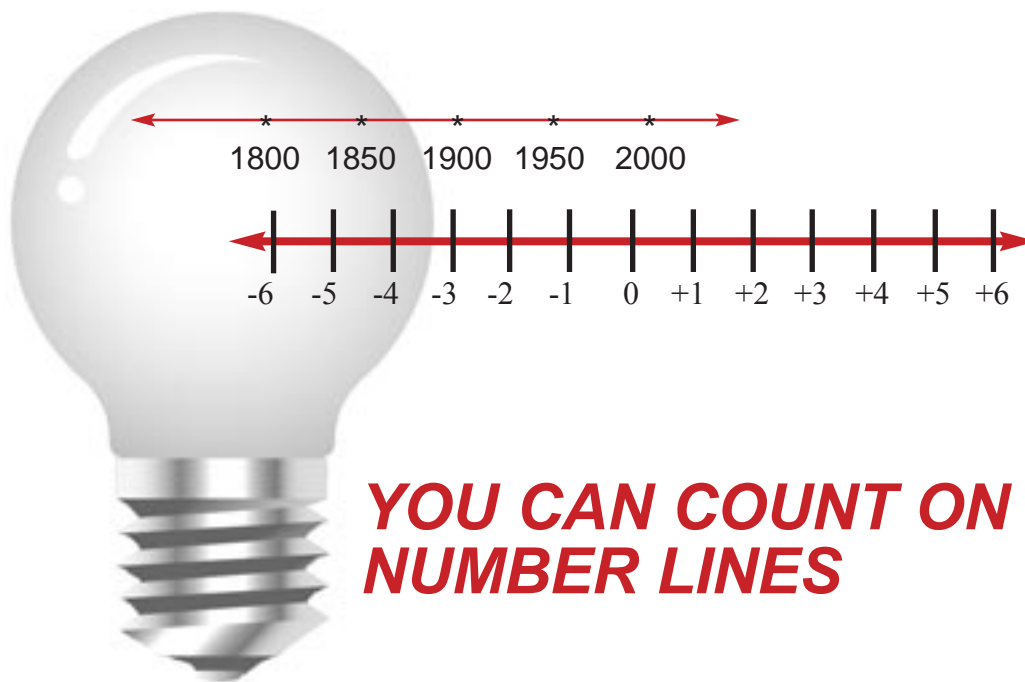


## Key Idea 2—Number and Numeration:

Students use number sense and numeration to develop an understanding of multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

### Overview:

Students begin developing number sense by counting, recognizing, and ordering whole numbers. Students relate counting to grouping and place value. Numeration expands to understanding, ordering, and comparing fractions and decimals. Intermediate students represent and use numbers in a variety of forms (integers, fractions, decimals, percents, exponents, expanded and scientific notation). Students learn to apply ratios, proportions, and percents, and develop an understanding of rational and irrational numbers. Commencement-level students apply the properties of real numbers to various subsets of numbers and gain proficiency at recognizing the hierarchy of the complex number system.



## **YOU CAN COUNT ON NUMBER LINES**

### Description:

The number line is an excellent tool for developing numerical understanding and making connections across number systems. Through the use of number lines, students can visually represent the relationships among whole numbers, integers, rational numbers, and irrational numbers. This representation gives each number a unique point on the line and a distance from zero depicting its magnitude and direction. Students use number lines to display operations, equivalents, and solutions.



## Elementary Performance Indicators

Students will:

- Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.
- Use concrete materials to model numbers and number relationships for whole numbers, fractions, and decimals.
- Recognize the order of whole numbers and commonly used fractions and decimals.

### PreK – K

1. *Number recognition:*  
Call out a number and have children move to or place a counter on the number line.
2. *Concept of **between**:*  
“Stand or place your counter on the number between 6 and 8.”
3. *Concepts of **after, before, and following**:*  
“Go to the number that follows the number 5.”
4. *Concept of **greater than or less than**:*  
“Stand on a number greater than 4.”  
“Move to the number that is 2 more than 3.”
5. *Awareness of **ordinal number** names from first to tenth:*  
“Move to the fourth spot on the number line.”

**NOTE:** Teachers may use large number lines on the floor or wall, or individual number lines on student desks. Students should develop the skills to construct their own number lines.

### Grades 1 – 2

1. *Compare and order whole numbers:*  
Students show **values** of whole numbers by locating numbers up to 999 on number lines.
2. *Justify answers:*  
Students **estimate** the value of a position on the number line by using number sense and mathematical reasoning (see Activity Sheet 1).
3. *Round numbers:*  
Students locate numbers on a number line to visually see the concept of **rounding** to the nearest 10 or 100.
4. *Skip count:*  
Students use number lines to **skip count** forward and backward by 2’s, 3’s, 5’s and 10’s.
5. *Fractions:*  
Students fold number lines to begin to develop the concept of fractions such as  $\frac{1}{2}$  or  $\frac{1}{4}$ . A number line from 0 to 10 folded in half shows that 5 is  $\frac{1}{2}$  of 10. Students use number lines to understand that **fractional parts** of a **whole number** are **equal**.
6. *Operations:*  
Students use number lines to show the concepts of addition and subtraction. For  $3 + 4 = 7$ , stand on the 3 and walk 4 steps forward to the 7; for  $7 - 4 = 3$ , put your counter on the 7 and move it back 4 steps.
7. *Construct number lines:*  
Students begin to construct number lines, paying special attention to the concept of **equal intervals**. Construct a number line from 20 to 40. Write the numbers 37 and 23 in their appropriate places on this number line.

## Grades 3 – 4

### *Whole Numbers:*

1. Students locate and order numbers to 1,000,000 on number lines.
2. Given a location on a number line, students estimate the number it represents.
3. Students use the number line to visually round numbers to the nearest 10, 100, 1000, etc.

### *Fractions and Decimals:*

1. Have students locate and label fractions and decimals on number lines to visualize the concept that there are numbers between whole numbers.
2. Have students use number lines to estimate whether a fraction or decimal is closer to 0 or 1.
3. Have students locate and label eighths on a number line to show concept of **equal parts**.
4. Have students fold a number line from 0 to 1 into fourths and label the parts. By folding again, into eighths, and labeling, the students visualize **equivalent fractions** of  $\frac{1}{4}$  and  $\frac{2}{8}$ ,  $\frac{1}{2}$  and  $\frac{4}{8}$ , etc.
5. Have students locate and label fractions with denominators of 2, 3, 4, 5, 6, 8, 10, and 12 on a number line to order and compare fractions.
6. Have students compare fractions by estimating whether a fraction is more or less than  $\frac{1}{2}$  (see Activity Sheet 2).

### *Operations:*

1. Have students use the number line to see that multiplication is **repeated addition**.

### *Negative Numbers:*

1. Have students use a thermometer as a number line that contains both **positive** and **negative** numbers.
2. Have students use a number line to demonstrate that  $5 - 7 = -2$ .

### *Measurement:*

1. Have students observe that both the **standard ruler** and the **meterstick** are finite number lines.

## Intermediate Performance Indicators

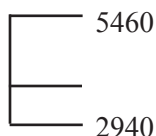
Students will:

- Understand, represent and use numbers in a variety of equivalent forms (integer, fraction, decimal, percent, exponential, and expanded notation).
- Understand and apply ratios, proportions, and percents.
- Develop an understanding of number theory (primes, factors, and multiples).
- Recognize order relations for decimals, integers, and rational numbers.

### Grades 5 – 6

1. *Compare and order numbers:*

Have students **estimate** a number on the number line and explain their reasoning.

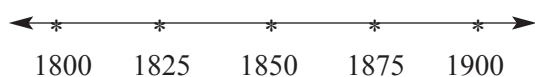


Have students draw a number line representing 0 to one million. Have students locate and label 165,000 on the number line and explain their reasoning.

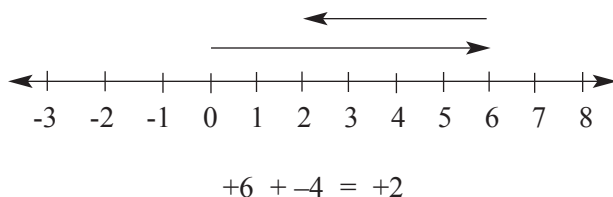
2. *Concept of equivalent fractions:*

Have students locate and label **whole numbers, proper and improper fractions, decimals, and exponents** on a number line (see Activity Sheet 3).

3. Have students develop an understanding that chronological timelines are a form of number lines.

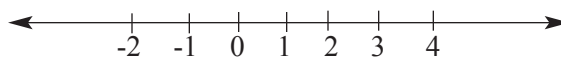


4. Have students use the number line to add and subtract **integers**.



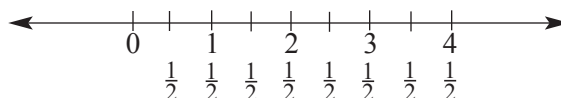
### Grades 7 – 8

1. Have students use a number line to locate and label **positive and negative integers, rational numbers, irrational numbers, square roots, and numbers written in scientific notation**.

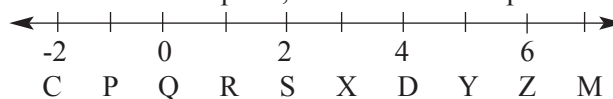


$\sqrt{0.6}$ ,  $2.6 \times 10^{-1}$ ,  $\sqrt{3}$ ,  $-\frac{19}{18}$ ,  $1\frac{2}{3}$

2. Have students use a double number line to understand the relationship between the **Fahrenheit** and **Celsius** scales (see Activity Sheet 4).
3. Have students use a number line to visually represent division of **mixed numbers** and fractions. If 4 divided by 2 means, “How many 2’s are in 4,” then 4 divided by  $\frac{1}{2}$  means, “How many halves are in 4?” (see Activity Sheet 5).



4. Have students show the **density property** by finding numbers between any two given numbers. For example, a number between  $\frac{2}{5}$  and  $\frac{4}{5}$  could be  $\frac{7}{10}$  because  $\frac{7}{10}$  is between  $\frac{6}{10}$  and  $\frac{8}{10}$ .
5. Have students use a number line to find the midpoint of a line segment.
- Find the midpoint of line segment CD.
  - If P is an endpoint of a line segment and X is the midpoint, find the other endpoint.





## Commencement Performance Indicators

Students will:

- Understand and use rational and irrational numbers.
- Recognize the order of real numbers.
- Apply the properties of real numbers to various subsets of numbers.

### Math A

1. Have students review the meaning of  $=$ ,  $\geq$ ,  $>$ ,  $\leq$ ,  $<$ , and  $\neq$  and graphically represent solutions to equations and **inequalities** (see Activity Sheet 6, Part 1, Items 1 – 6). Have students find the algebraic solutions for multistep **linear** inequalities and graph the solutions on the number lines provided. Students may use any number in their solution set to check their answer (see Activity Sheet 6, Part 2).
2. Have students review their solutions for **quadratic equations** of the form  $x^2 = 4$ , in which the solutions are both  $\pm 2$  (see Activity Sheet 6, Part 1, Items 7 and 8).
3. Have students find solutions for quadratic equations of the form  $x^2 = 2$  (in which the solutions are both  $\pm \sqrt{2}$ ) and  $x^2 = 3$  (in which the solutions are both  $\pm \sqrt{3}$ ). Have students use a calculator to find a decimal approximation for their solutions.
4. Have students find the exact location of  $\pm \sqrt{2}$  and  $\pm \sqrt{3}$  on the number line by **construction**. Have students explain how they could use their knowledge of special right triangles to perform this task (see Activity Sheet 7).

### Math B

1. Have students solve simple **quadratic inequalities** algebraically.  
Example:  $x^2 > 4$  which has solutions of  $x > 2$  or  $x < -2$   
Example:  $x^2 < 9$  which has solutions of  $x > -3$  and  $x < 3$  or  $-3 < x < 3$ .  
Next, have students represent their solutions graphically.
2. Have students solve algebraically quadratic inequalities that require **factorization**.  
Example:  $x^2 - 5x - 6 > 0$  which has solutions of  $x < -1$  or  $x > 6$ .  
Example:  $x^2 - 6x + 8 < 0$  which has solutions of  $x > 2$  and  $x < 4$  or  $2 < x < 4$ .  
Next, have students represent their solutions graphically (see Activity Sheet 8).
3. Have students solve **absolute value** inequalities algebraically and represent their solutions graphically.  
Example:  $|2x + 3| < 11$  which has solutions of  $x < 4$  and  $x > -7$  or  $-7 < x < 4$ .  
Example:  $|2x - 5| \geq 1$  which has solutions of  $x \geq 3$  or  $x \leq 2$ .

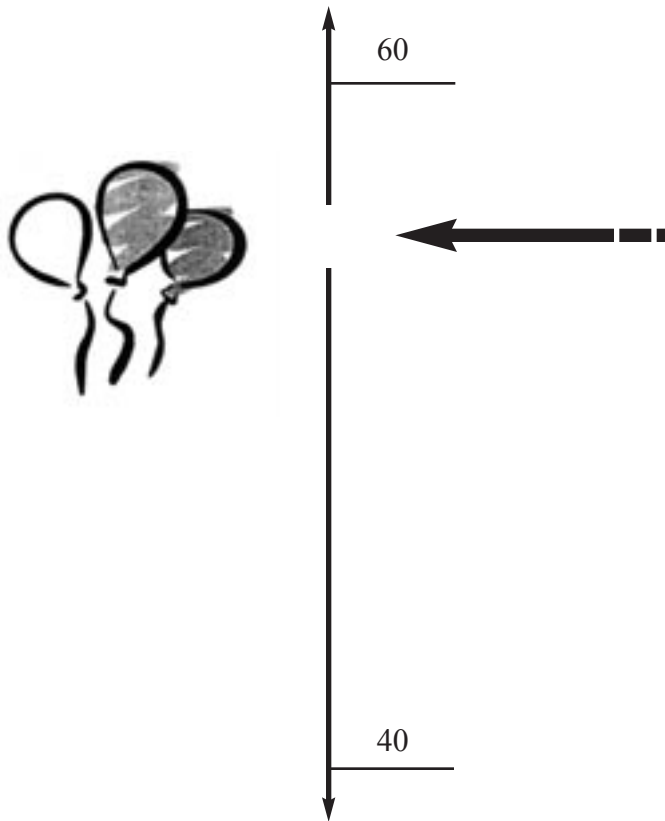




## Activity Sheet 1

### POP THE BALLOONS

Your task is to pop the balloons with the arrow by going through the hole in the number line. Estimate a number that could represent the opening on this vertical number line. Write the number on the arrow.



On the lines below, describe in words how you chose your number to pop the balloons.

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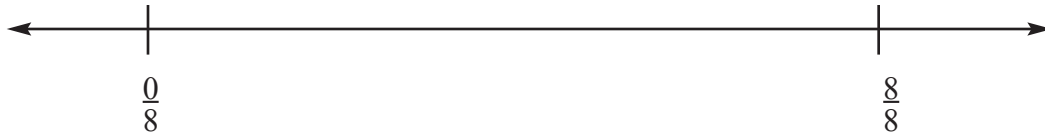
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## Activity Sheet 2

### COMPARING FRACTIONS



Label  $\frac{4}{8}$  on the number line above and explain why you placed it there.

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Is  $\frac{6}{8}$  greater than  $\frac{1}{2}$  or less than  $\frac{1}{2}$ ? How do you know? \_\_\_\_\_

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Label the following fractions on the number line above:  $\frac{10}{12}$ ,  $\frac{6}{12}$ ,  $\frac{4}{12}$ ,  $\frac{1}{12}$

Describe in words how you placed these fractions. \_\_\_\_\_

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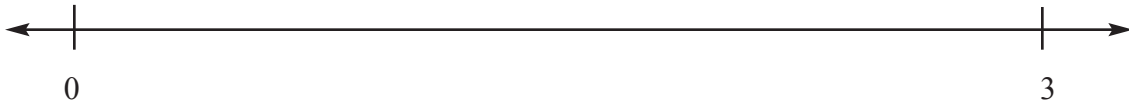
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### Activity Sheet 3

## NUMBER CHALLENGE

Locate 1 and 2 on this number line. Place the numbers below in the appropriate places on the number line. Some numbers are equivalent and will have to be put in the same place.



$\frac{3}{4}$	$\frac{22}{8}$	$2\frac{1}{4}$	$\frac{1}{20}$	$\frac{6}{12}$	$\frac{10}{10}$	$1\frac{3}{5}$
$\frac{25}{10}$	$\frac{19}{8}$	$\frac{2}{3}$	$\frac{8}{6}$	$\frac{36}{16}$	$\frac{5}{4}$	$\frac{1}{3}$
0.05	1.25	1.4	2.75	2.5	0.75	0.5
$1\frac{1}{2}$	$\frac{12}{8}$	$\frac{10}{11}$	$\frac{8}{5}$	$\frac{6}{3}$	$\frac{15}{9}$	$\frac{12}{4}$



## Activity Sheet 4

### DOUBLE STANDARDS

We can use a double number line to understand the relationship between the Fahrenheit and Celsius scales.

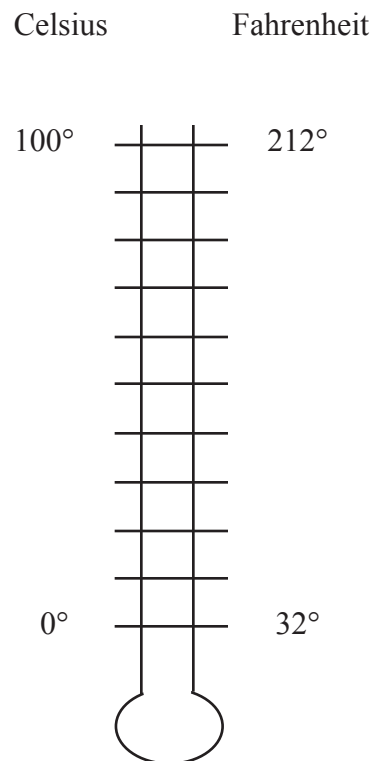
Water freezes at  $0^{\circ}\text{C}$  and boils at  $100^{\circ}\text{C}$ .

Water freezes at  $32^{\circ}\text{F}$  and boils at  $212^{\circ}\text{F}$ .

The difference between the boiling and freezing points on the Celsius scale is 100 degrees; the difference on the Fahrenheit scale is 180 degrees. The ratio of  $^{\circ}\text{C}$  to  $^{\circ}\text{F}$  is 100:180 or 5:9.

When we change from  $^{\circ}\text{F}$  to  $^{\circ}\text{C}$ , we subtract  $32^{\circ}$  from the Fahrenheit temperature and then take  $\frac{5}{9}$  of it.

To change from  $^{\circ}\text{C}$  to  $^{\circ}\text{F}$ , we take  $\frac{9}{5}$  of the Celsius temperature and add  $32^{\circ}$ .



1. Label the intervals on the thermometer above in degrees for both Fahrenheit and Celsius temperatures.
2. Find the Fahrenheit temperature when the Celsius temperature is  $20^{\circ}$ .
3. Normal body temperature in  $^{\circ}\text{F}$  is 98.6. What would normal body temperature be in  $^{\circ}\text{C}$ ?
4. When the Celsius scale indicates a temperature of  $10^{\circ}$ , what does the Fahrenheit scale read? Describe the procedure you used to arrive at your solution.



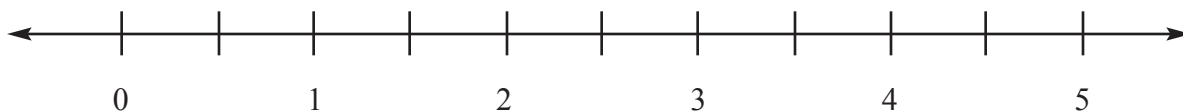


## Activity Sheet 5

### THE GREAT DIVIDE

Tom has  $4\frac{3}{4}$  feet of wire that he has to cut into  $\frac{1}{2}$  foot pieces for his technology project. How many  $\frac{1}{2}$  foot pieces will he be able to make? How much wire will he have left over?

You can use a number line to solve the problem. Represent  $4\frac{3}{4}$  feet on a number line. Use loops to indicate the number of  $\frac{1}{2}$  foot pieces that fit in  $4\frac{3}{4}$  feet.



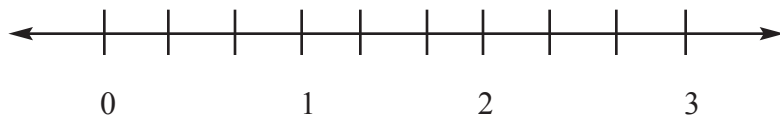
Verify that Tom can make 9 pieces of wire.

How much wire will Tom have left over? \_\_\_\_\_ Explain how you got your answer.

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1. Use a number line to divide:  $2\frac{1}{3} \div \frac{2}{3}$  \_\_\_\_\_



Verify your answer by using the division algorithm.

2. Use a number line to solve. How many stuffed animals can be made from  $3\frac{1}{4}$  yards of material if each animal requires  $\frac{3}{4}$  of a yard? \_\_\_\_\_

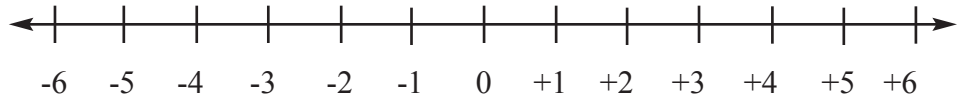


## Activity Sheet 6

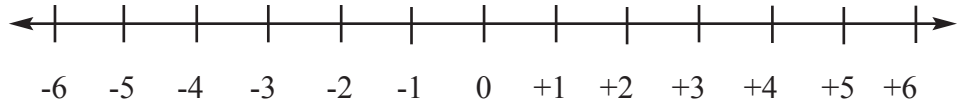
### PART 1: GRAPH SOLUTIONS TO INEQUALITIES AND EQUATIONS

Graph the solutions to each inequality.

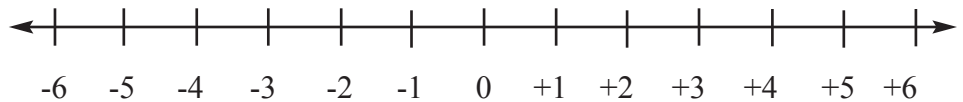
1.  $x \leq 4$



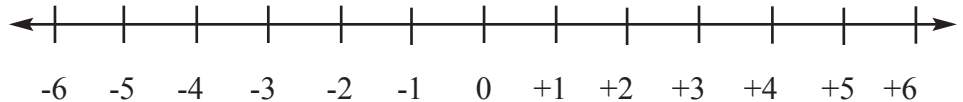
2.  $x > -3$



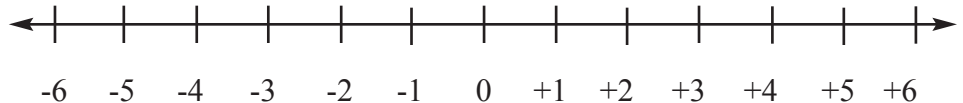
3.  $-5 \leq x < 3$



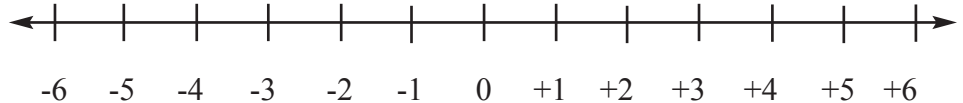
4.  $x \neq 2$



5.  $x < -2$  or  $x \geq 2$

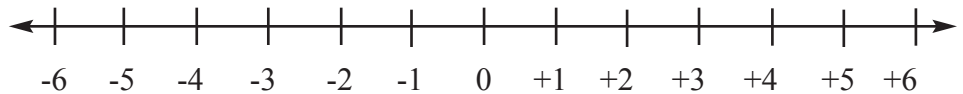


6.  $-4 < x \leq 4$

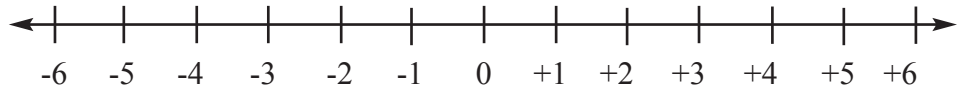


Solve each equation and graph the solution set on the number line:

7.  $x^2 = 9$



8.  $x^2 = 16$



Something to think about:

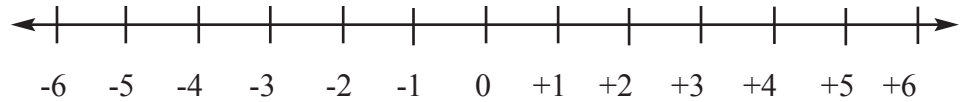
Can you think of a way to solve the equations  $x^2 = 2$  or  $x^2 = 3$  and graph the solutions on a number line?

Explain your reasoning.

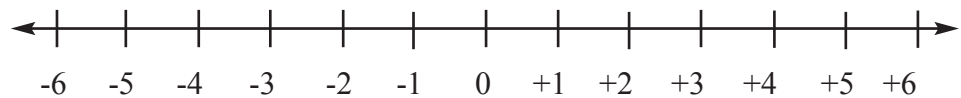
## Activity Sheet 6

### PART 2: SOLVE EACH INEQUALITY AND GRAPH THE SOLUTIONS

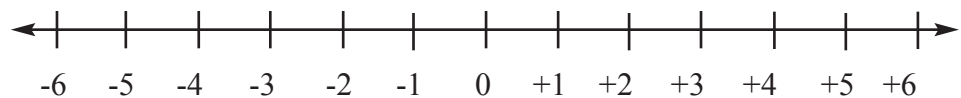
1.  $2x + 7 \leq 15$



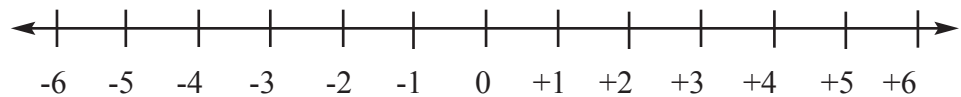
2.  $7 - 2x > 13$



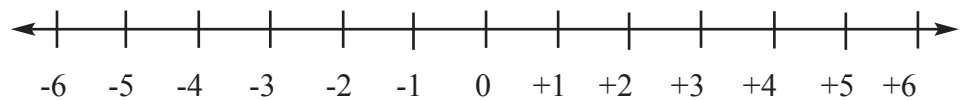
3.  $\frac{x}{2} + 7 < 10$



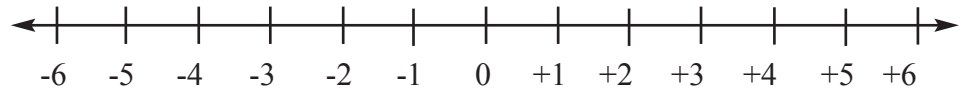
4.  $\frac{x}{3} - 2 \geq -3$



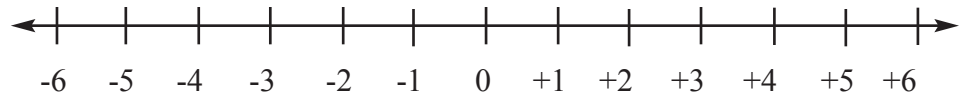
5.  $2 - \frac{x}{3} \geq 3$



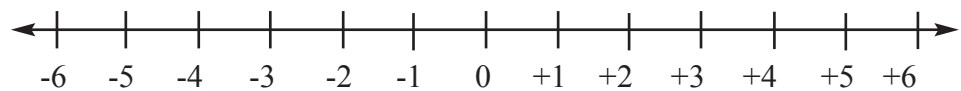
6.  $-5 < 2x + 3 \leq 5$



7.  $-5 \leq 3 - 2x < 5$



8.  $11 - 3x \leq 23$

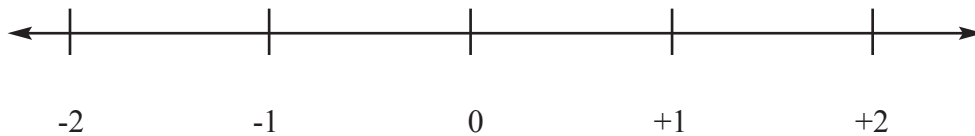


## Activity Sheet 7

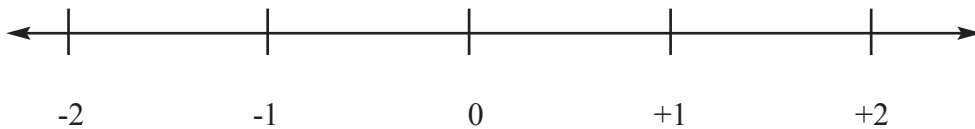
### SPECIAL RIGHT TRIANGLES

Use your knowledge of constructions and special right triangles to locate  $\pm\sqrt{2}$  and  $\pm\sqrt{3}$  on the number line.

1. Locate  $\pm\sqrt{2}$  on the number line. (Hint: Think of an isosceles right triangle.)

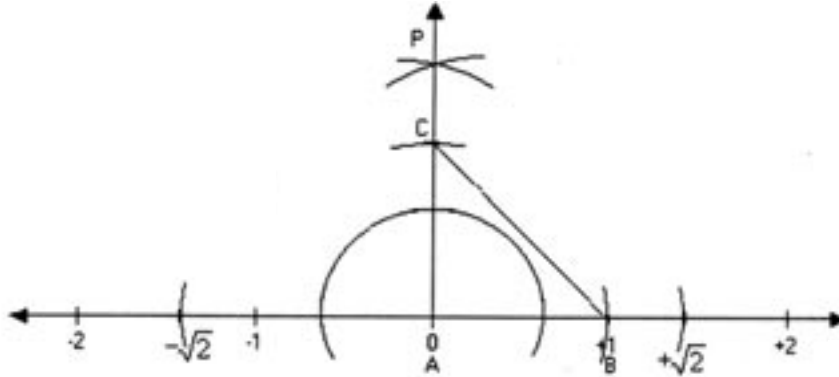


2. Locate  $\pm\sqrt{3}$  on the number line. (Hint: Think of a  $30^\circ - 60^\circ - 90^\circ$  triangle.)



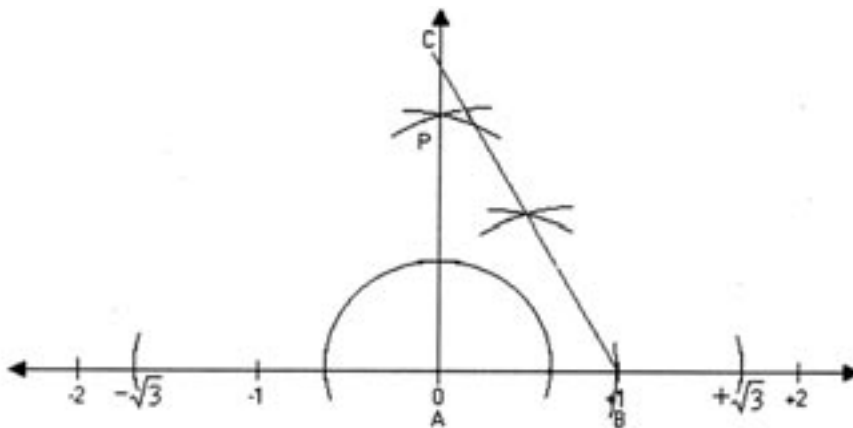
## Teacher Notes

1. Locate  $\pm\sqrt{2}$  on the number line. (Hint: Think of an isosceles right triangle.)



- |  |   |
|--|---|
| 1. Label point $A$ at 0 and point $B$ at +1  | 5. Mark off a segment equal to $BC$ on either side of point $A$ |
| 2. Construct $\overline{AP} \perp \overline{AB}$ at $A$  | 6. These points are $\pm\sqrt{2}$                               |
| 3. Measure $\overline{AB}$ and mark off $\overline{AC} \cong \overline{AB}$ on $\overline{AP}$ |   |
| 4. $BC = \sqrt{2}$   |   |

2. Locate  $\pm\sqrt{3}$  on the number line. (Hint: Think of a  $30^\circ - 60^\circ - 90^\circ$  triangle.)



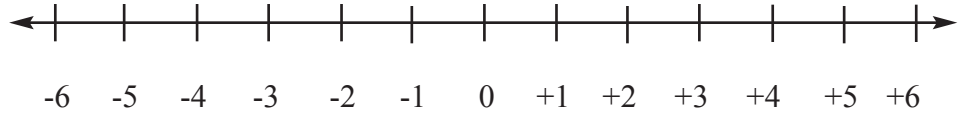
- |  |   |
|--|---|
| 1. Label point $A$ at 0 and point $B$ at +1                                    | 5. Draw $\overline{BC}$   |
| 2. Construct $\overline{AP} \perp \overline{AB}$ at $A$                        | 6. $BC = 2 AB$ and $AC = \sqrt{3}$                                |
| 3. Construct a $60^\circ$ angle at $B$ by constructing an equilateral triangle | 7. Mark off a segment equal to $AC$ on either side of the point 0 |
| 4. Label the point at which $\overline{AP}$ and $\overline{AB}$ intersect $C$  | 8. These points are $\pm\sqrt{3}$                                 |

## Activity Sheet 8

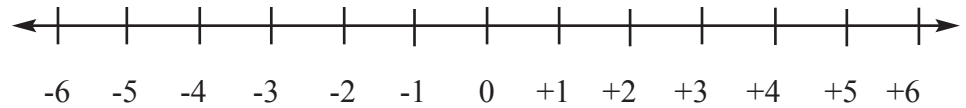
### GRAPHING QUADRATIC INEQUALITIES

Solve each quadratic inequality and graph the solutions.

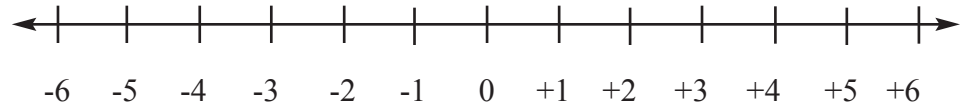
1.  $x^2 - 8x + 15 \leq 0$



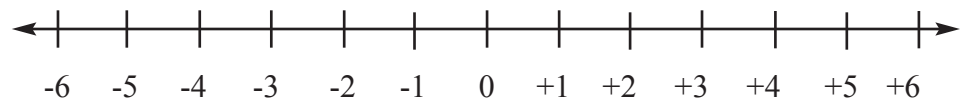
2.  $x^2 - x - 12 > 0$



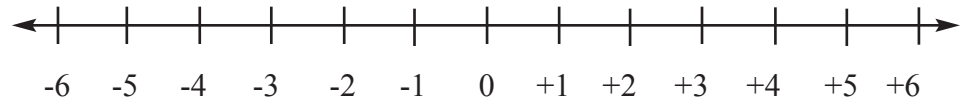
3.  $x^2 + 5x - 6 < 0$



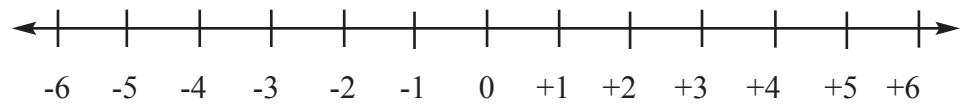
4.  $x^2 + 7x + 10 \geq 0$



5.  $x^2 + x - 6 < 0$



6.  $x^2 - 10x + 24 \leq 0$



7.  $x^2 + x - 30 \geq 0$

