacity, weight, volume, time, temperature, and angles.
5D. Estimate and find measures such as length, perimeter, area, and volume, using both nonstandard and standard units.
5E Collect and display data
5F Use statistical methods such as graphs, tables, and charts to interpret data

INVESTIGATION

Your class will be designing and making a patchwork quilt. The class will decide on a theme for the quilt. Each child will be given a 6x6 inch square of material on which he/she will make a design which represents that theme. Based on the number of children in the class, and the layout of the squares, the students will calculate the dimensions of the quilt. The end product will be on display in the classroom.

WHAT'S THE MATHEMATICS?

- explore connections between factors, multiplication facts and area
- find the area of a specific figure by counting units
- perimeter of polygons

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2f Extends and creates geometric patterns using concrete and pictorial models.
M2g Use basic ways of estimating and measuring in the real world, including area.

VOCABULARY: area perimeter yard pattern inch

SUGGESTED TIME: 4 – 5 days

MATERIALS: inch graph paper, fabric, needle, markers, paste, thread

DOING THE INVESTIGATION

- Read a book related to quilting (e.g., Sam Johnson and the Blue Ribbon Quilt by Lisa Campbell Ernst) to motivate discussion.
- Each child will lay out his/her design on graph paper.
- A sample of the class quilt will be made using these paper designs.
- Students will calculate the area and perimeter of the quilt in order to determine how much fabric is needed to make the quilt. The width of a bolt of fabric must be taken into consideration.

POSSIBLE SOLUTIONS

If you have 36 students in your class they could design a quilt of various dimensions. Some sample dimensions may be:

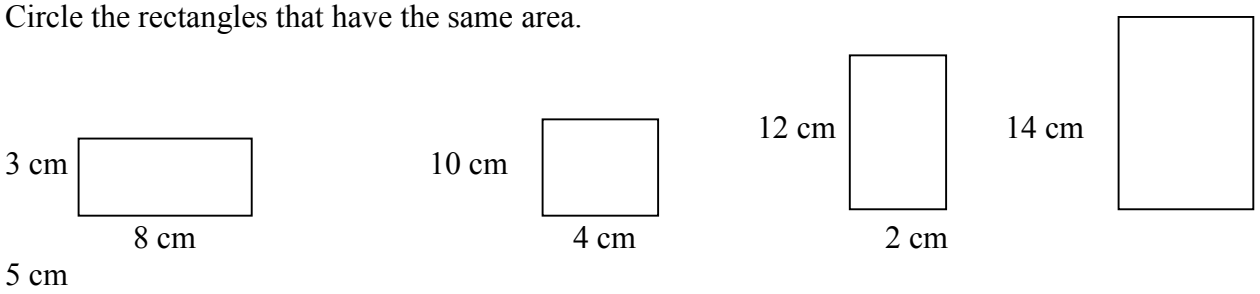
- six squares by six squares
- nine squares by four squares

EXTENDING THE ACTIVITY

Design a quilt that can be made from a piece of fabric 32 inches by 64 inches. Determine the size and number of squares you will need. Represent your quilt on graph paper.

ASSESSMENT 1

Circle the rectangles that have the same area.

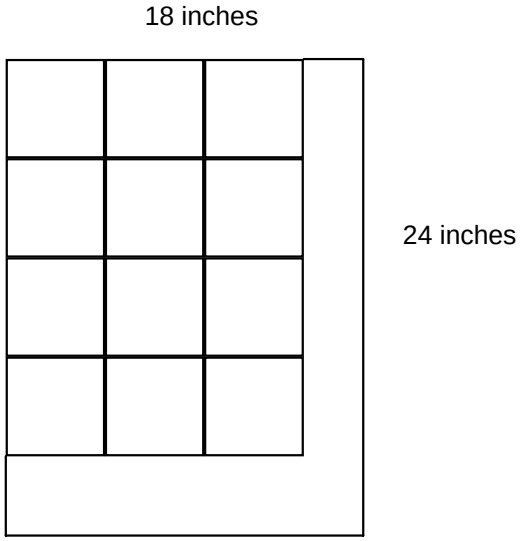


Ans. $3 \times 8 = 24\ cm^2$ and $2 \times 12 = 24\ cm^2$

ASSESSMENT 2

How many 5x5 inch squares can be cut from a piece of construction paper that measures 18 inches by 24 inches?
Show your work.

Ans. Twelve 5" x 5" squares can be cut from an 18" x 24" paper.



For additional information and activities, see pages 47 and pages 57-58 in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Make estimates to compare to actual results of both formal and informal measurement.
6B. Make estimates to compare to actual results of computations.
6C. Recognize situations in which only an estimate is required.
6D. Develop a wide variety of estimation skills and strategies.
6E. Determine the reasonableness of results.
6F. Predict experimental probability.
6G. Make predictions using unbiased random samples.
6H. Determine probabilities of simple events.

INVESTIGATION

Students will design a carnival game in which there are 4 possible outcomes. The likelihood of each outcome is as follows:
1/2, 1/4, 1/8, 1/8.
Write rules for this game so two players will have an equally likely chance of winning.

WHAT'S THE MATHEMATICS?

- express probability as a fraction
- use fractional notation to express probability of an occurrence
- find equivalent fractions

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1d Describe and compare quantities by fractions.
M4e Predict results and find out why some results are more likely, less likely, or equally likely.
M5a Formulation
M5b Implementation
M5c Conclusion
M7c Explain solutions to problems clearly and logically.

VOCABULARY: probability fair game unfair game
outcome equivalent

SUGGESTED TIME: 4 – 5 lessons

MATERIALS: Various manipulatives such as: pattern blocks/color tiles/ fraction strips/spinners/Cuisenaire rods, craft material such as construction paper, markers, tape, etc.

DOING THE INVESTIGATION

- Students will discuss the concept of a "fair game".
- Students will work in groups to find fractions of a set and fractional parts of a whole.
- Students will identify equivalent fractions.
- Students will demonstrate and apply their ability to add fractions.

  Students will work in groups to:
  - Design a game board and divide it into fractional parts. Each part should reflect one of the possible outcomes.
  - Discuss rules that will make their game fair.
  - Write rules for playing the game.

POSSIBLE SOLUTIONS

Design a spinner and spin it 25 times

- If it lands on 1/2, player A gets a point
- If it lands on anything else, player B gets a point

or

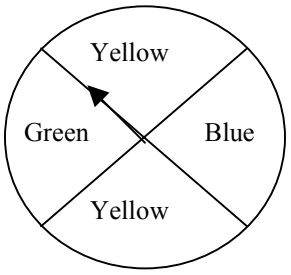
- If it lands on 1/2 no one gets a point
- If it lands on 1/4 player A gets a point
- If it lands on 1/8 player B gets a point

EXTENDING THE ACTIVITY

Design a game that has several possible outcomes.
Explain how you arrived at your answer

ASSESSMENT 1

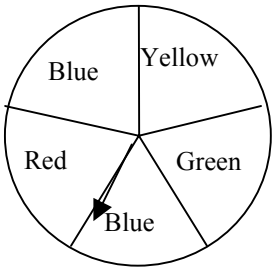
Using the spinner to the right, what is the probability of landing on yellow?



a) 1/2 c) 1/6
b) 1/4 d) 1/8

Ans. a) 2 of the 4 equal segments are yellow. The probability of landing on yellow is 2/4 = 1/2.

ASSESSMENT 2



Is this spinner fair?
Explain your answer.

Ans. The spinner is not fair. Two of the five equal segments are blue. The probability of landing on blue is 2/5. The probability of landing on red is 1/5, the probability of landing on yellow is 1/5, and the probability of landing on green is 1/5.

For additional information and activities, see pages 48-49 and 60 in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 4

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, extend, and create a wide variety of patterns.
7B. Represent and describe mathematical relationships.
7C. Explore and express relationships, using variables and open sentences.
7D. Solve for an unknown, using manipulative material.
7E Use a variety of manipulative materials to explore patterns.
7F. Interpret graphs.
7G. Explore and develop relationships among two- and three-dimensional geometric shapes.
7H. Discover patterns in nature, art, music and literature.

INVESTIGATION

Children create patterns that grow in predictable ways. They make conjectures about the design and predict the number of blocks used in the eighth term of the pattern.

WHAT'S THE MATHEMATICS?

- design patterns
- extend repeated patterns
- analyze number patterns and sequences
- make predictions about patterns

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2f Extend and create geometric patterns using concrete and pictorial models.
M3b Build iterations of simple non-linear patterns . . .
M7a Use appropriate mathematical terms, vocabulary and language.

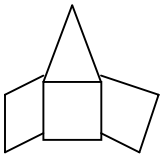
VOCABULARY:	pattern	rhombus	hexagon	growth pattern
	term	trapezoid	parallelogram	

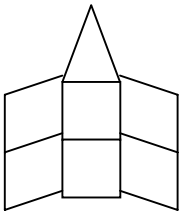
SUGGESTED TIME: 2 – 3 days
MATERIALS: pattern blocks

DOING THE INVESTIGATION

1. Teacher displays the first 2 terms of a simple growth pattern made with pattern blocks.
2. Students work in pairs as they:
   - discuss how the pattern changed from the 1st to 2nd term
   - use pattern blocks to build the 3rd and 4th terms of the pattern
   - make a chart to record the number of blocks used in each term
   - identify and describe the numerical pattern
   - generalize the pattern using numbers or words
- Students work independently as they:
   - predict how the 8th term of the pattern will look and the number of blocks needed to build it
   - draw a picture of the 8th term
   - write a description of the pattern and how it is growing
- Teacher leads a class discussion as students share their predictions, descriptions, and pictures.

POSSIBLE SOLUTIONS





Term	# Blocks
9. 4 10. 7 11. 10 12. 13 . . . 8	. . . 25

EXTENDING THE ACTIVITY

Students will create their own growth patterns and exchange them with each other. They will then describe, analyze and extend each other's patterns.

ASSESSMENT 1

In	Out
5. 5 6. 7 4 5	 ☐ 11

What number is missing in this table?

a) 8 b) 9 c) 10 d) 11

Ans. b) 9. The rule is 2 x IN +1.

ASSESSMENT 2

60, 48, 36, ☐

What is the next number in this pattern?
Explain the rule.

Ans. 24. The rule is subtract 12.

For additional information and activities, see pages 50-51 and 61-62
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 5

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Apply a variety of reasoning strategies.
1B. Make and evaluate conjectures and arguments, using appropriate language.
1C. Make conclusions based on inductive reasoning.
1D. Justify conclusions involving simple and compound (i.e., and/or) statements.

INVESTIGATION

SORTING AND CLASSIFYING

Students will be engaged in sorting and classifying geometric shapes of various colors and sizes. They will discuss the properties of these shapes and illustrate a range of classifications using Venn diagrams.

WHAT'S THE MATHEMATICS?

- Use Venn diagrams to demonstrate simple and compound statements (and/or).
- Include set ideas and terms such as element, subset, intersection, and union.
- Recognize properties of plane figures.
- Solve problems.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Is familiar with assorted two- and three-dimensional objects, including squares, triangles, other polygons, circles, cubes, rectangular prisms, pyramids, spheres, and cylinders.
M5a Formulation
M5b Implementation
M5c Conclusion
M6e Refers to geometric shapes and terms correctly.
M6g Reads and organizes data on charts and graphs, including scatter plots, bar, line, and circle graphs, and Venn diagrams; calculates mean and median.

M7a Uses mathematical language and representations with appropriate accuracy, including numerical tables and equations, simple algebraic equations and formulas, charts, graphs, and diagrams.

VOCABULARY: element member union intersection universal set

SUGGESTED TIME: 1 – 2 lessons

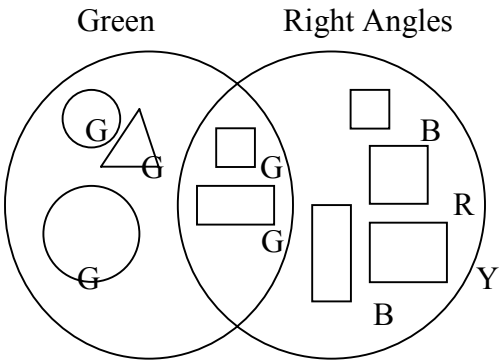
MATERIALS: Attribute blocks or a variety of geometric shapes cut out of construction paper of various colors and cut in a variety of sizes; several lengths of string each with their ends tied together so each length of string can be made to form a circle; index cards

DOING THE INVESTIGATION

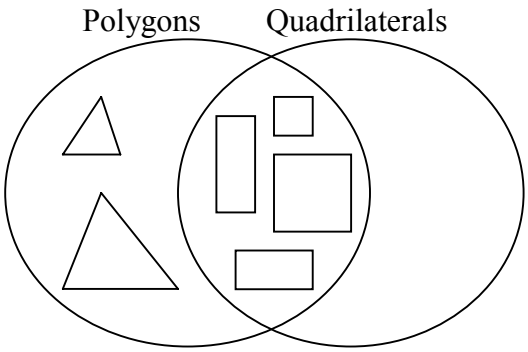
1. Distribute a variety of geometric shapes to each group of students and two of the strings that can be used to form large circles.
2. Ask each group to select one figure and describe its properties to the other groups.
3. Display one of the figures, such as a large red triangle. Ask the students to examine the figures they have and find one that is like the large red triangle in at least one way, i.e., they share at least one common property. (e.g., A red circle is like the red triangle in color, a green triangle is like the red triangle in shape.)
4. Select two distinct properties of the geometric figures, e.g., figures that are green and figures that have right angles. Have each group write each of these classifications on an index card. Ask the groups to sort their figures into these two classifications and place them inside the two string circles, labeling each circle with the appropriate index card.
5. After some discussion, students should realize that there will be some overlap in the two classifications, some green shapes will have right angles and therefore will belong in both groups. How can this overlap be shown?
6. Have the groups share their results by discussing what they put inside each circle and how they showed the shapes that belonged in both groups, i.e., the ones with a property common to both classifications.
7. Explain that we call this kind of illustration a Venn diagram and we use it to identify the elements of a set or sets. It shows what the elements of each set are and what, if anything, they have in common (the intersection of the two circles). If we consider all the elements of both sets together, we are looking at the union of the sets.
8. Have the groups sort their figures again, this time sorting the figures into those that are polygons and those that are quadrilaterals.
9. Have each group write a conclusion about quadrilaterals and polygons based on their Venn diagrams.

POSSIBLE SOLUTIONS

1st Activity:



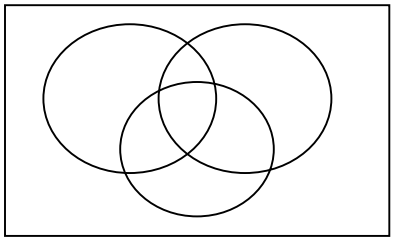
2nd Activity:



Conclusion: All quadrilaterals are polygons.

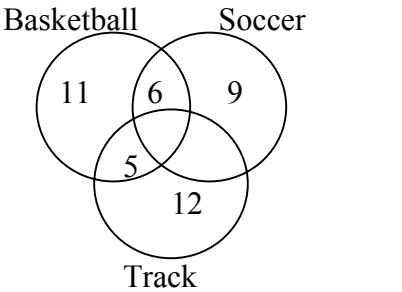
EXTENDING THE ACTIVITY

1. Distribute a copy of the following to each student or group of students:



2. Pose the following problem and ask the students to illustrate the information using the Venn diagram and to write a conclusion they can draw from it:

Winfield School has three sports teams, basketball, soccer, and track. Among the 5th graders, 11 are on the basketball team only, 9 are on the soccer team only, and 12 are on the track team only. Six students play both basketball and soccer and 5 students play basketball and are also on the track team. Fifteen students do not belong to any sports team.



(Many conclusions can be drawn.)

3. Discuss that the universal set includes all the elements, including the 15 students
who play no sports at all.

ASSESSMENT 1

Using the Venn diagram in Extending the Activity on the previous page, determine
how many students at Winfield School are on the track team?

(a) 11 (b) 12 (c) 17 (d) 23

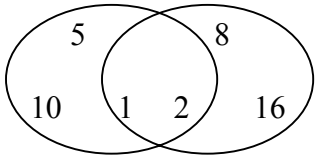
Ans. C

ASSESSMENT 2

a. Draw a Venn diagram that compares all the factors of 20 and 32.
b. Explain the difference between the union of those two sets and the intersection of those two sets.

Ans.

a. Factors of 20 Factors of 32



b. The union of the sets consists of all the factors of both 20 and 32. The intersection shows the factors they have in common.

For additional information and activities, see pages 66, 77, and 78

in the New York State Core Curriculum/Mathematics Resource Guide.
GRADE 5

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and expanded notation).
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.
2C. Develop an understanding of number theory (primes, factors, and multiples).
2D. Recognize order relations for decimals, integers, and rational numbers.

INVESTIGATION

IMPROPER FRACTIONS AND MIXED NUMERALS

How can we rename an improper fraction so it is easier to understand its value? Students will explore the values of improper fractions and their equivalent values renamed as mixed numbers. Pattern blocks or other representative materials will be used to model and compare equivalent values.

WHAT'S THE MATHEMATICS?

- understand proper and improper fractions
- change improper fractions to mixed numbers and vice versa
- compare size of fractions
- add/multiply with fractions
- solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1c Consistently and accurately applies and converts the different kinds and forms of rational numbers.
M5a Formulation
M5b Implementation
M5c Conclusion

M6h Uses recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions.

VOCABULARY: numerator denominator improper fraction mixed number

SUGGESTED TIME: 2 – 3 lessons

MATERIALS: pattern blocks (or other material to represent fractional parts)

DOING THE INVESTIGATION

1. Distribute a supply of pattern blocks to each group of students.
2. Display a yellow hexagon and tell the students that this represents 1 whole. Have the students find a pattern block that is 1/3 of the whole. (blue rhombus) Students must be able to explain why the blue rhombus can be called 1/3 of the hexagon.
3. Assign each group a different number of thirds, e.g., 7 thirds, 8 thirds, 9 thirds, 10 thirds. Ask each group to determine how many hexagons they can build with the number of thirds they have. How many thirds are left over, if any?
4. Have each group describe the process they used to rename their thirds as wholes (hexagons).
5. Have each group write the improper fraction and the equivalent mixed number shown with their pattern blocks.
6. Repeat this process using the hexagon as 1 whole and renaming as sixths using the green triangles.
7. Reverse the process by showing 3 hexagons. Have the groups determine how many sixths would be needed to be equivalent to the 3 wholes? Write the equivalent forms of the number, e.g., 3 = 18/6.
8. Have the groups explore ways to rename improper fractions and mixed numbers without the manipulatives. Have them describe the steps they would take and prove they are correct using the patterns blocks.

POSSIBLE SOLUTIONS

7/3 = 2 wholes and 1/3 left over or 2 1/3
8/3 = 2 wholes and 2/3 left over or 2 2/3
9/3 = 3 complete wholes or 3
10/3 = 3 wholes and 1/3 left over or 3 1/3

Students should be able to generalize that an improper fraction can be renamed as a mixed number by dividing the numerator by the denominator. The quotient represents the number of wholes and the remainder represents the fractional part left over.

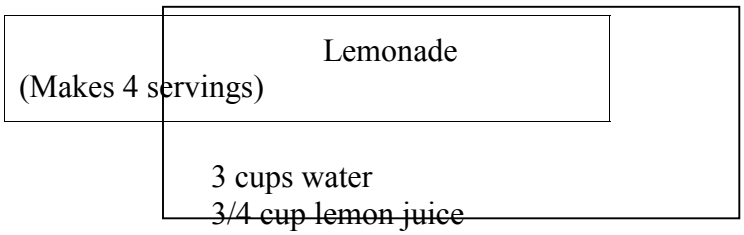
Conversely, the students should be able to explain that a mixed number can be changed to its equivalent improper fraction by multiplying the denominator and the whole number and adding to that product the numerator of the fraction part of the mixed number.

NOTE: Students may devise other equally valid explanations.

EXTENDING THE ACTIVITY

1. Pose the following problem:

Norma is making lemonade for the class picnic. She needs to make enough for 20 servings. How much of each ingredient will she need?



2. Have the students decide how many times Norma has to increase the recipe and what operation they should use to do so. (addition or multiplication)
3. Discuss with the students what the amount of each ingredient should be.
4. If the students did not convert the improper fractions to mixed numbers, discuss why an answer in mixed number form is more appropriate in this case. (It's easier to measure 3 3/4 cups of lemon juice than to think of it as 15/4 cups.)
5. Ask the students to put the ingredients in order from the least amount to the greatest amount and to prove they are correct by indicating each amount on a number line. (3 1/3 cups < 3 3/4 cups < 15 cups)
6. As a challenge, the students may wish to find out whether Norma has made more or less than a gallon of lemonade.

ASSESSMENT 1

Which of the following is not greater than 2 1/3?

a) $\frac{5}{2}$ b) $\frac{6}{2}$ c) $\frac{3}{2}$ d) $\frac{11}{4}$

Ans. c

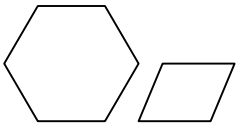
ASSESSMENT 2

Kira said that 1 1/3 is greater than 7/3. Maria said that 7/3 is greater than 1 1/3.

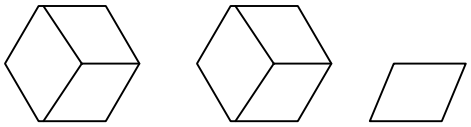
a. Who is correct? (Maria)

b. Use your pattern blocks to draw a picture that proves your answer.

1 1/3



7/3 or 2 1/3



c. Write a sentence to explain the picture you drew.

(7/3 is equal to 2 1/3 which is greater than 1 1/3.)

For additional information and activities, see pages 67, 68, and 79
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 5

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply, and divide fractions, decimals, and integers.
3B. Use grouping symbols (parentheses) to clarify the intended order of operations.
3C. Apply the associative, commutative, distributive, inverse, and identity properties.
3D. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).
3E. Develop appropriate proficiency with facts and algorithms.
3F. Apply concepts of ratio and proportion to solve problems.

INVESTIGATION

STORING CD'S

Reuben has 160 CD's in his collection. Each CD is 1/4" wide. The storage rack he wants to buy has 6 shelves that are each 9 1/2" wide. Will he have enough room for all his CD's on this rack? Show how you know you are correct.

WHAT'S THE MATHEMATICS?

- Add, subtract, multiply, and divide with fractions and mixed numbers.
- Add, subtract, multiply, and divide with whole numbers.
- Solve problems.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Consistently and accurately multiplies and divides rational numbers using appropriate methods.
M5a Formulation
M5b Implementation
M5c Conclusion
M6a Computes accurately with arithmetic operations on rational numbers.

VOCABULARY: length width

SUGGESTED TIME: 2 – 3 lessons

MATERIALS: None

DOING THE INVESTIGATION

1. After reading the problem, have groups of students discuss strategies and develop a plan for solving the problem.
2. Ask a spokesperson from each group to describe the steps his/her group has decided to use to solve the problems. Allow time for feedback on strategies by members of other groups.

   Strategies might include:
   - find out how many CD's fit on one shelf, multiply by 6 shelves and compare that product to 160
   - find out how many CD's fit on one shelf and divide 160 by that number to see how many shelves are needed
   - find the total storage space for the entire rack, then find the total space taken up by 160 CD's and compare the two amounts.
3. Allow time for the groups to arrive at a solution. Groups should compare answers and strategies. Discuss which strategy was most efficient.
4. Students should conclude that the rack would have enough room for Reuben's CD collection.

POSSIBLE SOLUTIONS

A. STEP 1: Find how many CD's fit on one shelf.

Divide the length of one shelf by the width of 1 CD. 9 1/2in. ÷ 1/4in = 38 CD's on 1 shelf	OR	Knowing there are four 1/4 in. in one inch, multiply 9 1/2 in. by 4. 9 1/2 in. x 4 CD's per inch = 38 CD's on 1 shelf

STEP 2: Find out how many CD's fit on 6 shelves.

38 x 6 = 228 CD's on 6 shelves

STEP 3: Compare the two products.
228 is greater than 160, so Reuben's CD's will fit on the rack.

B. STEP 1: Find how many CD's fit on one shelf. (See Step 1 on the previous page.)

STEP 2: Find how many shelves are needed for 160 CD's.
160 CD's ÷ 38 CD's per shelf = 4 complete shelves and 8 CD's on the 5th shelf

STEP 3: Compare number of shelves needed to number of shelves on rack.
The rack will hold his CD's since 5 shelves are needed and the rack has 6 shelves.

C. STEP 1: Find total storage space for entire rack.
9 1/2 in. x 6 shelves = 57 in. of storage space available

STEP 2: Find total storage space needed by 160 CD's.
160 x 1/4 in. = 40 in. of storage space needed

STEP 3: Compare the two products.
57 in. is greater than 40 in., so Reuben's CD's will fit on the rack.

EXTENDING THE ACTIVITY

1. Using the information from the initial investigation and the students' solutions, have them determine how many more CD's Reuben will need to fill his CD rack completely.
2. Have the students discuss appropriate strategies such as the following:
   - Students who solved the investigation using solution A on the previous page need only subtract 160 CD's from the 228 CD's they know will fit on the rack.
   - Students who solved the investigation using solution B must first find the number of CD's on 6 shelves (228) and then subtract 160 CD's from 228 CD's.
   - Students who solved the investigation using solution C may divide the total of 57" of storage space by 1/4" to find that 228 CD's will fit on the rack and then subtract 160 CD's from 228 CD's.
3. Finally, if CD's cost $15.00 each (including tax), what will be the total cost of all the CD's when the rack is full? ($3,420.00)

ASSESSMENT 1

A candy bar weighs 3 1/2 ounces. Ling ate 1/3 of the candy bar. How many ounces of candy did she eat?

a) 3 1/6 oz b) 1 1/6 oz c) 2 2/3 oz d) 10 1/2 oz

Ans. b)

ASSESSMENT 2

For a party Carla wants to order a hero sandwich for her party. The hero sandwiches come in lengths of 3 feet or 6 feet and are cut into pieces that are 1/4 foot long.

a. Which hero should Carla buy to feed 20 people if each person eats only one piece?

b. If there will be some left over, how many pieces will that be?

Show all your work.

Ans. a.

3 ft. ÷ 1/4 ft. = 3 x 4 = 12 slices
6 ft. ÷ 1/4 ft. = 6 x 4 = 24 slices

OR

36 in. ÷ 3 in. = 12 slices (not enough to feed 20 people)
72 in. ÷ 3 in. = 24 slices (enough for 20 people)

Carla needs to buy the 6-foot hero.

b. 24 slices – 20 slices needed = 4 slices left over

For additional information and activities, see pages 69, 80, and 81
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 5

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Visualize, represent, and transform two- and three-dimensional shapes.
4B. Use maps and scale drawings to represent real objects or places.
4C. Use coordinate plane to explore geometric ideas.
4D. Represent numerical relationships in one- and two-dimensional graphs.
4E. Use variables to represent relationships.
4F. Use concrete materials and diagrams to describe the operation of real-world processes and systems.
4G. Develop and explore models that do and do not rely on chance.
4H. Investigate both two- and three-dimensional transformations.
4I. Use appropriate tools to represent and verify geometric relationships.

INVESTIGATION

SYMMETRY

For each pattern block shape, determine whether it has line symmetry, rotational symmetry, or both kinds of symmetry. If the figure has line symmetry, determine how many lines of symmetry it has.

WHAT'S THE MATHEMATICS?

- recognize and demonstrate line symmetry
- recognize and demonstrate rotational symmetry
- recognize and name geometric figures
- understand congruence
- solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Is familiar with assorted two- and three-dimensional objects, including squares, triangles, other polygons, circles, cubes, rectangular prisms, pyramids, spheres, and cylinders.
M2e Recognizes similarity and rotational and bilateral symmetry in two- and three-dimensional figures.
M2k Models situations geometrically to formulate and solve problems.
M5a Formulation
M5b Implementation
M5c Conclusion
M5d Demonstrates mathematical reasoning by generalizing patterns, making conjectures and explaining why they seem true, and by making sensible, justifiable statements.
M6e Refers to geometric shapes and terms correctly.

VOCABULARY: symmetry rotational congruent horizontal vertical

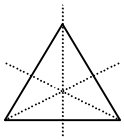
SUGGESTED TIME: 2 – 3 lessons

MATERIALS: pattern blocks, graph paper, rulers, crayons, scissors

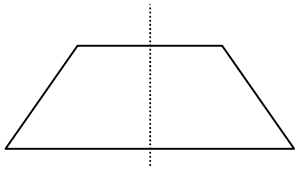
DOING THE INVESTIGATION

1. Give each pair of students two of the pattern block shapes, graph paper, rulers, crayons, and scissors.
2. Display the square and ask the students how we could determine whether it has a line of symmetry using the material we have. (We could trace it onto graph paper and use the ruler to draw a line that we think will be a line of symmetry. Cut the figure out and fold it on the line to see if the 2 parts fit exactly on one another.)
3. Have the students discuss a strategy for finding out if the square has rotational symmetry. (We could trace the square, cut it out and lay it on top of the pattern block square. Rotate it around a point at the center until a different vertex is at the top. If it fits exactly, i.e. is congruent, when a different vertex is at the top, it has rotational symmetry.)
4. Have each pair of students work together to determine whether the two pattern block shapes they were given have line symmetry and/or rotational symmetry. If a shape has line symmetry, the students should find out how many lines of symmetry it has.
5. Have students share the results of their investigations. They should examine each other's drawings and discuss generalizations about lines of symmetry.

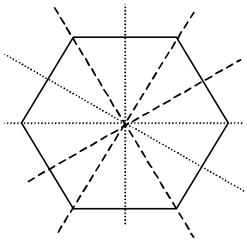
POSSIBLE SOLUTIONS



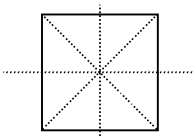
Line symmetry and rotational symmetry



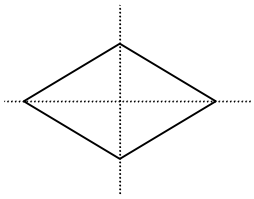
Line symmetry



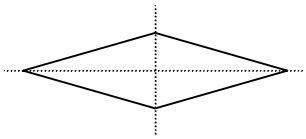
Line symmetry and rotational symmetry



Line symmetry and rotational symmetry



Line symmetry and rotational symmetry



Line symmetry and rotational symmetry

Generalizations may include: Equilateral triangles have 3 congruent sides and angles and 3 lines of symmetry; squares have 4 congruent sides and angles and 4 lines of symmetry, etc.

EXTENDING THE ACTIVITY

1.	Give each student a set of the 6 different pattern blocks, graph paper, crayons, and a ruler.
2.	Have the students draw a vertical line down the center of the graph paper. This will be a line of symmetry. On one side of the line, each student will use the pattern blocks to create a geometric design and color it.
3.	Collect all the designs and place them in a large box. Have each student select a design from the box (not his/her own) and create the symmetrical design on the other side of the line.

4. The designs may be taped together to form a large "quilt" of symmetrical designs that can be displayed in the room.

ASSESSMENT 1

How many of the letters below have both a horizontal and a vertical line of symmetry?

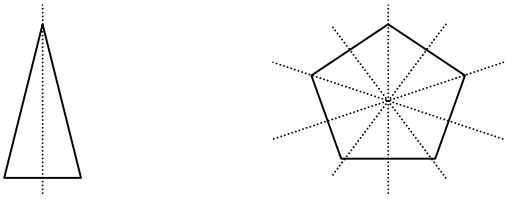
A E I O U

a) 5 b) 4 c) 3 d) 2

Ans. d

ASSESSMENT 2

a. Examine the isosceles triangle and the pentagon below. Draw as many lines of symmetry as you can for each.



(Lines of symmetry shown as dashed lines indicate answers.)

b. Predict which of the above shapes will also have rotational symmetry.

(the pentagon)

c. Explain why you believe your prediction is correct.

(Accept reasonable answers, e.g., the pentagon has more than one line of symmetry so I predict it will also have rotational symmetry OR the pentagon has 5 congruent sides and angles so I predict it will have rotational symmetry.

For additional information and activities, see pages 70, 71, 82, and 83
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 5

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Estimate, make, and use measurements in real-world situations.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.
5C. Develop measurement skills and informally derive and apply formulas in direct measurement activities.
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.
5E. Explore and produce graphic representations of data. (Calculators/computers may be used.)
5F. Develop critical judgment for the reasonableness of measurement.

INVESTIGATION

JESSICA'S AQUARIUM

Jessica's aquarium is 50 cm long, 20 cm wide, and 30 cm high. She wants to fill the bottom of the aquarium with gravel to a height of 4 cm.
How can we determine the amount of gravel she will need?

WHAT'S THE MATHEMATICS?

- Use a model to find the volume of a rectangular prism.
- Develop a formula to find the volume of a rectangular prism.
- Operate with units of measurement.
- Solve problems.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Determines and understands length, area, and volume (as well as the differences among these measurements), including perimeter and surface area; uses units, square units, and cubic units of measure correctly; computes areas of rectangles, triangles, and circles; computes volumes of prisms.
M2g Measures angles, weights, capacities, times, and temperatures using appropriate units.

M2k Models situations geometrically to formulate and solve problems.
M5a Formulation
M5b Implementation
M5c Conclusion
M6a Computes accurately with arithmetic operations on rational numbers.
M6d Measures length, area, volume, weight, time, and temperature accurately.
M6h Uses recall, mental computations, pencil and paper, measuring devices, mathematics texts, manipulatives, calculators, computers, and advice from peers, as appropriate, to achieve solutions.

VOCABULARY: volume capacity cubic centimeter (cm^3) milliliter

SUGGESTED TIME: 2 – 3 lessons

MATERIALS: centimeter cubes

DOING THE INVESTIGATION

1. Present the problem to the students. Have centimeter cubes available for the groups to use.
2. Discuss the measurements of the space the gravel will take up (50 cm by 20 cm by 4 cm) and tell the students that when we want to know how much space is in a rectangular prism, we are looking for the volume.
3. Have the students think about possible strategies they might use to find the volume of the space that will be filled by gravel; e.g., using the centimeter cubes to fill the space the gravel would take up. Why would this not be practical? (You would need too many centimeter cubes.)
4. Elicit that we could solve a simpler problem by making the dimensions smaller, e.g., 5 cm long by 2 cm wide by 2 cm high and see how we can apply what we find to solving the larger problem. Another strategy might be to use what we know about finding the area of a rectangle and figuring out how to account for the added dimension of height.
5. Students should use the cubes to model the problem and determine that, whether they built a smaller model or used their knowledge of area as length times width, the space inside a rectangular prism can be determined by multiplying the length times the width times the height. Jessica will need 4000 cm^3 of gravel.
6. Students may wish to verify the formula by using the cubes to build other models of rectangular prisms, counting the cubes used, and then checking it with the formula.
7. Discuss the unit of measure when finding volume, in this case, cubic centimeters. How is a cubic centimeter different from a square centimeter? (A square centimeter is 1 cm long and 1 cm wide. A cubic centimeter is 1 cm long, 1 cm wide, and 1 cm high.)

POSSIBLE SOLUTIONS:

a. Solve a simpler problem by building a rectangular prism that is smaller than the one in the original problem, e.g., one that is 5 cm by 2 cm by 2 cm. Count the number of cubes to find that the volume of this model is 20 cubes. Students should determine that the dimensions of the figure, when multiplied together, also produces an amount of 20 cm^3. The students can conclude that the formula volume = l x w x h can be used to find the volume (space inside) a rectangular prism.

OR

b. Students may use what they know about area to solve the problem. Multiplying length times width tells us that the number of cubes needed to cover one layer along the bottom of the tank will be 50 x 20 or 1000. Because these cubes are 1 cm high, we will need 4 layers like the original one to fill a space 4 cm high. Therefore, 4000 cubes would be needed to fill the space of the gravel. Jessica would need enough gravel to fill 4000 cubic centimeters.

EXTENDING THE ACTIVITY

1. After Jessica puts the gravel in her aquarium, how much water will she need to fill the aquarium up to 3 cm from the top?
2. Have the students discuss strategies they might use;
   e.g.,
   - Find the volume by subtracting 7 cm from the height of the of the aquarium (4 cm for the height of the gravel and 3 cm for the room left at the top without water) and multiply the new height of 23 cm with the other two dimensions. 50 cm x 20 cm x 23 cm = 23,000 cm^3.
   - Find the volume of the aquarium: 50 cm x 20 cm x 30 cm = 30,000 cm^3. Find the volume of the space at the top of the aquarium without water and add that amount to the 4,000 cm^3 of gravel:
     50 cm x 20 cm x 3 cm = 3,000 cm^3
     3,000 cm^3 + 4,000 cm^3 = 7,000 cm^3
   - Finally, subtract 7,000 cm^3 from the total volume (30,000 cm^3) to find the volume of 23,000 cm^3.
3. Have the students recall that 1 cm^3 holds 1 mL of water. Therefore, a volume of 23,000 cm^3 will hold 23,000 mL of water or 23 liters of water.
4. Discuss reasons why this measure of volume will be approximate.

ASSESSMENT 1

The sandbox at the playground is 4 m long, 3 m wide, and 1/2 m high. How much sand is needed to fill this sandbox?

a) 24 m^3 b) 12 m^3 c) 6 m^3 d) 7 1/2 m^3

Ans. c)

ASSESSMENT 2

The Acme Box Company wants to construct a new box with a volume of 24 in^3. They need to decide what the dimensions of this box should be. Find two different sets of dimensions for this box.

Ans. 6 in x 2 in x 2 in = 24 in^3 OR 4 in x 3 in x 2 in = 24 in^3
Accept other reasonable responses.

For additional information and activities, see pages 72, 73, and 84
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 5

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Use estimates to check the reasonableness of results obtained by computation, algorithms, or the use of technology.
6B. Use estimation to solve problems for which exact answers are inappropriate.
6C. Estimate the probability of events.
6D. Use simulation techniques to estimate probabilities.
6E. Determine probabilities of independent events.

INVESTIGATION

WHAT'S IN THE BAG?

Students will explore experimental probability to determine what color cubes are in a bag. Without looking, they will pick and tally cubes to draw conclusions about what is in the bag.

WHAT'S THE MATHEMATICS?

- make predictions based on sample data
- conduct and predict outcomes of experiments with independent events
- express probability as a fraction, a decimal, and/or a percent
- solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Collects data, organizes data, and displays data with tables, charts, and graphs that are appropriate, i.e., consistent with the nature of the data.
M4d Makes conclusions and recommendations based on data analysis.
M4h Represents and determines probability as a fraction of a set of equally likely outcomes and constructs sample spaces (including those described by numerical combinations and permutations).
M41 Makes predictions based on experimental or theoretical probabilities.
M4j Predicts the result of a series of trials once the probability for one trial is known.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY:

likely	impossible	favorable outcomes
unlikely	certain	theoretical probability
possible	sample space	experimental probability

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: colored cubes or tiles, paper bag

DOING THE INVESTIGATION

1. Display a wide variety of colored cubes or tiles.
2. Without showing the students, place 3 blue cubes, 1 white cube, and 1 red cube in the bag.
3. Tell the students there are 5 cubes in the bag. Ask the students what colors the cubes might be. (Accept all reasonable responses.)
4. Can we tell for certain whether any of your guesses are correct? Discuss with the students how we might be able to make a reasonable prediction about the colors of the cubes in the bag. Elicit from the students that we could pick and replace one cube at a time and tally the outcomes.
5. Perform the experiment where each student picks a cube, states the color, and replaces it in the bag. One student tallies all the outcomes on the board or on a chart. Repeat until 50 – 60 trials have been done.
6. Examine the results of the experiment as shown on the tally chart. How many times did each color come out?
7. Based on this data, have the students predict the colors of the 5 cubes in the bag and justify their responses. Check their predictions by actually looking at what was in the bag.
8. Compare the experimental probability of each color with the theoretical probability.
   e.g., Theoretical probability: P(white) = 1/5
   Experimental probability: Answers will vary depending on how many times white actually was chosen in the experiment.
9. Have the students express the theoretical probability of each color in the sample space as a fraction, a decimal, and a percent. (P(white) = 1/5 or 0.2 or 20%)
10. Show the students 3 green cubes and 1 yellow cube. Ask them to predict the likelihood of picking the yellow cube from a bag containing those 4 cubes. They should be able to express the answer as a fraction, a decimal, and a percent. (P(yellow) = 1/4, 0.25, or 25%)
11. Ask the students to predict how many times they think yellow would be picked in 20 trials, in 50 trials, and in 100 trials. Students should justify their responses.
12. Students may conduct the actual experiment and compare the results with their predictions.

POSSIBLE SOLUTIONS

1st Activity: Students should conclude that there are probably 3 blue cubes, 1 red cube, and 1 white cube in the bag. Conclusions should be based on the tallied outcomes.
P (white) = 1/5, 0.2, or 20% P(red) = 1/5, 0.2, or 20%
P(blue) = 3/5, 0.6, or 60%

2nd Activity: P(yellow) =1/4, 0.25, or 25%
Students should conclude that since yellow has a probability of 1/4, it will probably be picked about 5 times in 20 trials and about 25 times in 100 trials.

EXTENDING THE ACTIVITY

1. Prepare a variety of colored cubes or tiles, Ziploc bags, and a task card such as the following for each group:

> *Place cubes in a bag so that the probability of picking a red cube is twice as likely as the probability of picking a blue cube.*

2. Give a task card, cubes, and a Ziploc bag to each group of students. Have the students decide on what cubes to put in the bag so that the conditions on the task card are met. For the task card above, a group might put 2 red cubes and 1 blue cube, or 4 red cubes and 2 blue cubes, etc.

3. Have each group trade bags with another group and try to determine what the other group's task was. Students will find that many responses are possible, but only one will match the task. For example, using the above task, a group may put 4 red and 2 blue cubes in the bag. The other group may suggest that the task was to put cubes in the bag so that the probability of picking blue is 2 out of 6. Another response may be that the cubes were put in the bag so that it was more likely to pick a red cube. These responses are true based on the cubes in the bag, and should be discussed, but groups should continue until they figure out the task on the card.

ASSESSMENT 1

On a regular die, with faces from 1 through 6, what is the probability of rolling a composite number?

a) 0/6 b) 2/6 c) 6/6 d). 3/6

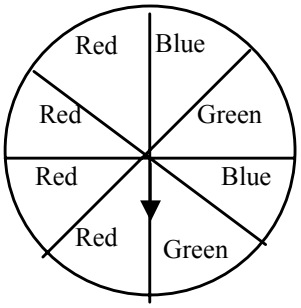
Ans. b) (The composite numbers are 4 and 6. 1 is neither prime nor composite.)

ASSESSMENT 2

Part A: Design a three-color spinner where one color is twice as likely to occur as each of the other two colors.

Part B: What is the probability of each color occurring?

Ans. Part A: Spinners may vary, one solution is shown at the right.



Part B: P(red) = 4/8 or 1/2 or 50%

P (blue) = 2/8 or 1/4 or 25%

P (green) = 2/8 or 1/4 or 25%

For additional information and activities, see pages 74 and 85
in the Core Curriculum/Mathematics Resource Guide

GRADE 5

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns.

PERFORMANCE INDICATORS:

7A. Recognize, describe, and generalize a wide variety of patterns and functions.
7B. Describe and represent patterns and functional relationships, using tables, charts and graphs, and verbal descriptions.
7C. Develop methods to solve basic linear equations.
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.
7E. Apply the concept of similarity in relevant situations.
7F. Use properties of polygons to classify them.
7G. Explore relationships involving points, lines, angles, and planes.
7H. Develop readiness for basic concepts of right triangle trigonometry.
7I. Use patterns and functions to represent and solve problems.

INVESTIGATION

SQUARE NUMBERS AND TRIANGULAR NUMBERS

Students will use pattern blocks to build increasingly larger squares and triangles. They will record the growth patterns on a chart and look for corresponding numerical patterns. The students will describe the patterns they find and use them to predict the size of larger squares and triangles without building them.

WHAT'S THE MATHEMATICS?

- recognize and extend geometric and numerical patterns
- identify square numbers and triangular numbers
- use a chart to record data
- solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2f Analyzes and generalizes geometric patterns, such as tessellations and sequences of shapes.
M2k Models situations geometrically to formulate and solve problems.
M5a Formulation

M5b Implementation
M5c Conclusion
M5d The student demonstrates mathematical reasoning by generalizing patterns, making conjectures and explaining why they seem true, and by making sensible, justifiable statements.
M6e Refers to geometric shapes and terms correctly.

VOCABULARY: square numbers triangular numbers

SUGGESTED TIME: 3 – 4 lessons

MATERIALS: pattern blocks

DOING THE INVESTIGATION

1. Distribute pattern blocks to each group of students and ask them to display one orange square.
2. Have each group use additional orange squares to build the next larger square.
   e.g., 
3. How many small squares were added to the original square to form the new, larger square? (3) How many small squares make up the new, larger square in all? (4) Have the students keep track of this information on a chart such as the following:

Square	No. of small squares added	Total number of small squares
1st	1	1
2nd	3	4

4. Have the groups build the next four larger squares and continue to record the growth information on the chart.
5. Have the students examine the numbers on the chart and discuss any patterns they notice.
   Provide time for the groups to share patterns they found with each other. Tell the students that the numbers in the third column are called square numbers. Ask why they think this is so.
6. Without having the students build any additional squares, ask them to tell how many squares they would have to add to the 6^{th} square to make the 7^{th} square. (13) How many small squares will there be in all in the 7^{th} square? (49) Have them continue the chart to the 10^{th} larger square. Discuss how the number patterns they noticed

helped them to predict what the next larger squares would look like.

Repeat this investigation using the green triangles to build increasingly larger triangles and describing the patterns found. Note that in building larger triangles, space is left between the smaller triangles in each new row, e.g.,

NOTE: If all the spaces are filled in, you will get a pattern of square numbers instead of

triangular numbers.

POSSIBLE SOLUTIONS:

Square	No. of small squares added	Total no. of small squares used	no. of small triangles used
1st	1	1	1
2nd	3	4	3
3rd	5	9	6
4th	7	16	10
10th	19	100	55

Triangle	No. of small triangles added	Total
1st	1	
2nd	2	
3rd	3	
4th	4	
10th	10	

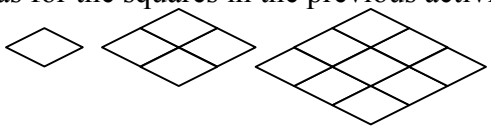
Patterns on the chart of increasingly larger squares may include the following as well as others:

- The 2nd column contains consecutive odd numbers beginning with 1.
- The 3rd column contains consecutive square numbers.
- Numbers in the 3rd column can be achieved by multiplying the corresponding number in the 1st column by itself.

Patterns on the chart of increasingly larger triangles may include the following as well as others:

- The second column contains consecutive whole numbers beginning with 1.
- The number of triangles added in each new row is the same as the number of the triangle in column 1.
- The number of triangles needed to form a given triangle is equal to the sum of consecutive whole numbers beginning with 1 and continuing to the given number.

EXTENDING THE ACTIVITY

1. Have the students use the blue rhombus pattern block as the base shape and build increasingly larger rhombi, recording the growth pattern on a chart as in the earlier investigation.
2. Students should discover that the pattern for increasingly larger rhombi is the same as for the squares in the previous activity.
   e.g., 

ASSESSMENT 1

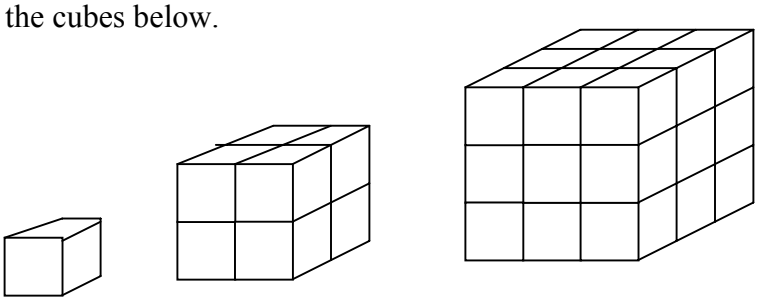
What number is missing from the pattern below?

1 3 7 15 ____ 63

a) 31 b) 21 c) 48 d) 26 Ans. a)

ASSESSMENT 2

Look at the cubes below.



a. How many small cubes would be needed to build the next larger cube? Ans. 64

b. Explain the mathematics you used to find your answer.

(Accept reasonable responses which indicate using the formula for finding the volume of a rectangular prism or which explain and continue the pattern of cubes.)

For additional information and activities, see pages 75, 76, 86, and 87 in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 6

KEY IDEA # 1: MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence and construct an argument.

PERFORMANCE INDICATORS:

1A. Apply a variety of reasoning strategies.
1B. Make and evaluate conjectures and arguments, using appropriate language.
1C. Make conclusions based on inductive reasoning.
1D. Justify conclusions involving simple and compound (i.e., and/or) statements.

INVESTIGATION

Justin and Daniel's Puzzles

Justin and Daniel were playing with the same 5 tangram pieces. Justin made a rectangle with his pieces. Daniel made a square with his pieces. Compare the areas and perimeters of Daniel's and Justin's shapes.

WHAT'S THE MATHEMATICS?

- understanding area of a rectangle
- understanding perimeter of a rectangle
- understanding area of a square
- understanding perimeter of a square
- using inductive reasoning
- using geometric figures to understand area and perimeter

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2d Determine and understand length, area, and volume.
M5a Formulation
M5b Implementation
M5c Conclusion
M5d Mathematical Reasoning
M6e Refer to geometric shapes and terms correctly.
M7d Exhibit developing reasoning abilities by justifying statements and defending work.
M7e Show understanding of concepts by explaining ideas to others.

VOCABULARY: tangrams area perimeter rectangle square

SUGGESTED TIME: 2 – 3 lessons

MATERIALS: Tangrams, Grandfather Tang's Story, graph paper

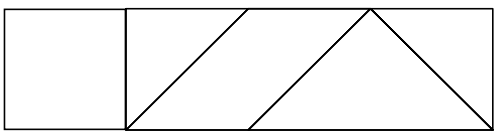
DOING THE INVESTIGATION

1. Read Grandfather Tang's Story.
2. Distribute 2 sets of tangrams to each group of 4 students. (Remove the 2 large triangles)
3. One pair in each group should use the tangram pieces to make a rectangle and the other pair should make a square.
4. Have students trace their shapes onto graph paper.
5. Elicit from students similarities in their two shapes (both are quadrilaterals, 4 right angles, same areas.)
6. Elicit from students differences in the two shapes (one has 4 equal sides and the other has 2 pairs of equal sides, different perimeters.)
7. Discuss meaning of area and perimeter.
8. Discuss the area and perimeter of each shape.
9. Elicit how we know the areas are the same (count all the squares, same pieces can only cover the same area.)
10. Make a rule for finding the perimeter of squares and rectangles.
11. Make a rule for finding the area of squares and rectangles.

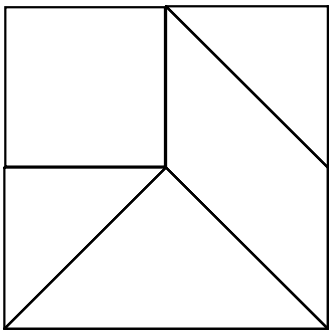
POSSIBLE SOLUTIONS

1. Use rulers.
2. Count boxes on the graph paper.
3. Show how all the shapes relate by using units (the small triangle is one unit for area, the side of square is a unit of length).
   Area = 8 square units (both shapes).
   Perimeter = 8 units (square) and 10 units (rectangle).

4a. Justin's puzzle



4b. Daniel's puzzle



EXTENDING THE ACTIVITY

1. Give students graph paper.
2. Have students draw rectangles with areas of 8 square units and show all of the possible perimeters. (P = 12, P = 18).
3. Repeat with areas of 14 (P =18, P = 30) and 20 (P = 18, P = 24, P = 42).
4. Exchange papers to check each other's perimeters.

ASSESSMENT 1

How many different rectangles can you draw that have a perimeter of 16 inches? (Use whole inches only)

a. 1 b. 4 c. 16 d. 2 Ans. b

ASSESSMENT 2

Peter has to put up a rectangular fence in his backyard to make a place for his new dog to run around. His father gave him 180 feet of fencing. What are the largest possible dimensions for the play area for Peter's dog? Show your work or explain in words how you determined the dimensions. Be sure to include the area where the dog can play in your work.

The answer is a square that is 45 feet on each side. The area is 2025 square feet.
Accept any reasonable explanation such as guess and check, drawing a picture, etc.

For addition information and activities see pages 66,77, and 78
in the New York State Core Curriculum/Mathematics Resource Guide.

KEY IDEA # 2: NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and expanded notation).
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.
2C. Develop an understanding of number theory (primes, factors, and multiples).
2D. Recognize order relations for decimals, integers, and rational numbers.

INVESTIGATION

THE m & m MYSTERY

How can you find out what percent of your favorite color of m & m's is in a bag of m & m's?

WHAT'S THE MATHEMATICS?

- interpreting percent
- displaying data in the form of a circle graph
- displaying data in the form of a bar graph
- understanding how to convert decimals, fractions and percents

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1e Interpret percent as part of 100.
M4a Organize and display data.
M5a Formulation
M5b Implementation
M5c Conclusion
M6g Read and organize data on charts and graphs.

VOCABULARY: circle graphs bar graphs percent

SUGGESTED TIME: 2 - 3 lessons

MATERIALS: small bags of m & m's, graph paper, drawing paper, crayons

DOING THE INVESTIGATION

1. Distribute a small bag of m & m's to each student.
2. Have students count out 25 m & m's without looking and eat the rest.
3. Have students separate m & m's by color and make a circle with them on the drawing paper.
4. Trace the circle of m & m's.
5. Using crayons, have the students color each fractional part of the circle with the appropriate color.
6. Have students write each fraction in on the circle graph, making sure it's labeled. List the fractions on the paper also.
7. Elicit that the fractions must add up to 25/25.
8. Elicit a definition of percent.
9. Have each student pick the color with the most m & m's.
10. Elicit if any student has half a circle of that color.
11. Discuss why not. (You can't have 12 1/2 of one color.)
12. Elicit if any student has 50% of his circle that color.
13. Discuss why not.
14. Elicit that 50% and 1/2 are the same.
15. Elicit how to find percent when the denominator is 25.
16. Have student convert their fractions to percents.
17. Elicit the students favorite colors.
18. Elicit the percent of their favorite color of m & m's.

POSSIBLE SOLUTIONS

Graphs will vary depending on the contents of each bag of m & m's.

EXTENDING THE ACTIVITY

1. Have students draw bar graphs to match their m & m circle graphs using percent.
2. Make a class circle graph.
3. Elicit how to figure out percent for the class graph.

ASSESSMENT 1

There are 5 black, 4 white, 3 red, 2 blue and 6 green marbles in a bag. What percent of the marbles are green?

a). 30% b) 6% c) 20% d). 60% Ans. a

ASSESSMENT 2

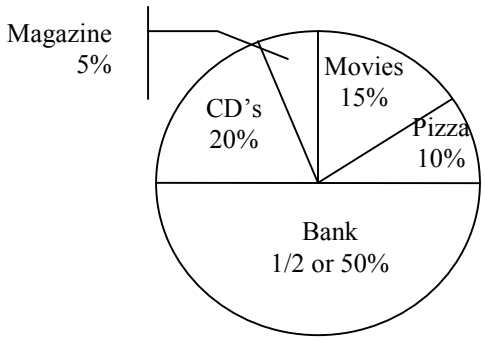
Jesse got $50 for his birthday. He put 1/2 in the bank. He spent 20% on CD's. He went to the movies and spent 15% there. He spent 5% on a magazine and the rest he spent on pizza.

Part A: Draw a circle graph showing how Jesse spent his birthday money using percents.

Part B: Draw a conclusion based on the circle graph

Ans. Accept all reasonable answers.

Ans. Part A Jesse's Spending



For additional information see pages 67,68 and 79
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 6

KEY IDEA # 3: OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply, and divide fractions, decimals, and integers.
3B. Use grouping symbols (parentheses) to clarify the intended order of operations.
3C. Apply the associative, commutative, and distributive properties, and inverse and identity elements.
3D. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).
3E. Develop appropriate proficiency with facts and algorithms.
3F. Apply concepts of ratio and proportion to solve problems.

INVESTIGATION

Using the rules you know for order of operations, solve the following number sentence.

$5 \times 6 - 8 \times 2 \div 4 + 4^2 =$ ☐

Add parentheses to the original equation to find two additional solutions.

WHAT'S THE MATHEMATICS?

- use conventional order of operations
- add, subtract, multiply and divide whole numbers
- operate with exponents

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Consistently and accurately add, subtract, multiply, and divide rational numbers.
M5a Formulation
M5b Implementation
M5c Conclusion
M6a Compute accurately with arithmetic operations on rational numbers.
M6b Know and use the correct order of operations for arithmetic computations.
M6f Use equations, formulas, and simple algebraic notation appropriately.

M6h Use calculators, as appropriate, to achieve solutions.
M7b Organize work, explain a solution orally and in writing, and use other techniques to make meaning clear to the audience.
M7d Exhibit developing reasoning abilities by justifying statements and defending work.

VOCABULARY: exponent base parentheses

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: calculators

DOING THE INVESTIGATION

1. Present the equation for solution.
2. During your discussion of the correct answer, elicit that the students needed to use the rules for order of operations.
3. Discuss the importance of understanding the rules for order of operations so that we can always get the same answer.
4. Give students time to study the equation without the solution.
5. Students should place parentheses in that equation to come up with two different solutions.

POSSIBLE SOLUTIONS

Activity 1

$5 \times 6 - 8 \times 2 \div 4 + 4^2 = \_\_\_$
$5 \times 6 - 8 \times 2 \div 4 + 16 = \_\_\_$
$30 - 16 \div 4 + 16 = \_\_\_$
$30 - 4 + 16 = \_\_\_$
$26 + 16 = 42$

Activity 2

1) $5 \times (6 - 8) \times 2 \div 4 + 4^2 = 11$
2) $5 \times 6 - 8 \times 2 \div (4 + 4^2) = 29\ 1/5$

EXTENDING THE ACTIVITY

Have students write five number sentences that equal 24. Each number sentence must use the conventional order of operations. The first sentence must use two numerals. The second sentence must use three numerals. The third sentence must use four numerals. The fourth sentence must use five numerals and the fifth sentence must use six numerals. (Students may use the same numeral more than once.)

One possible sets of solutions is:

1) $8 \times 3 = 24$
2) $12 \times 3 - 12 = 24$
3) $6^2 \div 4 \times 2 + 6 = 24$
4) $(10 - 4) \times 8 \div 4 + 12 = 24$
5) $4 + 4 + 4 + 4 + 4 + 4 = 24$

Accept all possible solutions.

ASSESSMENT 1

What is the solution to the following equation:

$$6^2 \times (4 - 2) - 3 \times 2 = \square$$

a) -36 b) 278 c) 66 d) 272 Ans. c)

ASSESSMENT 2

Part A: Place the parentheses in the correct place to make the statement true.

$4 \times 10 - 8 + 6 \div 2 - 4 = 29$

Part B: Explain your answer.

Ans. $4 \times 10 - (8 + 6) \div 2 - 4 = 29$
Accept all reasonable explanations.

For additional information and activities, see pages 69, 80, and 81
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 6

KEY IDEA # 4: MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Visualize, represent, and transform two- and three-dimensional shapes.
4B. Use maps and scale drawings to represent real objects or places.
4C. Use the coordinate plane to explore geometric ideas.
4D. Represent numerical relationships in one- and two-dimensional graphs.
4E. Use variables to represent relationships.
4F. Use concrete materials and diagrams to describe the operation of real-world process and
Systems.
4G. Develop and explore models that do and do not rely on chance.
4H. Investigate both two- and three-dimensional transformations.
4I. Use appropriate tools to represent and verify geometric relationships.

INVESTIGATION

ADRIENNE'S PATTERN BLOCK ANGLES

Adrienne took a bucket of pattern blocks and removed one of each color. She couldn't find her protractor but wanted to know the measure of each angle on each pattern block. Can you help her find the angle measurements by just using your pattern blocks?

WHAT'S THE MATHEMATICS?

- knowledge of properties of polygons
- knowledge of right angles
- use of concrete materials to measure angles

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Be familiar with assorted two- and three-dimensional objects.
M2g Measure angles, weights, capacities, times, and temperature using appropriate units.
M2j Reason proportionally with measurements.
M5a Formulation
M5b Implementation
M5c Conclusion
M6d Measure accurately.

M6e Refer to geometric shapes and terms correctly.
M7b Organize work, explain a solution orally and in writing, and use other techniques to make meaning clear to an audience.

VOCABULARY: right angle, acute angle, obtuse angle, opposite angles, square, triangle, trapezoid, parallelogram, hexagon

SUGGESTED TIME: 2 - 3 lessons

MATERIALS: Pattern blocks

DOING THE INVESTIGATION

1. Distribute one of each pattern block to each student.
2. Elicit the name of each shape.
3. Elicit complete definition of a square (making sure to include that it contains 4 right angles which measure 90° each.)
4. Elicit that all quadrilaterals contain 360°.
5. Discuss what shapes make a quadrilateral. (Two triangles always make a quadrilateral.)
6. Elicit that the sum of the measure of the angles in a triangle always equals 180° because two triangles put together form a quadrilateral.
7. Allow students to discover that the pattern block triangle is equilateral.

POSSIBLE SOLUTIONS

Square: 90° in each angle
Triangle: 60° in each angle
Hexagon: 120° in each angle
Trapezoid: base angles are 60°, other angles are120°
Tan Parallelogram: 30°, 150°
Blue Parallelogram: 60º, 120º

Allow students to explore and explain how they arrived at their solutions.

EXTENDING THE ACTIVITY

1. Discuss concept of regular polygons with students.
2. Allow students to discover the angle measurement of regular polygons with 3-8 sides.
3. Review with students the knowledge that triangles contain 180º is important to their discovery.
4. Lead them to come up with a chart such as the following:

# sides	angle measurement	sum of angle measures
3	60º	180º
4	90º	360º
5	108º	540º

6	120º	720º
7	128.57º	900º
8	135º	1080º
n	(n-2)(180º)/n	(n-2)180º

ASSESSMENT 1

If the yellow hexagon is 1 whole, which pattern blocks would you use to model the sentence 2/3 + 1/6?

a) two oranges, 1 tan
b) two oranges, 1 green
c) two reds, 1 green
d) two blues, 1 green

Ans. d)

ASSESSMENT 2

Part A: Choose two pattern blocks. Place them next to each other so that the sides touch completely. Trace around the new figure. What is the name of the shape you made and what is the sum of the measures of the angles?

Part B: Explain or show how you got your answers.

Ans. Accept all reasonable responses.

KEY IDEA # 5: MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Estimate, make, and use measurements in real-world situations.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.
5C. Development measurement skills and informally derive and apply formulas in direct measurement activities.
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.
5E. Explore and produce graphic representations of data. (Calculators/computers may be used.)
5F. Develop critical judgement for the reasonableness of measurement.

INVESTIGATION

HOW TALL IS THE AVERAGE SIXTH GRADER?

Students will measure each other and record the data in inches. Students will use a stem and leaf plot to display data. Figure out the average height of the students in your class.

WHAT'S THE MATHEMATICS?

- mean
- median
- mode
- add, subtract, multiply and divide whole numbers
- stem and leaf plot

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Consistently and accurately add, subtract, multiply, and divide rational numbers.
M4a Organize and display data.
M4b Analyze data with respect to frequency and distribution.
M4c Analyze central tendencies of data.
M4d Make conclusions and recommendations based on data analysis.
M5a - Formulation

M5b - Implementation
M5c - Conclusion
M6a - Compute accurately with arithmetic operations on rational numbers.
M6g - Read and organize data on charts and graphs.

VOCABULARY: mean median mode average data stem and leaf plot

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: yard sticks, tape measures, meter sticks, calculators

DOING THE INVESTIGATION

1. Introduce the problem.
2. Elicit that the mean height is the average height.
3. Have each student measure his/her partner using a tape measure or yard stick. (Use total inches.)
4. Organize data on a stem and leaf plot on chart paper for the class.

e.g.,

stem	leaf
5	5 5 6 7
6	0 0 0 8 9
7	1 1

(Using 11 students)

(Stem is tens place and the leaf is the ones place for height in inches.)

5. Elicit the average (MEAN) height of the students.

POSSIBLE SOLUTIONS

Answer for the example is 62 inches. Answers will vary from class to class.

EXTENDING THE ACTIVITY

Extension 1: Eliciting the median and mode of the data.

1. Elicit from students which height appears most often. (In example the answer is 60 inches. Answer in your classes will vary.)
2. Discuss that this number is the MODE.
3. Elicit the importance of understanding the mode. (e.g., knowing what you need to order if you work in a store or restaurant.)
4. Elicit from the students which height is the middle. (in the example above, 60 inches.)
5. Discuss that this number is the MEDIAN.
6. Discuss the difference between mean, median and mode.

Extension 2: Do this same activity using metric measurement.

ASSESSMENT 1

Your math scores so far this year have been: 80, 100, 92 and 87. What is the lowest score you can get on your final test in order to still receive an A. (A is 90% or more)

a) 90 b) 85 c) 91 d) 89 Ans. c)

ASSESSMENT 2

Part A: Organize the following test scores on a stem and leaf plot:
86, 43, 92, 67, 90, 87, 86, 71, 94, 73, 90, 86, 77, 71, 100.

Part B: What is the mean, mode, and median of these test scores?

Ans. a)

stem	leaf
4	3
6	7
7	1 1 3 7
8	6 6 6 7
9	0 0 2 4
10	0

Ans. b) Mode = 86
Median = 86
Mean = 80.87

For additional information and activities, see pages 72, 73, and 84 in the Core Curriculum/Mathematics Resource Guide.

KEY IDEA # 6: UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS:

6A. Uses estimation to check the reasonableness of results obtained by computation, algorithms or the use of technology.
6B. Use estimation to solve problems for which exact answers are inappropriate.
6C. Estimate the probability of events.
6D. Use simulation techniques to estimate probabilities.
6E. Determine probabilities of independent events.

INVESTIGATION

STACI'S NEW CAR

Staci and her parents went shopping for a new car. The following choices were available:

2 door or 4 door
Blue, white, red or silver
CD player or cassette

How many different choices did they have?

WHAT'S THE MATHEMATICS?

- use tree diagrams to find all possible combinations
- make organized lists
- discover the multiplication principle

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4a Organize and display data.
M4h Represent and determine probability, recognize equally likely outcomes, and construct sample spaces.
M5a Formulation.
M5b Implementation
M5c Conclusion.

VOCABULARY: arrangement combinations tree diagram multiplication principle

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: None

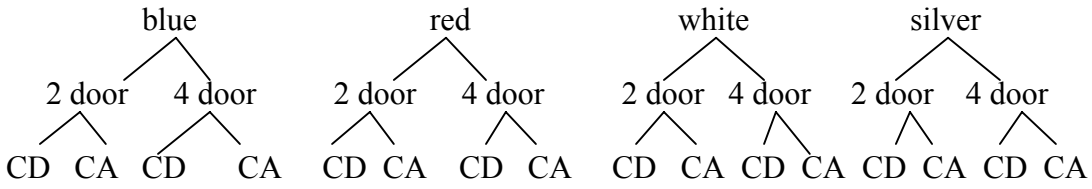
DOING THE INVESTIGATION

1. Present the shopping problem.
2. Discuss with the students how they might go about finding all of the possible choices. (Tree diagram, organized list or drawings.)
3. Discuss how to make an organized list as opposed to just a list.
4. Draw a tree diagram showing all possibilities.
5. Count and number all possibilities.
6. Elicit from students if there is an equation they can come up with to figure out the answer.
7. This equation demonstrates the multiplication principle.

POSSIBLE SOLUTIONS

Organized list: blue-2dr-CD, blue-2dr-CA, blue-4dr-CD, blue-4dr-CA, white-2dr-CD, white-2dr-CA, white-4dr-CD, white- 4dr-CA, red-2dr-CD, red-2dr-CA, red-4dr-CD, red-4dr-CA, silver-2dr-CD, silver-2dr-CA, silver-4dr-CD, silver-4dr-CA.

Tree Diagram:



The multiplication principle: 4 x 2 x 2 = 16

EXTENDING THE ACTIVITY

1. Prepare drawing paper and crayons for students to draw aliens.
2. Each student will design a set of possible attributes for his own alien.
   e.g., Head (one eye, two eyes, three eyes)
   Body (one leg, two legs, four legs)
   Color (blue, green, red)
3. Students will design their own different alien combinations and cut them into squares so various heads can be combined with different bodies and in different colors.
4. Exchange attribute cards with a partner and find out how many possible aliens you can make.
5. You may have students turn their designs into a flip book to share with younger students.

ASSESSMENT 1

You attend a friend's birthday party. You may have chicken, a hot dog or a hamburger. You may have cheese fries or plain fries. You can also have either a chocolate, vanilla, or strawberry shake. How many possible meal combinations are there?

a) 8 b) 12 c) 3 d) 18 Ans. d)

ASSESSMENT 2

A)

Make an organized list showing all the possible outfits that include 1 tee shirt, 1 pair of shorts and 1 pair of sneakers that you may wear on July 4th. You have a red shirt, white shirt and a blue shirt. You also have red shorts, white shorts and blue shorts. You only have white sneakers.

B)

What is the probability that you will choose an outfit that is red, white and blue?

Ans. A:

red shirt red shorts white sneakers	red shirt white shorts white sneakers	red shirt blue shorts white sneakers
white shirt red shorts white sneakers	white shirt white shorts white sneakers	white shirt blue shorts white sneakers
blue shirt red shorts white sneakers	blue shirt white shorts white sneakers	blue shirt blue shorts white sneakers

Ans. B: 2/9

For additional information and activities, see pages 70, 74, 83 and 85
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 6

KEY IDEA # 7: PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

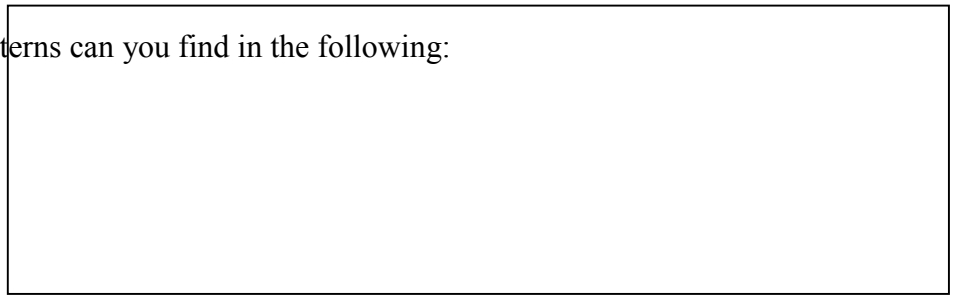
PERFORMANCE INDICATORS

7A. Recognize, describe, and generalize a wide variety of patterns and functions.
7B. Describe and represent patterns and functional relationships, using tables, charts and graphs, and verbal descriptions.
7C. Develop methods to solve basic linear equations.
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.
7E. Apply the concept of similarity in relevant situations.
7F. Use properties of polygons to classify them.
7G. Explore relationships involving points, lines, angles, and planes.
7H. Develop readiness for basic concepts of right triangle trigonometry.
7I. Use patterns and functions to represent and solve problems.

INVESTIGATION

What patterns can you find in the following:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1



WHAT'S THE MATHEMATICS?

- exploring patterns
- understanding Pascal's Triangle
- using patterns to explore probability.

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3a Discover, describe, and generalize patterns, and represent them with variables and expressions.
M3b Represent relationships.
M3c Analyze tables to determine functional relationships.

M5a Formulation
M5b Implementation
M5c Conclusion
M5d Mathematical Reasoning.
M7b Organize work, explain a solution orally and in writing, and use other techniques to make meaning clear to the audience.

VOCABULARY: pattern Pascal's Triangle

SUGGESTED TIME: 1 – 2 lessons

MATERIALS: color tiles, cubes, coins

DOING THE INVESTIGATION

1. Display the first five rows of Pascal's Triangle.
2. Have students discuss in their groups what patterns they see in the rows of numbers.
3. Elicit student responses.
4. Discuss that these numbers form what is called Pascal's Triangle.
5. Elicit the next 5 rows or more.

POSSIBLE SOLUTIONS

1st Activity

1 is always the first and last numeral in each row.
One diagonal consists of counting numbers.
Another diagonal consists of triangular numbers.

Accept all possible correct answers.

2nd Activity

1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1

EXTENDING THE ACTIVITY

1. Distribute pennies to each group of students.
2. Elicit the possible outcomes when you flip a coin (1 head, 1 tail).
3. Elicit the possible outcomes when you flip two coins (HH, HT, TH, TT).
4. Have students look at the 2^{nd} and 3^{rd} rows of Pascal's Triangle to see if they can discover a relationship between the numerals and the possible outcomes from flipping 1 coin and then two coins.
5. Look at the 4^{th} row (1 3 3 1) and elicit what they think these numerals may represent in relation to flipping coins. (When flipping three coins the possible outcomes are: 1 way of getting 3 heads, 3 ways of getting 2 heads and 1 tail, 3 ways of getting 2 tails and 1 head, and 1 way of getting 3 tails.)
6. Give students time to explore further with their coins.
7. Ask students to write a sentence or two explaining the relationship they discovered between the outcome of flipping coins and Pascal's triangle.

ASSESSMENT 1

What are the next 4 numbers in the following pattern: 1, 1, 2, 3, 5, 8, 13

a) 21, 34, 55, 89
b) 15, 18, 21, 22
c). 21, 22, 23, 25
d). 13, 18, 22, 23

Ans. a)

ASSESSMENT 2

Part A: Design your own number pattern and make up a multiple-choice question that goes with your pattern.

Part B: Describe your pattern in words.

Accept all correct solutions and explanation

For additional information and activities, see pages 75, 76, 86 and 87 in the Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Apply a variety of reasoning strategies.
1B. Make and evaluate conjectures and arguments, using appropriate language.
1C. Make conclusions based on inductive reasoning.
1D. Justify conclusions involving simple and compound (i.e., and/or) statements.

INVESTIGATION

WHAT'S THE NUMBER?

Students will explore number sentences using a variety of strategies making each equation a true sentence.

WHAT'S THE MATHEMATICS?

- using patterns
- using trial and error
- using a simpler but related problem
- working backwards

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3d Finds solutions for unknown quantities in linear equations.
M5a Formulation.
M5b Implementation.
M5c Conclusion.
M5d Mathematical reasoning.

VOCABULARY:	equation	possibilities	patterns
	strategies	simpler problem	trial and error
	working backwards	remainder	

SUGGESTED TIME: 1 – 2 lessons

MATERIALS: calculator

DOING THE INVESTIGATION

1. Show students the following equations:

   1. $\square$ 2 ÷ $\square$ = $\square$
   2. 4$\square$ ÷ 6 = $\square$ R $\square$
   3. 89 ÷ $\square$ = 9 R $\square$
   4. 3$\square$ ÷ 7 = $\square$ R2

2. Explain that all the squares must be filled in using the digits 0-9 once and only once so that each equation will be a true sentence.
3. Guide them to look at the four equations to determine which equation might be the best to begin with.
4. Have students explain and discuss why a certain equation was selected.
5. Discuss the possible remainders when dividing by any given number, e.g. 1,2,3,4,5 are the only possible remainders when dividing by 6.
6. Look at the first equation and list all the possible solutions using three different digits.
7. Repeat this process for each of the other equations.
8. Guide your students to look at the results of each individual equation to solve all four equations without repeating a digit.
9. Ask them to record their work and be able to explain their strategies.

POSSIBLE SOLUTIONS

Do the third equation first because there is only one possible solution. 89 ÷ 9 = 9 R8
Do the fourth equation next. There are two possible solutions. 30 ÷ 7 = 4 R2
OR 37 ÷ 7 = 5 R2
Let's now look at all the possibilities of the first two equations.

First equation:
12 ÷ 2 = 6
12 ÷ 3 = 4
32 ÷ 4 = 8 (8 has already been used)
42 ÷ 6 = 7
72 ÷ 9 = 8 (8 and 9 have already been used)
46 ÷ 6 = 7 R4
47 ÷ 6 = 7 R5 (The digit 7 is used twice.)
49 ÷ 6 = 8 R1 (8 has already been used)

Second equation:
40 ÷ 6 = 6 R4
41 ÷ 6 = 6 R5
43 ÷ 6 = 7 R1
44 ÷ 6 = 7 R2
45 ÷ 6 = 7 R3

Using trial and error, only the first solution for the fourth equation will enable the other equations to become true sentences. Therefore, the four equations would be:

1. $12 \div 2 = 6$
2. $45 \div 6 = 7$ R3
3. $89 \div 9 = 9$ R8
4. $30 \div 7 = 4$ R2

EXTENDING THE ACTIVITY

Have students work in groups to make up their own number sentences. Each of the digits 0 - 9 must be used once and only once. Have groups present their number sentences and ask their classmates to try to solve the problems. Have students explain their strategies.

ASSESSMENT 1

Look at the number sentence.

$\square 3 \times 7 = \square\square 1$

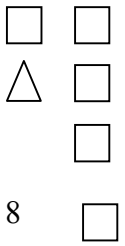
If three different digits replace the blanks, what could the product be?

a. 231 b. 371 c. 441 d. 721

Ans. b (53 x 7, Choice "a" can't work because 33 x 7 = 231 and it repeats the 3's in the factors and the answer. Choice "c" repeats the 4 in the product. Choice "d" uses a three digit factor to obtain the product.)

ASSESSMENT 2

Explain how you would solve the following equation to find the value of the □ and of the △, where the same shape contains the same number and different shapes contain different numbers.



Ans. □ = 5 and △ = 2. To add 3 □ 's and end up with a □ in the ones place, only a 5 is possible. Once realizing that, the tens column is 5 + 1 + △ = 8, therefore, △ is 2.

For additional information and activities see pages 102-103
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded, and scientific notation).
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.
2C. Develop an understanding of number theory (primes, factors, and multiples).
2D. Recognize order relations for decimals, integers, and rational numbers.

INVESTIGATION

MORE THAN 4 - LESS THAN 5

Students will combine decimals and fractions to find sums which are greater than 4 and less than 5.

WHAT'S THE MATHEMATICS?

- reading and writing integers, rational and irrational numbers
- understanding the relationship between terminating and repeating decimals
- describing the equivalent relationships among representations of rational numbers (fractions, decimals, and percents) and use these representations in estimation, computation, and applications
- calculating fraction, decimal and percent equivalents

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Consistently and accurately adds rational numbers using appropriate methods.
M1c Consistently and accurately applies and converts the different kinds and forms of rational numbers.
M1g Orders numbers...recognizes relationships to "benchmark" numbers 1/2 and 1.
M5a Formulation.
M5b Implementation.
M5c Conclusion.

VOCABULARY:	fraction	decimal	equivalent
	sum	greater than	less than
	mixed number	improper fraction	

SUGGESTED TIME: 1 class lesson

MATERIALS: fraction circles and/or fraction strips, etc., calculator

DOING THE INVESTIGATION

1. Show the students the fractions and decimals:
   3/4, 5.2, 15/9, 1.4545 . . ., 1/5, 34/7.
2. Discuss with the students the approximate value of each fraction and decimal (closest to 0, 1/2 or 1).
3. Show your students the following numbers:
   1/2, 3.6, 13/3, 7/4, 1.8, 2 3/4, 2.3, 5/6.
4. Guide your students to look at the largest numbers to see if any of them can be combined to have a sum > 4 and < 5
5. Create four sums >4 and <5 using all eight numbers.

POSSIBLE SOLUTIONS

Any two of the four largest numbers, 3.6, 13/3, 2 3/4 and 2.3 when combined will be larger than 5. Therefore, each one of them must be added to a smaller number. One strategy is to create a chart with the remaining numbers and one of the larger numbers.

Trial Chart					Satisfy the Conditions
3.6	+ 1/2	5/6	1.8	7/4	3.6 + 1/2 or 3.6 +5/6
13/3	+ 1/2	5/6	1.8	7/4	13/3 + 1/2
2 3/4	+ 1/2	5/6	1.8	7/4	2 3/4 + 1.8 or 2 3/4 +7/4
2.3	+ 1/2	5/6	1.8	7/4	2.3 + 1.8 or 2.3 + 7/4

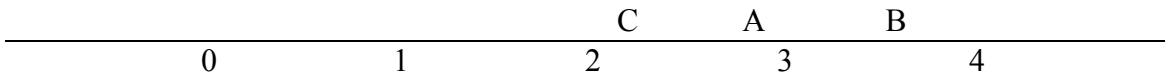
Since 13/3 can only be added to 1/2, 13/3 + 1/2 must be one pair of numbers. Realizing this, 3.6 can only be added to 5/6 as another pair of numbers. 2 3/4 and 2.3 have two possible solutions. Therefore, there are 2 possible solutions containing 2 pairs of numbers:

Solution I: 3.6 + 5/6; 13/3 + 1/2; 2 3/4 + 1.8; and 2.3 + 7/4 OR
Solution II: 3.6 + 5/6; 13/3 +1/2; 2 3/4 + 7/4; and 2.3 + 1.8

EXTENDING THE ACTIVITY

Look at the numbers: 1.37; 7/3; 2; 7/11; 0.96; and 1 3/4

The six numbers can be matched into three pairs whose sum will match a letter on the number line below. Find the pairs that will match each letter.



A = the sum of
B = the sum of
C = the sum of

Explain your thinking.

ASSESSMENT 1

The decimal portion of which number is furthest from 1/2?

a. 12/5 b. 5/9 c. 7.49 d. 5 8/11

Ans. d (12/5 = 2.4, 5/9 = 0.555... 5 8/11 = 5.7272...)

ASSESSMENT 2

Part A: Circle the numbers below that are greater than or equal to 5 1/2 and less than 6 3/4

5.49999 33/6 6 11/16 22/4 4 13/8

Part B: Explain your thinking.

Ans. The numbers circled are: 33/6, 6 11/16, 4 13/8. (5.4999 is too small).

One explanation is that the student changed everything to decimals while stating that 5 1/2 = 5.5 and 6 3/4 = 6.75.

For additional information and activities see page 104
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply, and divide fractions, decimals, and integers.
3B. Explore and use the operations dealing with roots and powers.
3C. Use grouping symbols (parentheses) to clarify the intended order of operations.
3D. Apply the associative, commutative, and distributive properties, and inverse and identity elements.
3E. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc).
3F. Develop appropriate proficiency with facts and algorithms.
3G. Apply concepts of ratio and proportion to solve problems.

INVESTIGATION

PLACING PARENTHESES

Students will investigate placing parentheses and using the order of operations appropriately to make true sentences.

WHAT'S THE MATHEMATICS?

- using the order of operations within a problem
- understanding the use of parentheses and their relationship to the order of operations

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a Consistently and accurately adds, subtracts, multiples, and divides rational numbers
M1d Is familiar with characteristics of numbers and with properties of operations
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: order of operations parentheses

SUGGESTED TIME: 1 lesson

MATERIALS: calculators

DOING THE INVESTIGATION

1. Discuss the order of operations with your students.
2. Use different calculators, e.g. algebraically programmed and non-algebraic to discuss the different answers.
3. Place parentheses in the following equations to make true sentences:

1) $2 + 2 - 2 \times 2 = 1$
$2 + 2 \div 2 \times 2 \quad 2$

2) $2 + 2 - 2 \times 2 = 4$
$2 + 2 \div 2 \times 2$

3) $2 + 2 - 2 \times 2 = 2$
$2 + 2 \div 2 \times 2$

4) $2 + 2 - 2 \times 2 = 0$
$2 + 2 \div 2 \times 2$

POSSIBLE SOLUTIONS

1) $2 + (2 - 2) \times 2 = 1$
$2 + (2 \div 2) \times 2 \quad 2$

2) $(2 + 2 - 2) \times 2 = 4$
$(2 + 2) \div (2 \times 2)$

3) $2 + (2 - 2) \times 2 = 2$
$(2 + 2) \div (2 \times 2)$

4) $(2 + 2) - 2 \times 2 = 0$
$2 + 2 \div 2 \times 2$

Notice that in example #4, there are several options for the denominator. One option is to leave it as it is without parentheses. Any other number that will not yield a 0 is also good.

EXTENDING THE ACTIVITY

Using the number 1, 2, 3, 4 and 5 once in each problem solve the equations to make true sentences:

$(\square + \square) \times (\square) \div (\square) - (\square) = 30$

$((\square) + (\square)) \times (\square) \div (\square) - (\square) = 33$

$((\square) + (\square)) \times (\square) \div ((\square) - (\square)) = 32$

$((\square) + (\square)) \times (\square) \div ((\square) - (\square)) = 35$

ASSESSMENT 1

Which answer shows the parentheses placed correctly to make the equation below a true sentence?

$5^2 - 7 \div 6 \times 8 = 3$
$9 \times 5 + 3 - 2^6$

a. $5^2 - 7 \div (6 \times 8)$
$9 \times 5 + (3 - 2^6)$

b. $5^2 - (7 \div 6) \times 8$
$9 \times 5 + 3 - 2^6$

c. $5^2 - 7 \div 6 \times 8$
$9 \times (5 + 3) - 2^6$

d. $(5^2 - 7) \div 6 \times 8$
$9 \times (5 + 3) - 2^6$

Ans. d

ASSESSMENT 2

Lynn placed parentheses in the expression $3 + 15 \div 3 - 1 + 2 \times 4$ to make an equation with the largest possible answer. She said that the largest number is 17. However, Casey said that he placed parentheses in the expression $3 + 15 \div 3 - 1 + 2 \times 4$ and got 18 1/2. Show how they both arrived at their answers.

Ans. Lynn's answer is $(3+15) \div (3-1) + 2 \times 4$. Casey's answer is $3 + 15 \div (3-1) + 2 \times 4$.

For additional information activities, see pages 105-106
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Visualize, represent, and transform two-and three-dimensional shapes.
4B. Use maps and scale drawings to represent real objects or places.
4C. Use the coordinate plane to explore geometric ideas.
4D. Represent numerical relationships in one-and two-dimensional graphs.
4E. Use variables to represent relationships.
4F. Use concrete materials and diagrams to describe the operation of real-world processes and systems.
4G. Develop and explore models that do and do not rely on chance.
4H. Investigate both two-and three-dimensional transformations.
4I. Use appropriate tools to construct and verify geometric relationships.
4J. Develop procedures for basic geometric constructions.

INVESTIGATION

WHAT'S IN THE BAG?

Students will construct an appropriate sample space to solve a problem.

WHAT'S THE MATHEMATICS?

- constructing an appropriate sample space
- exploring the range of probabilities

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M5a formulation
M5b implementation
M5c conclusion

VOCABULARY: sample space tree diagram probability product

SUGGESTED TIME: 1-2 lessons

MATERIALS: bag of tiles numbered 1 through 8

DOING THE INVESTIGATION

1. Show students a bag with tiles.
2. Show students that there are eight tiles and each one has a different number, from one through eight.
3. Have them select a tile, record its number and then put it back into the bag.
4. Have them repeat this with a second tile and then with a third tile.
5. Have them predict the probability that the product of all three tiles selected will be an odd number.
6. Have students work in groups and experiment selecting 3 tiles with replacement and then record the results. Have each group do it 30-40 times.
7. Record the data from all the groups.
8. Ask groups to think of a way to determine the probability.

POSSIBLE SOLUTIONS

One solution is to list the sample space and another is to make a tree diagram.
Let E represent a tile with an even number and O represents a tile with an odd number.

E x E x E = E	O x E x E = E
E x E x O = E	O x E x O = E
E x O x E = E	O x O x E = E
E x O x O = E	O x O x O = O

Of the eight possible outcomes, only one results in an odd product. Therefore, the probability is 1/8.

EXTENDING THE ACTIVITY

Experiment with the same problem to determine the results if a tile were selected and not replaced. What would the probability be if three tiles were selected and not replaced that the product of the numbers would be odd?

ASSESSMENT 1

How many different products are there which can be pulled out of the bag, if two tiles are pulled out with replacement?

a. 8 b. 16 c. 32 d. 64

Ans. d (8 x 8)

ASSESSMENT 2

What is the probability that the product is odd when two tiles are selected from a bag with replacement? Show your work.

Ans. 1/4
Sample Space: E x E = E E x O = E O x E = E O x O = O

For additional information and activities, see pages 107-108 and 114
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Estimate, make, and use measurements in real-life situations.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.
5C. Develop measurement skills and informally derive and apply formulas in direct measurement activities.
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.
5E. Explore and produce graphic representations of data (calculators or computers may be used).
5F. Develop critical judgment for the reasonableness of measurement.

INVESTIGATION

RECTANGULAR GARDENS

Students will create rectangular gardens with a given perimeter. They will generalize how to maximize the area.

WHAT'S THE MATHEMATICS?

- knowing and applying formulas for perimeter and area of polygons
- understanding length, area, and volume and make relationships between the measurements

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Is familiar with assorted two dimensional objects, including rectangles.
M2d Determines and understands length, area, and computes areas of rectangles.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: rectangle perimeter area maximize minimize

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: grid paper, square tiles, scissors

DOING THE INVESTIGATION

1. Discuss with your class the concepts of area and perimeter.
2. Have students use square tiles to create a rectangular garden with a perimeter of 24 feet where the length of the sides are positive integers. Let a side of the tile represent one foot.
3. Have students record their work on grid paper.
4. Have students work in groups to create five other rectangular gardens with a perimeter of 24 feet, but with different areas. They should record their work on grid paper and cut the shapes.
5. Have students arrange the gardens in order from largest area to smallest area.
6. Pose the following question to your students: Describe the relationship between perimeter and area. Given a perimeter, how can the area be maximized? minimized? Explain.
7. Allow time for students to try out their conjectures by investigating rectangles with different perimeters, such as a perimeter of 18 units. Explain why your observations are the same or different for a rectangular garden with either perimeter.

POSSIBLE SOLUTIONS

In order of size from largest to smallest area the dimensions (in feet) of the rectangular gardens are: 6x6, 5x7, 4x8, 3x9, 2x10, and 1x11. The areas (in square feet) are: 36, 35, 32, 27, 20 and 11, respectively.
The closer to a square the rectangle is, the area is maximized (largest).
The longer the rectangle is, the area is minimized (smallest).
This observation is verified with a rectangle whose perimeter is 18 feet. Four rectangles can be created whose dimensions are 1x8, 2x7, 3x6 and 4x5. Their areas are 8, 14, 18 and 20 square feet, respectively. Thus, the shape closest to a square (4x5) has the largest, maximum, area.

EXTENDING THE ACTIVITY

Use 36 tiles and grid paper to draw rectangles whose area is 36 square units.
Explain how to maximize and how to minimize the perimeter.

ASSESSMENT 1

What is the largest possible perimeter, using integers, for a rectangular region with an area of $16x^2$?

a. 34x b. 20x c. 16x d. 8x

Ans. a (The dimensions of the rectangle are x by 16x).

ASSESSMENT 2

Joshua said that his rectangular garden has a perimeter of 24 meters.
Kristina said that her rectangular garden has a perimeter of 26 meters.
If both have maximized the area, whose garden is larger? Explain.

Ans. Kristina has the garden with the largest area. The dimensions of her garden that maximizes the area is 6 x 7. The dimensions of Joshua's garden is 6 x 6.

For additional information and activities, see pages 109 and 115
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with every day situations.

PERFORMANCE INDICATORS:

6A. Use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.
6B. Use estimation to solve problems for which exact answers are inappropriate.
6C. Estimate the probability of events.
6D. Use simulation techniques to estimate probabilities.
6E. Determine probabilities of independent and mutually exclusive events.

INVESTIGATION

DESIGN YOUR OWN SPINNER

Students will design unbiased spinners given various conditions.

WHAT'S THE MATHEMATICS?

- understanding and using empirical and theoretical probability, using the formula P(E) = f/n

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4h Represents and determines probability as a fraction of a set of equally likely outcomes; recognizes equally likely outcomes and constructs sample spaces.
M4i Making predictions based on experimental or theoretical probabilities.
M4j Predicting the result of a series of trials once the probability for one trial is known.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY:

outcome	equally likely	prime
composite	perfect squares	multiples
biased	unbiased	fair
unfair	triangular numbers	odd
even	consecutive numbers	

SUGGESTED TIME: 1 lesson

MATERIALS: circular disks, rulers, protractors

DOING THE INVESTIGATION

1. Discuss with your students the meaning of biased and unbiased outcomes (not equally likely and equally likely outcomes: unfair and fair).
2. Have students give examples from real-life situations that are biased and ones that are unbiased.
3. Discuss with your students the meaning of consecutive numbers.
4. Ask your students to design fair spinners using consecutive numbers less than 10 with the following conditions:

    Spinner 1: odd and even numbers
    Spinner 2: prime and composite numbers
    Spinner 3: perfect squares and non perfect squares
    Spinner 4: multiples of 3 and prime numbers
    Spinner 5: triangular numbers and prime numbers

POSSIBLE SOLUTIONS

Spinner 1: Five different types of solutions (zero can be used on any spinner when appropriate).

Solution 1: a spinner with two equal regions and one odd number and one even number in each, e.g., 1 and 2 or 2 and 3 or 3 and 4, etc.
Solution 2: a spinner with four equal regions having two sections with even numbers and two sections with odd numbers, e.g., 1, 2, 3 and 4 or 2, 3, 4 and 5 or 3, 4, 5 and 6, etc.
Solution 3: 6 equal regions on the spinner using three odd and three even numbers
Solution 4: 8 equal regions on the spinner, four odd and four even numbers
Solution 5: 10 equal regions using all single digits, 0-9

Spinner 2: There are several solutions for using only two numbers. You can also use all eight numbers, 2 - 9
Spinner 3: Perfect squares are 0, 1, 4 and 9 with 2 regions: 1 and 2; 3 and 4; 4 and 5 or with 4 regions: 1, 2, 3, 4 or 0, 1, 2, 3
Spinner 4: There are three ways to do this. The spinner would consist of two equal regions with 5 on one half and 6 on the other half; 6 on one half and 7 on the other half; or 3 on one half and 2 one the other half.
Spinner 5: This may be accomplished with 1 and 2; 2 and 3; 5 and 6 or 6 and 7.

EXTENDING THE ACTIVITY

Design a spinner with 8 equal regions so that each region has a different number, but 1/4 of the region has two digit prime numbers, 3/8 has square numbers, 1/8 has a multiple of 11 and 1/4 has non even composite number. Each number can be used to fulfill only one condition.

ASSESSMENT 1

Which condition has an equally likely chance of happening on a spinner that is divided into four regions with the numbers 41, 42, 43 and 44?

a. prime numbers and multiples of 3
b. odd numbers and multiples of 4
c. composite numbers and multiples of 11
d. prime numbers and multiples of 2

Ans. d (41 and 43 are prime numbers)

ASSESSMENT 2

A spinner is divided into 6 equal regions. The numbers on the spinner are: 24, 36, 48, 81 96 and 121. What conditions would make this spinner a fair spinner? What conditions? would make it unfair?

Ans. Fair spinner: multiples of 24 and perfect squares (three of each on the spinner)
Unfair spinner: multiples of 3 and perfect squares (all but 121 are multiples of 3 and there are only 3 perfect squares)

Note: There could be other solutions.

For additional information and activities, see pages 111 and 115
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 7

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, and generalize a wide variety of patterns and functions.
7B. Describe and represent patterns and functional relationships, using tables, charts, graphs, algebraic expressions, rules and verbal descriptions.
7C. Develop methods to solve basic linear and quadratic equations.
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.
7E. Verify results of substituting variables.
7F. Apply the concept of similarity in relevant situations.
7G. Use properties of polygons to classify them.
7H. Explore relationships involving points, lines, angles, and planes.
7I. Develop and apply the Pythagorean principle in the solution of problems.
7J. Explore and develop basic concepts of right triangle trigonometry.
7K. Use patterns and functions to represent and solve problems.

INVESTIGATION

LOOKING FOR TRIANGLES

Students will investigate a series of triangles, looking for patterns and generalizing what they have found.

WHAT'S THE MATHEMATICS?

- identifying, describing, representing, extending, and creating patterns
- describing functions and generalizing by the use of rules and algebraic expressions
- describing and representing numerical and geometric patterns and functions using equations, graphs, and tables
- organizing and analyzing data resulting in function applications through use of a table of values, sentence, formula, graph, and prediction
- examining a situation and determining if the quantities vary directly or indirectly, and representing that variation graphically, in a table and in an equation
- identifying the input and the output in a relationship between two variables and determining whether the relationship is a function
- using patterns and functions to solve problems

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3a Discovering, describing, and generalizing patterns, including linear, exponential and simple quadratic relationships.
M3b Representing relationships with tables, graphs in the coordinate plane, and verbal or symbolic rules.
M3c Analyzing tables, graphs, and rules to determine functional relationships.
M5a Formulation
M5b Implementation
M5c Conclusion

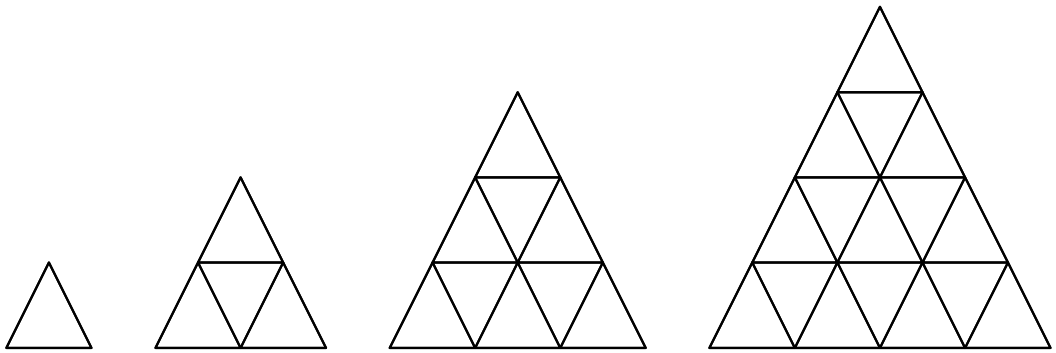
VOCABULARY: pattern generalization

SUGGESTED TIME: 1 lesson

MATERIALS: pattern blocks

DOING THE INVESTIGATION

1. Discuss with your students the meaning of the word pattern.
2. Have your students suggest some patterns, e.g. number patterns, geometric patterns and other patterns
3. Guide your students to look at growing patterns.
4. Have your students create their own growing patterns with the pattern blocks.
5. You may draw these designs or have your students use pattern blocks. Have your students examine the following designs (patterns):



Design 1 Design 2 Design 3 Design 4

6. Pose the problem, "If the above pattern continues, how many small triangles must be added to design 17 to make design 18. Show your work."

7. Using the information they have, ask your students to work in groups to give a general rule which will find the number of small triangles that are needed in any design number.

POSSIBLE SOLUTIONS

The answer is 35. One strategy is to keep drawing triangles. Another is to create a chart and look for patterns that will help.

Design (F)	Total Number of Triangles (T)	Difference From this Design to the Next (D)
1	1	3
2	4	5
3	9	7
4	16	9
5	25	11
6	36	13

Notice the differences from any design to the next design is $D=2F + 1$ where D is the difference, F is the figure number of the current shape and T the total number of triangles. Therefore, the difference between the 17th design and the 18th design is $2 \bullet 17 + 1 = 35$.

To find the number of triangles needed in any design, look at the chart. Notice that the total number of triangles is the design number squared: T= F squared. Squaring design 17 (289) and design 18 (324) is another way of determining the number of triangles that need to be added
(324 - 289).

EXTENDING THE ACTIVITY

Have them create a similar growing design 1, 2 and 3 using their hexagons.

Part A: If the pattern continues, how many small hexagons must be added to the 17th design to make the next design. Show your work.
Part B: Give a general rule. Explain.

ASSESSMENT 1

In the growing design using triangles the number of triangles varies directly as the design number. What would be the graph of this generalization?

a. a straight line
b. a parabola
c. the positive side of the parabola
d. an "s" curve

Ans. c (Since we are only concerned with positive numbers.)

ASSESSMENT 2

If the difference in the number of triangles between two designs in the investigation is 23, what are the two design numbers? Explain.

Ans. The design numbers are 11 and 12. One strategy to use is to work backwards from the generalization $D = 2F + 1$, solve for F where $D = 23$.

For additional information and activities, see pages 111 and 116
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 8

KEY IDEA # 1 MATHEMATICAL REASONING

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS:

1A. Apply a variety of reasoning strategies.
1B. Make and evaluate conjectures and arguments, using appropriate language.
1C. Make conclusions based on inductive reasoning.
1D. Justify conclusions involving simple and compound (i.e., and/or) statements.

INVESTIGATION

WHAT'S THE NUMBER?

Students will explore exponents and look for patterns that will help to determine the ones place when the base is raised to any power.

WHAT'S THE MATHEMATICS?

- using trial and error
- seeking general solutions
- identifying patterns in a number sequence
- applying strategies and results from simpler problems to more complex situations

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M5a Formulation
M5b Implementation
M5c Conclusion
M5d Mathematical Reasoning

VOCABULARY: exponents base pattern trial and error number sequence simpler problem

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: calculator, computer

DOING THE INVESTIGATION

1. Pose the question: What is the units digit in 7^{86}?
2. Discuss the meaning of exponents.
3. Discuss raising bases to different powers.
4. Allow your students time to work on a calculator.
5. Discuss the calculator results (except for a graphing calculator, depending on the calculator used, it will give an error message or express the solution in scientific notation).
6. If available, discuss the possibility of developing a computer program to solve the problem.
7. Even if a computer program is possible, guide your students to begin raising the 7 to a number of different exponents either with paper and pencil or with the calculator while recording their results.
8. After a few minutes, guide students to examine their results. You may wish to record the results on the board.
9. Guide your students in realizing that looking for a pattern may assist in finding a generalized solution.

POSSIBLE SOLUTIONS

One possible solution would be to devise a computer program that would solve this problem.
Another solution would be to use a simpler but related problem and look for patterns so as to find a general solution.
Students can use the calculator or pencil and paper to find the following:

$7^1 = 7$	Try the next four exponents:	$7^5 = 16{,}807$
$7^2 = 49$		$7^6 = 117{,}649$
$7^3 = 343$		$7^7 = 823{,}543$
$7^4 = 2{,}401$		$7^8 = 5{,}764{,}801$

Notice, that there is a pattern in the one's place with the digits 7, 9, 3 and 1 repeating every fourth exponent. If this is the case, the one's place will continue with this pattern.

7^9	will have a 7 in the one's place	7^{13}	will have a 7 in the one's place
7^{10}	will have a 9 in the one's place	7^{14}	will have a 9 in the one's place
7^{11}	will have a 3 in the one's place	7^{15}	will have a 3 in the one's place
7^{12}	will have a 1 in the one's place	7^{16}	will have a 1 in the one's place

A student could go on until reaching 86 or realize that $86 \div 4 = 21$ R 2. This means that 7 exponent 86 would follow the second (R2) number in the pattern and the one's place would be a 9.

EXTENDING THE ACTIVITY

Ask students to investigate other single digit bases to determine if any patterns exist when raising these numbers to consecutive powers. Show all work and explain your conclusion(s).

ASSESSMENT 1

What will be the one's place in 27 raised to the 35 power?

a. 7 b. 9 c. 3 d. 1

Ans. c (It will follow the same pattern as the 7's did because 27 = 20 + 7.)

ASSESSMENT 2

Part A: Raise 12 to consecutive powers from 1-8. Show your work.

Part B:

Explain why raising 12 to consecutive exponents will yield the same digits in the one's place as raising 2 to consecutive exponents from 1-8. Explain your thinking.

Ans. :Part A:

$12^1 = 12$	$12^5 = 248{,}832$
$12^2 = 144$	$12^6 = 2{,}985{,}984$
$12^3 = 1{,}728$	$12^7 = 35{,}831{,}808$
$12^4 = 20{,}736$	$12^8 = 429{,}981{,}696$

Part B Raising the 12 to powers is really raising 10 + 2 to the same powers. 10 to any power will always yield a 0 in the one's place. 2 to the same power is then added to the answer. With 0 in the one's place, it is always the result of two to a power that is added and the one's digit will thus be the same as 2 to any power.

GRADE 8

KEY IDEA # 2 NUMBER AND NUMERATION

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS:

2A. Understand, represent, and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, expanded, and scientific notation).
2B. Understand and apply ratios, proportions, and percents through a wide variety of hands-on explorations.
2C. Develop an understanding of number theory (primes, factors, and multiples).
2D. Recognize order relations for decimals, integers, and rational numbers.

INVESTIGATION

WHAT DOES IT COST?

Students will investigate percents as related to discounts by comparing different markings on the same sale item.

WHAT'S THE MATHEMATICS?

- calculating fraction, decimal and percent equivalents
- interpreting percent as part of 100, using a variety of manipulatives
- finding the percent of a number, calculating the percent of increases and decreases, rate, commissions, taxes, and simple interest

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1a consistently and accurately adds, subtracts, multiples, and divides rational numbers using appropriate methods
M1c consistently and accurately applies and converts the different kinds and forms of rational numbers
M1e interprets percent as part of 100 and as a means of comparing quantities of different sizes
M1f uses ratios and rates to express "part-to-part" and "whole-to-whole" relationships
M5a formulation
M5b implementation
M5c conclusion

VOCABULARY: percent mark down discount sale price
original price

SUGGESTED TIME: 1 class lesson

MATERIALS: calculator graph paper

DOING THE INVESTIGATION

1. Discuss with your students the meaning of percent.
2. Use 10 x 10 grids (graph paper) to have them shade in 4%, 35%, 1.2% and 125%
3. Discuss with your students what the above percents mean if there is a discount on an item that costs $100.
4. Guide your students to realize that on $100, a 1.2% discount is only $1.20 off and 125% discount means that the store owes you money and the item is free.
5. Ask your students to look at the following signs:

Store A	Store B	Store C
SALE Take 50% off original price	SALE 30% off original price, then take another 25% off	SALE 20% off original price Then marked down another 20% Today only, take off an additional 20%

The three stores have the same dress on sale. The original cost of the dress was $79.

Part A: If you wanted to pay the least amount, in which store would you buy the dress? Show your work.

Part B: In which store is the dress the most expensive? Explain.

POSSIBLE SOLUTIONS

Part A:

Store A: 50% of $79 = $39.50. The dress would cost $39.50 (1/2 of $79).

Store B: 30% of $79 = $23.70 $79 - 23.70 = $55.30
25% of $55.30 = $13.825 $55.30 - $13.825 = $41.475
The dress would cost $41.48. (The store will always round up.)

Store C:	20% of \$79 = \$15.80	\$79 - 15.80 = \$63.20
	20% of \$63.20 = \$12.64	\$63.20 - 12.64 = \$50.56
	20% of \$50.56 = \$10.112	\$50.56 - 10.112 = \$40.448
		The store will charge \$40.45

Store A charges the least for the dress.

Part B: Store B charges the most. Taking a percent and then taking another percent is not the same as taking the total of the two percents. The first percent is being taken of a larger number. The next percent is of a smaller number.

EXTENDING THE ACTIVITY

Suppose a sports jacket cost \$98.

Sale I:	50% off	Sale III:	25% off then 25% off
Sale II:	40% off then 10% off	Sale IV:	30% off then 20% off

Part A: Which is the best buy? Show your work.

Part B: Generalize a rule for taking one or more percents off.

ASSESSMENT 1

Which would be the best buy?

a. 75% off

b. 50 % off then 25% off

c. 25% off, then 25% off, then another 25% off

d. 40% off then 35% off

Ans. a (Taking the percent off the total is always better.)

ASSESSMENT 2

Use grid paper to demonstrate why/why not the following "percents off" would make sense when buying an article of clothing.

1/4% 35% 70% 105%

Explain.

Ans. 1/4% and 125% would not make sense. 1/4% is 25¢ on $100 and 125% means you would get the article for free and then get some extra money from the store.

35% and 70% are more realistic. 70% is usually used only on a close out or going out of business sale, but it is possible.

For additional information and activities see page 104
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 8

KEY IDEA # 3 OPERATIONS

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS:

3A. Add, subtract, multiply, and divide fractions, decimals, and integers.
3B. Explore and use the operations dealing with roots and powers.
3C. Use grouping symbols (parentheses) to clarify the intended order of operations.
3D. Apply the associative, commutative, and distributive properties, and inverse and identity element.
3E. Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc).
3F. Develop appropriate proficiency with facts and algorithms.
3G. Apply concepts of ratio and proportion to solve problems.

INVESTIGATION

THE LOCAL BAGEL SHOP

Students will investigate unit pricing to find which cup of coffee is a better buy.

WHAT'S THE MATHEMATICS?

- using ratio and proportions to solve problems involving a change of scale in drawings, maps, recipes, etc.
- determining the unit cost of items to compare prices

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M1f Use ratio and rates to express "part-to-part" and "whole-to-whole" relationships, and reason proportionally to solve problems, equivalent fractions, equal ratios, or constant rates.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: unit price ratio and proportions

SUGGESTED TIME: 1 lesson

MATERIALS: calculators

DOING THE INVESTIGATION

1. Discuss with your students the meaning of unit price.
2. Discuss with your students the meaning of ratios and the meaning of proportions.
3. Tell your class the following story:

   Freddie stopped in the local bagel shop to buy coffee. A 10-ounce cup of coffee costs 95¢ and a 16-ounce cup costs $1.05.

4. Ask them to determine the following:

Part A: Which size cup is the better buy? Show your work.

Part B: What is the least amount of coffee, using whole ounce cups, Freddie must get in the larger cup to make buying it a better buy than the small cup? Explain your answer.

POSSIBLE SOLUTIONS

One strategy could be using ratio and proportions to find the cost of 1 ounce:

Part A:

10/.95 = 1/x	16/1.05 = 1/y
x = 0.095	y = 0.065625
x = 9.5 ¢ per ounce	y = 6.5625¢ per ounce

The larger cup of coffee is the better buy.

Part B:

Using similar proportions, buying 11 ounces for $1.05 costs 9.54545¢ per ounce. This is slightly higher than buying the smaller cup of coffee. Buying 12 ounces for $1.05 costs 8.75¢ per ounce. As long as Freddie wants 12 or more ounces of coffee, the larger cup is the better buy.

EXTENDING THE ACTIVITY

A 10 ounce (small) cup of coffee costs 95¢.

Part A: Design a medium and a large cup of coffee. Price them so that the coffee in each of the three cups costs the same amount per ounce. Show your work.

Part B: Then price them so that the coffee in the medium cup costs less per ounce than the small cup and that the large cup costs less per ounce than the medium cup. Explain your thinking.

ASSESSMENT 1

Marc is making a model airplane. The scale he is using is 3 meters = 1 centimeter. Which proportion will help to find the length of Marc's model airplane, if the plane is 98 meters long?

a. 3:1 = x:98 b. x/1 = 3/98 c. x:98 = 1:3 d. 98/3 = 1/x

Ans. c (Both ratios are in the form centimeters:meters.)

ASSESSMENT 2

A baker's dozen is 13 donuts. At the local donut shop a baker's dozen costs $3.00. They also advertise:

1/2 dozen donuts = $1.80 and 2 free donuts with a coupon with a limit of 3 coupons.

Shelly bought 6 dozen donuts at the baker's dozen price plus 1/2 dozen with a coupon

Alan bought 4 baker's dozen and 4 half dozen using the 3 coupon limit

Donna bought 5 baker's dozen and 3 half dozen with coupons.

Who had the best buy per donut? Show your work.

Ans. Donna (She bought 89 donuts at a cost of $20.40 or $0.229 per donut. Shelly's unit price was $0.230 per donut and Alan's was $0.234 per donut.)

For additional information activities, see pages 105-106
in the New York State Core Curriculum/Mathematics Resource Guide.

GRADE 8

KEY IDEA # 4 MODELING/MULTIPLE REPRESENTATION

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS:

4A. Visualize, represent, and transform two-and three-dimensional shapes.
4B. Use maps and scale drawings to represent real objects or places.
4C. Use the coordinate plane to explore geometric ideas.
4D. Represent numerical relationships in one-and two-dimensional graphs.
4E. Use variables to represent relationships.
4F. Use concrete materials and diagrams to describe the operation of real-world processes and systems.
4G. Develop and explore models that do and do not rely on chance.
4H. Investigate both two-and three-dimensional transformations.
4I. Use appropriate tools to construct and verify geometric relationships.
4J. Develop procedures for basic geometric constructions.

INVESTIGATION

LOOKING AT POLYGONS

Students will plot points and investigate polygons.

WHAT'S THE MATHEMATICS?

- locating a point, using ordered pairs of integers on the coordinate plane
- comparing geometric measurements and computations on coordinate axes as they are applied to parallel lines, congruent and similar figures
- locating the quadrant in which an ordered pair of integers is located
- developing geometric ideas such as measurement formulas, using geoboards and graph paper

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Is familiar with assorted three-dimensional objects, including rectangular prisms, pyramids, and cylinders.
M2b Uses transformations in the coordinate plane.

M2d Determines and understands length, area, and volume including perimeter and surface area.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: polygon rhombus square trapezoid
parallelogram rectangle coordinate pairs quadrants
x-axis y-axis origin image transformational geometry rotation reflection

SUGGESTED TIME: 1 lesson

MATERIALS: grid paper

DOING THE INVESTIGATION

1. Talk to students about coordinate geometry. Use terms like quadrants, coordinate pairs,
   x-axis, y-axis and origin.
2. Review plotting ordered coordinate pairs.
3. Ask students to plot the coordinates of each of the following polygons, connect the points to create a polygon and label each polygon 1, 2, 3, 4 or 5.
   Polygon 1: (11,7); (7,5); (3,7); and (7,9)
   Polygon 2: (3, -6); (6,-6); (6,-9); and (3,-9)
   Polygon 3: (-10,-2); (-5,-2); (-6,2); and (-9,2)
   Polygon 4: (-9,-7); (-4,-7); (-2,-4); and (-7,-4)
   Polygon 5: (5,1); (5,-3); (-1,-3); and (-1,1)
4. Then have your students select all the polygon number(s) which describe the shapes listed:

   rectangle: _____ square: _____
   trapezoid: _____ parallelogram: _____
   rhombus: _____

5. Explain.

POSSIBLE SOLUTIONS

Use graph paper for plotting of points
rectangle: – 2 and 5 square: – 2
trapezoid: – 3 parallelogram:– 1, 2, 4 and 5

rhombus: – 1 and 2

EXTENDING THE ACTIVITY

Part A: Which polygon has the largest area? smallest area?
Which two polygons have the same area?
Explain your thinking.
Part B: What are the coordinates of polygon 3 rotated around the y-axis?

ASSESSMENT 1

Use grid paper to plot the points: A(5,1); B(8,3) and C(4,4). Label and connect the points. What is the area?

a. 12 square units .
b. 6 1/2 square units
c. 5 1/2 square units
d. 4 square units

Ans. c (Enclose the shape inside a rectangle. Find the area of the rectangle and the three outside triangles. Add the areas of the triangles and subtract from the area of the rectangle. This will give the area of the shape.

ASSESSMENT 2

Then draw the image of the shape reflected over the x-axis. What are the coordinate points of this shape?

Ans. The first points plotted are of a triangle in the first quadrant. When it is reflected over the x-axis the new points are: (5,-1); (8,-3) and (4,-4).

For additional information and activities, see pages 107-108 and 114
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 8

KEY IDEA # 5 MEASUREMENT

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS:

5A. Estimate, make, and use measurements in real-life situations.
5B. Select appropriate standard and nonstandard measurement units and tools to measure to a desired degree of accuracy.
5C. Develop measurement skills and informally derive and apply formulas in direct measurement activities.
5D. Use statistical methods and measures of central tendencies to display, describe, and compare data.
5E. Explore and produce graphic representations of data (calculators/computers may be used).
5F. Develop critical judgment for the reasonableness of measurement.

INVESTIGATION

ON THE SURFACE

Students will examine solids to determine which has the least surface area.

WHAT'S THE MATHEMATICS?

- understanding the uses of units, square units, and cubic units
- knowing and applying formulas for perimeter and area of polygons, volume or rectangular solids, circumference, and area of circles
- deriving and using formulas for surface areas of solids, volume of right circular cylinders, spheres, cones, and pyramids.
- understanding length, area, and volume and make relationships between the measurements
- finding the measure of the sides and angles of a right triangle, using the Pythagorean theorem and trigonometric ratio

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M2a Is familiar with assorted three-dimensional objects, rectangular prisms, pyramids, cylinders
M2c Identifies three dimensional shapes from two dimensional perspectives
M2d Determines and understands volume, including perimeter and surface area

M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY:	surface area	rectangular solid	pyramid
cylinder			
	volume	height	rectangular prism
square			
	area	base	altitude
radius		circumference	
diameter	vertex		
	lateral area		face

SUGGESTED TIME: 1-2 lessons

MATERIALS: geoblocks: rectangular solid, cylinder and pyramid with square base

DOING THE INVESTIGATION

1. Allow your students to work in groups to examine geometric solids.
2. Discuss with your students the various properties of geometric solids.
3. Have your students closely examine the rectangular solid, the cylinder and the pyramid with a square base.
4. Give them the dimensions for each solid. The rectangular solid (in inches) is 3 x 4.75 x 7. The cylinder (in inches) has a diameter of 4.5 and a height of 7. The pyramid (in inches) has a base 5 on a side and the triangular faces have an altitude of 11.
5. Pose the problem that Madelaine is buying a gift for a friend and wants to buy the solid that will require the least amount of wrapping paper. Which solid should she buy? Show your work.
6. If wrapping paper is purchased by the square foot, how many square feet of paper are needed to wrap all three solids? Explain.

POSSIBLE SOLUTIONS

The surface area of the rectangular solid is the sum of the area of each face or twice the sum of the area of the three different faces. Thus, 3 x 4.75 = 14.25, 3 x 7 = 21 and 4.75 x 7 = 33.25. The surface area = 2 x (14.25 + 21 + 33.25) = 137 square inches.

The surface area of the cylinder is twice the area of the base plus the lateral area (the circumference of the base times the height). Each base is a circular region whose total areas are 2 x (π x 2.25^2) = 2 x (π x 5.0625) = 31.80862562. The circumference of the base is $\pi d = \pi$ x 4.5 = 14.14. This is multiplied by the height, 7 x 14.14 = 98.98. Therefore, the surface area is 130.79 square inches.

The surface area of the pyramid is the area of the four triangles plus the area of the square base. The area of the base is 25 square inches. The area of the triangles is 4 x (1/2 x b x h) = 110 square inches. The surface area is 135 square inches.

The sum of the surface area of the three solids is 137 + 130.8 + 135 = 402.8 square inches.
Note: Depending on decimals places and the use of calculators, the answers will vary.

EXTENDING THE ACTIVITY

If Madelaine would like to choose the gift that holds the most sand, which should she choose? Show your work.

ASSESSMENT 1

The dimensions of a rectangular solid are x, 2x, and 3x. If all the dimensions are doubled, what is the relationship between the surface area of the original rectangular solid and the new surface area?

a. eight times larger
b. six times larger
c. four times larger
d. two times larger

Ans. c (The surface area of the first solid is $2(x \cdot 2x + 2x \cdot 3x + x \cdot 3x) = 22x^2$. The surface area of the second solid is $2(2x \cdot 4x + 4x \cdot 6x + 2x \cdot 6x) = 88x^2$.

ASSESSMENT 2

Create two cylinders from two different pieces of 8 1/2 inches by 11 inches paper. Make one cylinder long and thin and the other short and fat (hold the paper lengthwise and make the first cylinder and then use the width to make the second cylinder. Which cylinder has the greater volume? Explain.

Ans. The shorter cylinder has the greater volume. (The dimensions of the longer cylinder are circumference 8.5 inches and height 11 inches. The dimensions of the shorter cylinder are circumference 11 inches and height 8.5 inches. A formula can be used to find the volume of each, but since the lateral areas of both cylinders are equal in measure, it is only necessary to look for the cylinder with the larger diameter.

For additional information and activities, see pages 109 and 115
in the New York State Core Curriculum/Mathematics Resource Guide
GRADE 8

KEY IDEA # 6 UNCERTAINTY

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with every day situations.

PERFORMANCE INDICATORS:

6A. Use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.
6B. Use estimation to solve problems for which exact answers are inappropriate.
6C. Estimate the probability of events.
6D. Use simulation techniques to estimate probabilities
6E. Determine probabilities of independent and mutually exclusive events.

INVESTIGATION

WHAT ARE MY CHANCES?

Students will investigate theoretical and experimental probability by conducting a series of experiments with multiple trials, comparing results, combining results and making conclusions.

WHAT'S THE MATHEMATICS?

- conducting a variety of simulations to represent an experiment that can not be determined by theoretical probability or is not practical to determine experimentally
- understanding and using empirical and theoretical probability, using the formula $P(E) = f/n$
- expressing probabilities as fractions, percents, or decimals
- predicting the results of a series of trials once the probability for one trial is known

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M4h Represents and determines probability as a fraction of a set of equally likely outcomes; recognizes equally likely outcomes and constructs sample spaces.
M4i Makes predictions based on experimental or theoretical probabilities.
M4j Predicting the result of a series of trials once the probability for one trial is known.
M5a Formulation
M5b Implementation
M5c Conclusion

VOCABULARY: theoretical probability experimental probability

equally likely	unbiased	biased
fair	unfair	

SUGGESTED TIME: 1 lesson

MATERIALS: spinner divided into three equal regions labeled red, blue and green,

paper cup

DOING THE INVESTIGATION

1. Discuss with your students the meaning of theoretical probability and experimental probability.
2. Have students look at a spinner with three equal regions labeled red, green and blue (three possible outcomes).
3. Discuss with your students that the probability of spinning any one color is 1/3.
4. Have your students predict how many of each color should be spun when spinning 100 times.
5. Pose the problem: Maxine spun the spinner 100 times and landed on blue 41 times, on red 29 and green 30. Ask your students to explain the results.
6. Richard decided to toss a paper cup. He predicted that the cup could land an equal amount of times on its side, on the open side (top) and on the closed side (bottom). That means that there is 1/3 chance of landing any of those ways. Predict if Richard is correct. Why/why not?
7. Work in a small group and do Richard's experiment. Each person should toss the cup 100 times and record the results. What were each person's results? What were the group results (all results combined)? What can you say about Richard's prediction? Explain.

POSSIBLE SOLUTIONS

Maxine only spun the spinner 100 times and she was close to obtaining 1/3 on each color. If all the results of all the students were combined, the probability would be closer to 1/3 for each color. The sample space with just Maxine's results was not large enough. Each region had an equally likely chance of being landed upon.

Richard did not have an equally likely chance of tossing the cup and landing as predicted. However, the only way to truly find out the probability is to try the experiment to see if there really is a 1/3 chance of landing as predicted. Just as Maxine, 100 trials may not be enough of a sample to determine the results. Thus, each student must toss the cup 100 times and the results must be combined.

EXTENDING THE ACTIVITY

Select an object like the paper cup that might not have an equal chance of landing equally on all surfaces. Predict what your results might be. Where you correct? Explain.

ASSESSMENT 1

When tossing a fair die with numbers 1, 2, 3, 4, 5 and 6 each written on a face, which result
would not be expected to have an equally likely outcome?

a. tossing an even number or an odd number
b. tossing any face of the die
c. tossing a prime number or a composite number
d. tossing a multiple of 3 or a factor of 5

Ans. c (3 ways to get a prime number, 2, 3 and 5 and only two ways of getting a composite number, 4 and 6)

ASSESSMENT 2

A spinner is divided into five equal regions. Two regions are marked red, two regions are marked blue and one region is marked green. Give an example of and explain which colors have an equally likely chance of being landed on and which colors do not have an equally likely chance?

Ans. Red and blue have an equally likely chance of being spun (2/5 each). Red and green or blue and green have an unequally likely chance of being spun. The green accounts for only 1/5 of the space.

For additional information and activities, see pages 110 and 115
in the New York State Core Curriculum/Mathematics Resource Guide

GRADE 8

KEY IDEA # 7 PATTERNS/FUNCTIONS

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS:

7A. Recognize, describe, and generalize a wide variety of patterns and functions.
7B. Describe and represent patterns and functional relationships, using tables, charts, graphs, algebraic expressions, rules and verbal descriptions.
7C. Develop methods to solve basic linear and quadratic equations.
7D. Develop an understanding of functions and functional relationships: that a change in one quantity (variable) results in change in another.
7E. Verify results of substituting variables.
7F. Apply the concept of similarity in relevant situations.
7G. Use properties of polygons to classify them.
7H. Explore relationships involving points, lines, angles, and planes.
7I. Develop and apply the Pythagorean principle in the solution of problems.
7J. Explore and develop basic concepts of right triangle trigonometry.
7K. Use patterns and functions to represent and solve problems.

INVESTIGATION

HOW MANY TOOTHPICKS?

Students will investigate a series of designs looking for patterns and generalizing what they have found.

WHAT'S THE MATHEMATICS?

- identifying, describing, representing, extending, and creating patterns
- describing functions and generalizing by the use of rules and algebraic expressions
- describing and representing numerical and geometric patterns and functions using equations, graphs, and tables
- organizing and analyzing data resulting in function applications through use of a table of values, sentence, formula, graph, and prediction

RELATED NEW YORK CITY PERFORMANCE STANDARDS

M3a Discovering, describing, and generalizing patterns, including linear, exponential and simple quadratic relationships.
M3b Representing relationships with tables, graphs in the coordinate plane, and verbal or symbolic rules.
M3c Analyzing tables, graphs, and rules to determine functional relationships.
M5a Formulation
M5b Implementation
M5c Conclusion

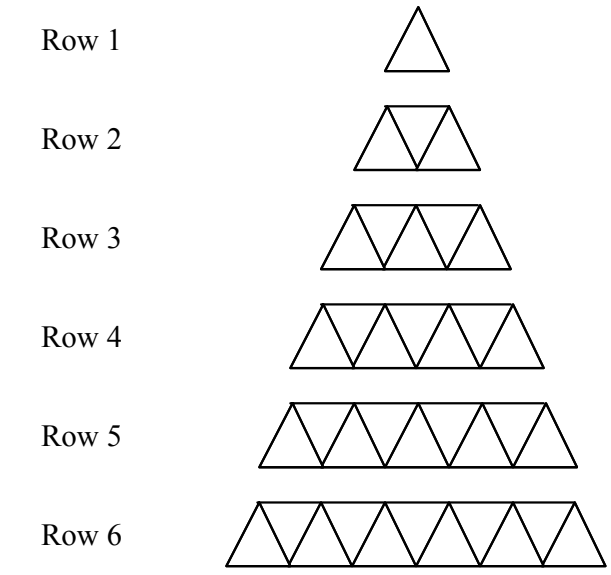
VOCABULARY: pattern generalization

SUGGESTED TIME: 1 - 2 lessons

MATERIALS: toothpicks

DOING THE INVESTIGATION

1. Discuss with your class the meaning of patterns.
2. Have students give examples of patterns, numerical, geometrical, etc.
3. Have your students use toothpicks to create a shape.
4. Have your students repeat the shape so that it extends lengthwise.
5. Draw these designs. Have your students examine the following designs (patterns):



6. Use the designs to fill in the chart below:

Length of Design	Number of Toothpicks
1	3
2	7
3	11
4	
5	
6	
7	

7. Describe at least three patterns seen in the table.
8. Write a statement that will always give your the number of toothpicks if the length of the design is known.

POSSIBLE SOLUTIONS

The number of toothpicks in the table from the beginning are: 3, 7, 11, 15, 19, 23 and 27. Some patterns are: the length of the design increases by consecutive numbers; the number of toothpicks increases by 4; the diagonal from left to right, e.g. 1 to 7, 2 to 11, etc. increases by multiples of 3; the diagonal from right to left, e.g. 3 to 2, 7 to 3, etc. increases by 3. There are other patterns.

Four times the length of the design minus one will always give the number of toothpicks.

EXTENDING THE ACTIVITY

Take another look at the investigation. Different students suggested formulas to generalize a rule for the number of toothpicks to use. They developed their formulas by looking at a design with a length of 4. They let T = number of toothpicks and L = length of the design.

Joan looked at the design like this:

She gave the following formula: $(L-1)\bullet 4 + 3 = T$

Debbie looked at the design like this:

She gave the following formula: $(L\bullet 3) + (L-1) = T$

Josephine looked at the design like this:

She gave the following formula: $L + (L-1) + 2L = T$

Explain each student's reasoning.

ASSESSMENT 1

Look at the design 3 of a pattern created using toothpicks:

How many toothpicks will be in the 8th design?

a. 19 b. 23 c . 31 d. 39

Ans. c (Students can either create a chart to look for a pattern or rebuild the design.)

ASSESSMENT 2

Use the design in assessment 1 to generalize a rule. Explain.

Ans. Let D = the design number and T=the number of toothpicks
4D - 1 = T (There are other ways of generalizing this rule.)

For additional information and activities, see pages 111 and 116
in the New York State Core Curriculum/Mathematics Resource Guide