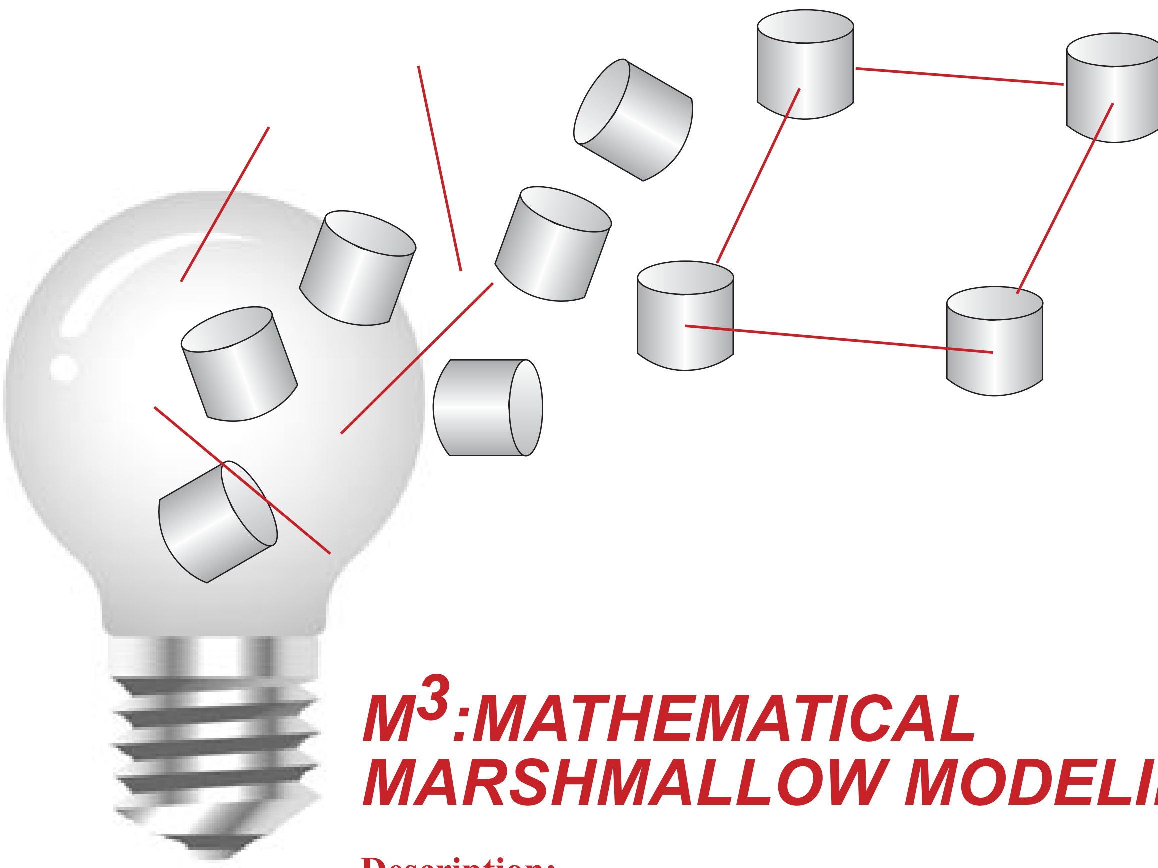
Key Idea 4—Modeling/Multiple Representation:

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Overview:

Students can assimilate abstract concepts by first constructing their own models of the concepts. They use concrete materials to model spatial relationships. Physical materials, pictures, and diagrams are used to explain mathematical ideas and processes and to demonstrate geometric concepts. Constructing models supports exploration of geometric concepts at an abstract level and verifies geometric conjectures. Students will collect data, use geometric formulas, determine the best curve-fitting model, and make predictions.



M³:MATHEMATICAL MARSHMALLOW MODELING

Description:

Students will construct concrete objects such as triangles, squares, rectangles, pyramids, and cubes with toothpicks and miniature marshmallows to create two- and three-dimensional objects. They will use these models to explore various mathematical concepts from elementary to commencement level.

Elementary Performance Indicators

Students will:

- Use concrete materials to model spatial relationships.
- Construct tables, charts, and graphs to display and analyze real-world data.
- Use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.
- Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.

PreK – K

1. Have students use attribute blocks or precut models to sort and identify **circles**, **squares**, **rectangles**, and **triangles**.
2. Have students sort the models into sets according to size. Discuss the concept of **larger** and **smaller** (see Activity Sheet 1).

Grades 1 – 2

1. Have students construct models of a triangle and a square from marshmallows and toothpicks.
2. Have students compare the number of **sides** and **vertices** of each model. Discuss the similarities and differences.
3. Have students "match up" their squares and triangles. **Congruent polygons** have the same size and shape.
4. Have students communicate their findings in writing (see Activity Sheet 2).

Grades 3 – 4

1. Have students use toothpicks and marshmallows to demonstrate representations of geometric concepts: **point**, **line segment**, **parallel**, **intersecting**, **perpendicular**, and **angle**.
2. Have students construct models of a square and a triangle, describing each polygon with appropriate geometric vocabulary.
3. Using each of the models, students should construct models of a **tetrahedron** (triangular pyramid) and a **cube**. Students will describe and discuss the geometric concepts of **face**, **edge**, and **vertex**.
4. Have students communicate their findings in writing (see Activity Sheet 3).

Intermediate Performance Indicators

Students will:

- Visualize, represent, and transform two- and three-dimensional shapes.
- Use concrete materials and diagrams to describe the operation of real-world processes and systems.
- Use appropriate tools to construct and verify geometric relationships.

Grades 5 – 6

1. Have students plot points to form a polygon in the **coordinate plane**.
2. Have students construct the shapes with marshmallows and toothpicks.
3. Have students perform **transformations**, such as **reflections** (flips) and **translations** (slides), and discuss the changes to the **coordinates** of the figures.
4. Have students discuss coordinate rules for reflections and translations in the coordinate plane.
5. Have students communicate their findings in writing (see Activity Sheet 4).
6. The teacher and/or students can demonstrate these transformations with a graphing calculator or appropriate computer software.

Grades 7 – 8

1. Using marshmallows and toothpicks, have students construct **polygons** whose areas can be found by subdividing them into triangles or rectangles.
2. Have students brainstorm ways to calculate the area of the polygon they constructed. (One way is to divide the polygon into smaller, familiar polygons, calculate the areas of the smaller polygons using area **formulas**, and add the areas of the smaller polygons to find the total area of the original polygon.)
3. Have students use a **nonstandard measurement**, such as the length of a toothpick, to express the area of the original polygon.
4. Have students generalize their **conjectures** and explain a method for finding the areas of polygons.
5. Have students find the area of a polygon whose dimensions cannot be measured in toothpicks (irrational). Use the **Pythagorean theorem** and the subdividing procedure developed in this activity (see Activity Sheet 5).

Commencement Performance Indicators

Students will:

- Represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs.
- Choose appropriate representations to facilitate the solving of a problem.
- Manipulate symbolic representation to explore concepts at an abstract level.
- Use learning technologies to make and verify geometric conjectures.
- Use graphing utilities to create and explore geometric and algebraic models.

Math A

1. Have students construct **similar** squares, rectangles, and triangles using marshmallows and toothpicks in order to investigate the relationships between the **corresponding sides**, the **perimeters**, and the **areas** of the **similar figures**.
2. Have students find the lengths of sides, perimeters, and areas of the similar figures, and determine the **ratios** of the corresponding sides, the perimeters, and the areas of the similar figures.
3. Have students make generalizations by analyzing their results, forming conjectures, and explaining the relationships that are evident in the activity.
4. Have students verify their findings by making a construction that illustrates what they discovered about the relationships between the corresponding sides, the perimeters, and the areas of the similar figures (see Activity Sheet 6).

Math B

1. Have students enter **data** in a graphing calculator and **plot** the data (see Activity Sheet 7).
2. Have students calculate and analyze the **correlation coefficient (*r*)** on the basis of the model. Students should understand that r provides the following information:
   a) how close the data are to the graph of the **regression equation**
   b) the strength of the relationship between the two variables.
3. Convey to the students that correlation does not imply a cause-and-effect relationship.
4. Have students look at data and consider the type of curve-fitting model to use (**linear**, **exponential**, **logarithmic**, or **power**).
5. Have students produce the corresponding equation that fits the data (linear, exponential, logarithmic, or power). This analysis should include making certain that what is being done makes sense.
6. Have students select and use appropriate **statistical methods** to analyze data, determine the best curve-fitting model to make predictions, discuss r as it relates to the data, and determine the best mathematical model for the given data.

Activity Sheet 1

FIND MY SHAPE

Students identify shapes by size, color, and number of sides.

Materials:
Attribute blocks or paper models replicating a set of attribute blocks

Procedure:
1. Divide students into groups of two or three and give each group a set of attribute blocks.

2. Allow students appropriate time to explore the blocks. Listen to the conversations among the students as they talk about them. Encourage the students to describe a shape to the others in their group. All the blocks are then put back into the center of the group.

3. Ask each student to find a shape from the pile as described by the teacher.

   Possible descriptions include:

   a) a big shape
   b) a small shape
   c) a shape with three sides
   d) a shape with four sides

Activity Sheet 2

MAKE MY SHAPE (TRIANGLES AND SQUARES)

Students construct and identify polygons by counting the number of sides and the number of vertices.

Materials:
Round toothpicks
Miniature marshmallows (These are better when they are a day old and have been exposed to air.)
Pencil and paper for recording

Procedure:
1. Divide students into groups of two or three.

2. In the center of the work surface (table or grouped desks), each group will have sufficient marshmallows and toothpicks for construction of models.

3. Demonstrate how to connect a toothpick to a marshmallow without pushing the toothpick all the way through the marshmallow.

4. Ask each student to construct a triangle from three toothpicks and three marshmallows.

5. Have some students describe their models to the class.

6. Ask each student to construct a square from four toothpicks and four marshmallows.

7. Have students explain the difference between a square and a triangle.

8. Allow students in each group to compare their triangles and squares to find congruent shapes. (They can match the length of the sides.)

9. Have each student write a letter to someone describing how the shapes were constructed and how a square is different from a triangle. Encourage the students to draw pictures to help in their explanations.

Activity Sheet 3

MAKE MY SHAPE (CUBES AND TETRAHEDRONS)

Students construct and identify polygons and geometric solids (polyhedra) by counting the number of edges, faces, and vertices.

Materials (per student):
18 round toothpicks
12 miniature marshmallows
Pencil and paper for recording (or a chart designed by the teacher)

Procedure:

1. As a review, have each student use two toothpicks to demonstrate: parallel line segments, intersecting line segments, perpendicular line segments, and a right angle.

2. Demonstrate how to connect a toothpick to a marshmallow without pushing the toothpick all the way through the marshmallow.

3. Ask each student to construct a triangle from toothpicks and marshmallows. Explain that the toothpicks will serve as sides and the marshmallows as vertices. Ask some students to describe their procedure, noting the number of sides and angles.

4. Ask each student to construct a square from toothpicks and marshmallows. Have some students describe a square. Encourage the use of comments such as “opposite sides are parallel” and “there are four right angles.”

5. Have each student use the triangle as a base and construct a tetrahedron with three more toothpicks and one more marshmallow. This is a geometric solid with four triangular faces. On a piece of paper (or chart), have each student describe and record the shape of the faces and count the number of faces, edges, and vertices.

6. Have each student construct two squares and then construct a cube by connecting the two squares with four more toothpicks. On a piece of paper (or chart), have each student describe and record the shape of the faces and count the number of faces, edges, and vertices.

7. Have each student write a letter to someone describing how the shapes were constructed and how a cube is different from a tetrahedron.

Activity Sheet 4

TRANSFORMATIONS

Students use marshmallows and toothpicks to explore transformations in the coordinate plane.

1. Draw and label the x-axis and the y-axis. In quadrant I, make any shape whose vertices are on points of intersection.

2. Label the coordinates of the vertices. (This is your starting position.)

3. Using marshmallows and toothpicks, construct a model over your drawing.

4. Reflect (flip) your model in the x-axis. Trace the image and label the coordinates of its vertices.

5. What happened to the coordinates of the vertices after the reflection in the x-axis? Write a sentence explaining what happened. Look at your neighbors' work. Did they have different shapes and different starting positions? Compare explanations.

6. Place your shape back on your starting position. Reflect your shape in the y-axis. Trace the image and label the coordinates of its vertices.

7. What happened to the coordinates of the vertices after the reflection in the y-axis? Write a sentence explaining what happened. Look at your neighbors' work. Did they have different shapes and different starting positions? Compare explanations.

8. Place your shape back on your starting position. Translate (slide) your shape three units to the right and five units up. Trace the image and label the coordinates of its vertices.

9. What happened to the coordinates of the vertices after the translation? Write a sentence explaining what happened. Look at your neighbors' work. Did they have different shapes and different starting positions? Compare explanations.

10. What would happen if you moved to the left and/or down? Try it! How did it affect the coordinates of your image? Write a sentence explaining what happened.

11. Challenge: Compare your findings with your neighbors' work. Discuss and write the coordinate rules for translations and reflections in the x-axis and y-axis.

Activity Sheet 5

AREA

Students use marshmallows and toothpicks to explore areas of polygons.

1. Using exactly 22 marshmallows and 22 toothpicks, construct a block capital letter E.
2. Using the length of one toothpick as your unit of measure, find the area of the model.
3. Do you know a formula for finding the area of a capital letter? ____________________________
4. What area formula(s) do you know that may be helpful in finding the area of the model?
   ____
5. Explain how you might use the formula(s) to solve the problem. ___________________________
   ____
6. What is the perimeter of your model? _____________________ Are you surprised? ___________
7. Using exactly 14 marshmallows and 14 toothpicks, construct a right trapezoid with two sides, each five toothpicks long. What are the lengths of the other two sides?
   ____
8. Do you know a formula for finding the area of a trapezoid? ___________________ If you do, then use it to find the area of the trapezoid (using the length of one toothpick as your unit of measure). ____
9. Can you find the area another way?__________
   Please try. You may use other area formulas, if you wish.
10. What is the perimeter of the trapezoid? ___________________ Are you surprised? ____________
11. From your experiences with marshmallows and toothpicks in steps 1 – 8 above, write a method for finding the area of a polygon when you do not know a formula.
    ____
    ____
    ____

Activity Sheet 6

SIMILARITY

Students use marshmallows and toothpicks to explore the ratios of the perimeters and the ratios of the areas of similar figures.

1. Using at most 20 marshmallows and 20 toothpicks, construct two squares of different sizes.

2. Explain why the squares are similar. ____
   ____
   ____

3. Using the length of one toothpick as your unit of measure, find the lengths of sides, the perimeter, and the area of each square. ____
   ____
   ____

4. What is the ratio of the lengths of the corresponding sides of the two squares? __________

5. What is the ratio of the perimeters of the two squares? __________

6. What is the ratio of the areas of the two squares? __________

7. Using at most 36 marshmallows and 36 toothpicks, construct two similar rectangles.

8. Explain why the rectangles are similar. ____
   ____
   ____

9. Using the length of one toothpick as your unit of measure, find the lengths of the sides, the perimeter, and the area of each rectangle. ______________________________________
   ____
   ____

10. What is the ratio of the lengths of the corresponding sides of the two rectangles? __________

11. What is the ratio of the perimeters of the two rectangles? __________

12. What is the ratio of the areas of the two rectangles? __________

13. Using at most 45 marshmallows and 45 toothpicks, construct two similar triangles.

14. Explain why the triangles are similar. ____

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15. Using the length of one toothpick as your unit of measure, find the lengths of the sides, the perimeter, and the area of each triangle. ____

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16. What is the ratio of the lengths of the corresponding sides of the two triangles? __________

17. What is the ratio of the perimeters of the two triangles? __________

18. What is the ratio of the areas of the two triangles? __________

19. From your investigations in steps 1 – 18, make a conjecture about the relationship between the ratio of the lengths of the corresponding sides and the ratio of the perimeters of similar figures.

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20. From your investigations in steps 1 – 18, make a conjecture about the relationship between the ratio of the lengths of the corresponding sides and the ratio of the areas of similar figures.

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Activity Sheet 7

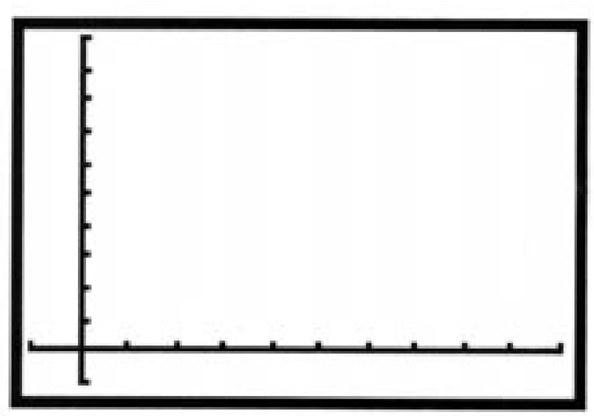
REGULAR POLYGONS

A regular polygon has congruent sides and congruent interior angles.

1. Plot the following data on your graphing calculator with the independent variable being the number of sides (3 – 12) and the dependent variable being the measure of each interior angle (to the nearest hundredth) of the regular polygon.

Name of Polygon	Number of Sides	Measure of Interior Angle
Equilateral Triangle	3	60°
Square	4	90°
Regular Pentagon	5	108°
	6	120°
	7	128.57°
	8	135°
	9	140°
	10	144°
	11	147.27°
	12	150°
	. . .	
	20	
	. . .	
n-gon	n	

2. Plot the data on the grid below. Label the *x*- and *y*-axes with their appropriate variable names.



3. What is the best mathematical model that would be appropriate for the data? _______________
The equation is ____

4. What is the correlation coefficient? Explain it in the context of the problem. _______________
____
____
____
____

5. Using the equation, what is the measure of each interior angle of a regular polygon with 20 sides? __________

6. Using the equation, what is the measure of each interior angle of a regular polygon with 30 sides? __________

7. If the measure of the interior angle of a regular polygon is 165°, how many sides does the polygon have? __________

8. What is the mathematical formula to find the measures of the interior angles of a regular polygon? ____

9. Does the mathematical formula relate to the mathematical model? Explain your reasoning.
____
____
____
____
____