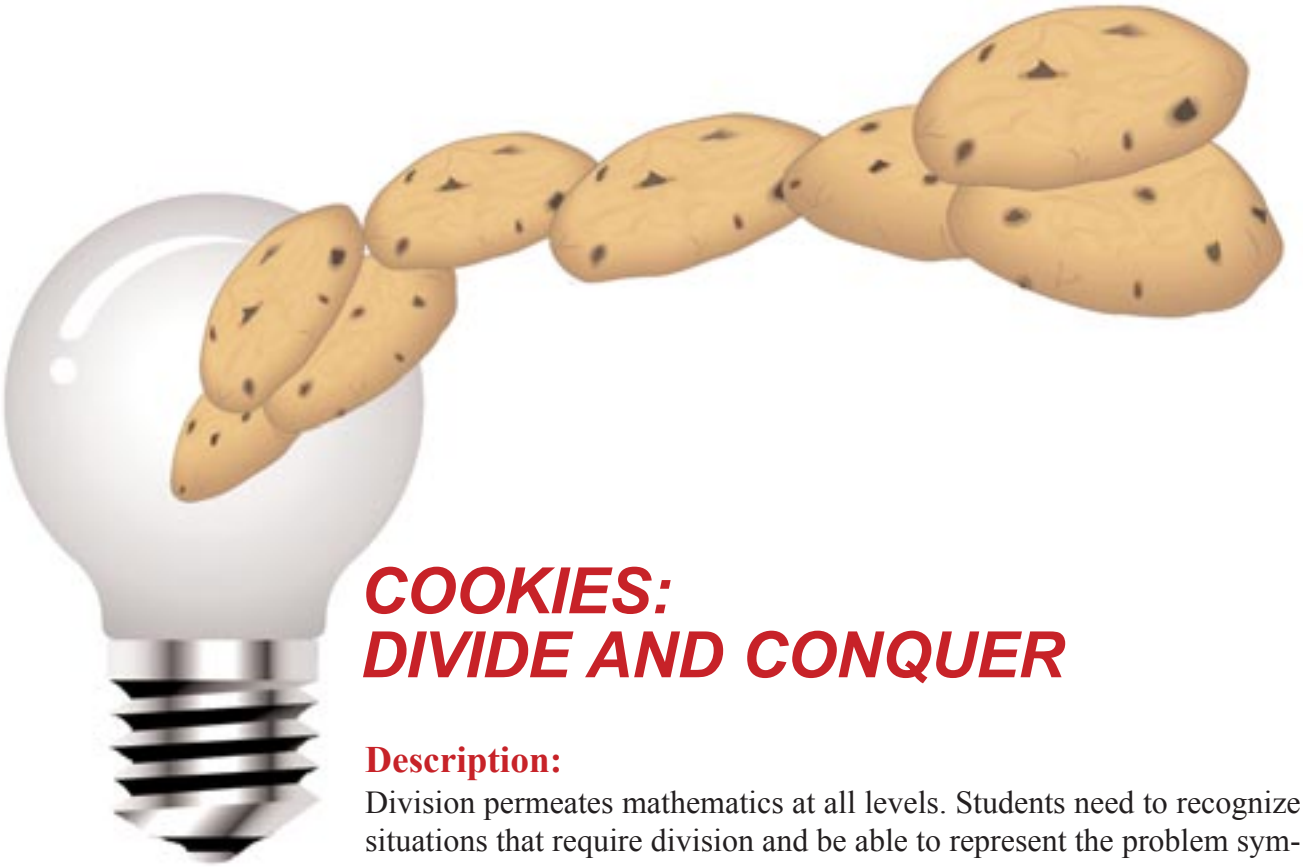
Key Idea 3—Operations:

Students use operations and relationships among them to understand mathematics.

Overview:

At the elementary level, mathematical operations begin with joining, separating, comparing, equalizing, and grouping objects. Students learn to model these relationships with numerals and symbols. Abstraction and generalization of these relationships lead to the basic number facts and properties associated with the four operations of addition, subtraction, multiplication, and division. At the intermediate level, the concepts are expanded to include fractions, decimals, percents, and integers. Students increase proficiency with algorithms (mechanical procedures) as their conceptual understanding develops. At the commencement level, students apply their knowledge of algebraic abstractions in other areas of study such as geometry, probability, and statistics. Students demonstrate proficiency in operations through applications to and solutions of real-life problems and situations.



COOKIES: DIVIDE AND CONQUER

Description:

Division permeates mathematics at all levels. Students need to recognize situations that require division and be able to represent the problem symbolically. As skill development increases, students apply their knowledge to problem solving. Relating the concept of division to everyday experiences through the use of a cookie theme helps students make connections to the real world. Activities include sharing cookies, purchasing cookies, modifying a cookie recipe, and analyzing cookie package parameters.

Elementary Performance Indicators

Students will:

- Add, subtract, multiply, and divide whole numbers.
- Develop strategies for selecting the appropriate computational and operational method in problem-solving situations.
- Know single-digit addition, subtraction, multiplication, and division facts.
- Understand the commutative and associative properties.

PreK – K

1. Pose the following cookie problem: "There are four cookies to be shared by you and a friend. How many cookies will you each get?" Have students act out this scenario with real cookies or manipulatives (links, unifix cubes, etc.).
2. Pose similar scenarios, such as using six cookies and three friends or eight cookies and four friends.
3. Discuss the concept of **fair share** when a quantity is separated into equivalent groups. Have students also discuss the number of groups.
4. Introduce the concept of **leftovers** by posing the problem with five cookies instead of four: "You and a friend are to share five cookies."
5. Pose similar *leftover* situations, using manipulatives or real cookies.

Grades 1 – 2

1. Pose the following cookie problem: "There are eight cookies to be shared by four students. If each student is to have an equal share, how many cookies will each student get?" Have students divide a plate of cookies among themselves through the use of construction paper cookies or manipulatives (links, unifix cubes, etc.).
2. Pose the problem: "Four students are to share 13 cookies." Explore what happens now.
3. Discuss the concept of fair share, **groups**, and **remainders**.
4. Using other number combinations, students should explore different scenarios with manipulatives or paper cookies and discuss the results.
5. Have students create their own story problem and draw a picture to represent that story.

Grades 3 – 4

1. Have students model the division scenario as it develops in the story **The Doorbell Rang**, by Pat Hutchins, using construction paper cookies or manipulatives (links, unifix cubes, etc.) and small paper plates (to represent each child in the story). As a follow-up, have students create similar number stories that they can act out.
2. Have students use manipulatives to solve problems with remainders. "If four students are sharing 10 cookies, how many cookies will each student receive?"
3. Introduce students to the symbolic representation of **division** (÷). Extract symbolic representation of division from life experiences. "If four students share eight cookies, it is written as 8 ÷ 4 (eight cookies divided into four groups)."
4. Have students compare the meaning of related fact family problems. "If two students share eight cookies, it is written as 8 ÷ 2 (eight cookies divided into two groups)." Have students represent both facts pictorially. Using manipulatives, students should explore similar related problems such as 12 ÷ 3 and 12 ÷ 4, etc.
5. Incorporate the language and meaning of **divisor**, **dividend,** and **quotient** into problems.
6. Have students progress to an abstract level (without manipulatives) using their knowledge of the multiplication facts to solve problems.
7. At an appropriate date later in the year, have students use base ten materials when working with three-digit and four-digit dividends (see Activity Sheet 1).
8. Introduce division with two-digit divisors beginning with divisors that are multiples of 10.

NOTE: Examples of children's literature that illustrate the concept of division.

A Remainder of One by Elinor Prinzes
Divide and Ride by Stuart Murphy
Monster Math by Grace Maccarone
One Hundred Hungry Ants by Elinor Prinzes
17 Kings and 42 Elephants by Margaret Mahy

Intermediate Performance Indicators

Students will:

- Add, subtract, multiply, and divide fractions, decimals, and integers.
- Explore and use the operations dealing with roots and powers.
- Use grouping symbols (parentheses) to clarify the intended order of operations.
- Apply the associative, commutative, distributive, inverse, and identity properties.
- Demonstrate an understanding of operational algorithms (procedures for adding, subtracting, etc.).
- Develop appropriate proficiency with facts and algorithms.
- Apply concepts of ratio and proportion to solve problems.

Grades 5 – 6

1. Have students revisit the meaning of division with whole numbers using two- or three-digit **dividends** and one- or two-digit **divisors**.
2. Have students create story problems that require mental calculations. For example, "36 cookies will be shared among four students. How many cookies does each student get?"
3. Have students solve division problems that require interpretation of **remainders**. Discuss the implications for each of the following scenarios:
   a) "Cookies are sold in packages of 24. There are 179 students. How many packages need to be purchased so that each student gets one cookie?"
   b) "There are 24 students in the class. 179 cookies were purchased. How many cookies can each student have?"
4. Have students use cookie problems to apply the divisibility rules for 2, 3, 4, 5, 6, 9, and 10 to determine whether there will be a remainder.
5. Have students divide decimals to the hundredths place using whole number divisors (see Activity Sheet 2).
6. Have students divide common fractions and mixed numbers using **multiplicative inverse (reciprocal)** (see Activity Sheet 2).

Grades 7 – 8

1. Have students determine the unit cost of items to compare prices. For example, "A 12-ounce bag of chocolate chips costs $2.89. A 20-ounce bag costs $4.49. Explain which is the better buy." Ask students to explore other *best buy* situations.
2. Have students use **ratios** and **proportions** to solve scaling problems involving recipes (see Activity Sheet 3).
3. Ask students to collect recipes and develop problems, such as the one on Activity Sheet 3.

Commencement Performance Indicators

Students will:

- Use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions.
- Develop an understanding of and use the composition of functions and transformations.
- Explore and use negative exponents on integers and algebraic expressions.

Math A

1. Have students use their knowledge of division and three-dimensional objects to investigate real-world packaging problems. Students should decide what information is needed to solve the problems, write an equation involving division, and find its solution (see Activity Sheet 4, Items 1 – 4).
2. Have students solve problems involving the division of algebraic expressions (see Activity Sheet 4, Item 5).

Math B

1. After students have simplified **rational expressions** and **complex fractions**, have them explore algebraic **functions** that involve division both algebraically and graphically (see Activity Sheet 5). Incorporate the use of graphing calculators.
2. Have students explore the concept of division by zero as it relates to algebraic fractions and functions (see Activity Sheet 5, Item 2c).
3. Have students explore the concepts of **domain** and **range** as they relate to problems involving division of functions.

Activity Sheet 1

TRAYS OF COOKIES

- Give each student (or pair of students) a place value mat, base ten blocks, and trays.
- Pose the problem: "We have 452 cookies to be placed on three trays. How many cookies should be placed on each tray?"
- Start by having the students place 452 on their place value mats to represent the cookies (four flats, five rods, and two units).
- Ask the students to separate the blocks into three groups.

> **Solution:** Each tray (group) gets one flat. Trade the extra flat for 10 rods. There are now 15 rods. Each tray (group) gets five rods. There are still two units remaining. The final answer will be that each tray will have 150 cookies, and there will be a remainder of two cookies.

- Students need to use the base ten materials and become comfortable with them before they can move on to the connecting step of manipulating the material and writing the algorithm. Students may be at this level for quite some time before going to the symbolic or abstract level.

Activity Sheet 2

THE COOKIE CRISIS

There are 20 students in a class.

1. The teacher wishes to treat the class by purchasing cookies for them. If a package of cookies costs $1.80 and there are 30 cookies in the package, how much does each cookie cost? (Show your work and explain how you arrived at your answer.)

2. Unfortunately, the teacher forgot to go to the store, but she does find 6 cookies left over in the cookie jar. If each student eats only $\frac{1}{2}$ of a cookie, how many students will get $\frac{1}{2}$ a cookie? (Draw a picture to illustrate your solution.)

   How many of the 20 members of the class are left without a cookie?

3. When they realize they have to share 6 cookies, several members of the class complain that $\frac{1}{2}$ of a cookie is just not enough! If each person wants $1\frac{1}{2}$ cookies, how many students will now get cookies?

   How many students will not get any cookies?

Activity Sheet 3

CHOCOLATE CHIP COOKIE RECIPE

You decide you want to bake a batch of cookies for your class. You find a recipe that makes 90 chocolate chip cookies, but you only need 36 cookies so that each of your 18 students can have two cookies.

To make 90 cookies you will need:	To make 36 cookies you will need:
$5\frac{5}{8}$ cups flour	_____cups flour
$1\frac{7}{8}$ cups sugar	_____cups sugar
$1\frac{7}{8}$ cups brown sugar	_____cups brown sugar
5 eggs	_____eggs
$2\frac{1}{2}$ teaspoons vanilla	_____teaspoons vanilla
$2\frac{1}{2}$ cups butter	_____cups butter
$1\frac{1}{4}$ teaspoons baking soda	_____teaspoons baking soda
$1\frac{1}{4}$ teaspoons salt	_____teaspoons salt
30 oz. chocolate chips	_____oz. chocolate chips

1. Adjust the recipe by figuring out how much of each ingredient you will need to make exactly 36 cookies.

2. Explain the process that you used to convert the amount of each ingredient for the recipe.

Activity Sheet 4

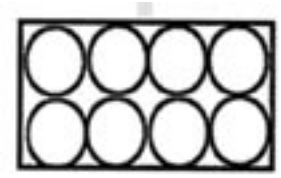
THE TOO GOOD CHOCOLATE CHIP COOKIE COMPANY

1. The Too Good Chocolate Chip Cookie Company plans to package its cookies in a cylindrical container. If the cookie has a maximum diameter of 3" and a thickness of $\frac{1}{4}$", how many whole cookies will fit into a cylindrical container whose interior height is 8.1" and whose interior radius is 1.52"?

2. Generalize a formula for determining the number of cookies, c, that can be packaged in a cylindrical container, if h represents the interior height of the container and t represents the thickness of the cookie. Assume that the radius of the cookie does not exceed the radius of the container.

3. If the company utilizes the container described in problem #1, how many containers are needed to package 260,000 cookies?

4. The cylindrical cookie containers described in problem #1 are being packaged in a shipping carton that contains 8 containers arranged as shown in the diagram at the right. What are the smallest interior dimensions of the carton, to the nearest inch, assuming an allowance of 0.1" for thickness of the wall of the cylindrical containers?



5. If the area of the bottom of the cookie carton is expressed algebraically $24x^3y^2z^5$ and the length is represented by $8x^3z^2$, represent the width of the bottom of the carton.

Activity Sheet 5

THE TOO GOOD CHOCOLATE CHIP COOKIE COMPANY

1. The Too Good Chocolate Chip Cookie Company plans to package its cookies in a cylindrical container. The maximum diameter of the cookies will not exceed the inner diameter of the container. The thickness of each cookie is represented by $\frac{a+2}{4}$, and the interior height of the container is represented by $\frac{3a^2+6a}{2}$. Represent in simplest form the number of cookies in each container in terms of a.

2. The Too Good Chocolate Chip Cookie Company also manufactures miniature cookies. The function $f(x) = \frac{x^2-1}{2x}$ represents the number of possible ounces of cookie dough in each mini-cookie. The function $g(x) = \frac{x^2-1}{2}$ represents the number of possible ounces of cookie dough contained in each carton of mini-cookies.

   a. What does $\frac{g(x)}{f(x)}$ represent?

   b. Find $\frac{g(x)}{f(x)}$ algebraically.

   c. Are there any values of x for which $\frac{g(x)}{f(x)}$ is undefined?

   d. Using your graphing calculator, graph the function $\frac{g(x)}{f(x)}$, choosing an appropriate window. List the window variables and justify your choice of window.

   e. Find the maximum value of $\frac{g(x)}{f(x)}$ on the interval $1 < x \leq 7$.