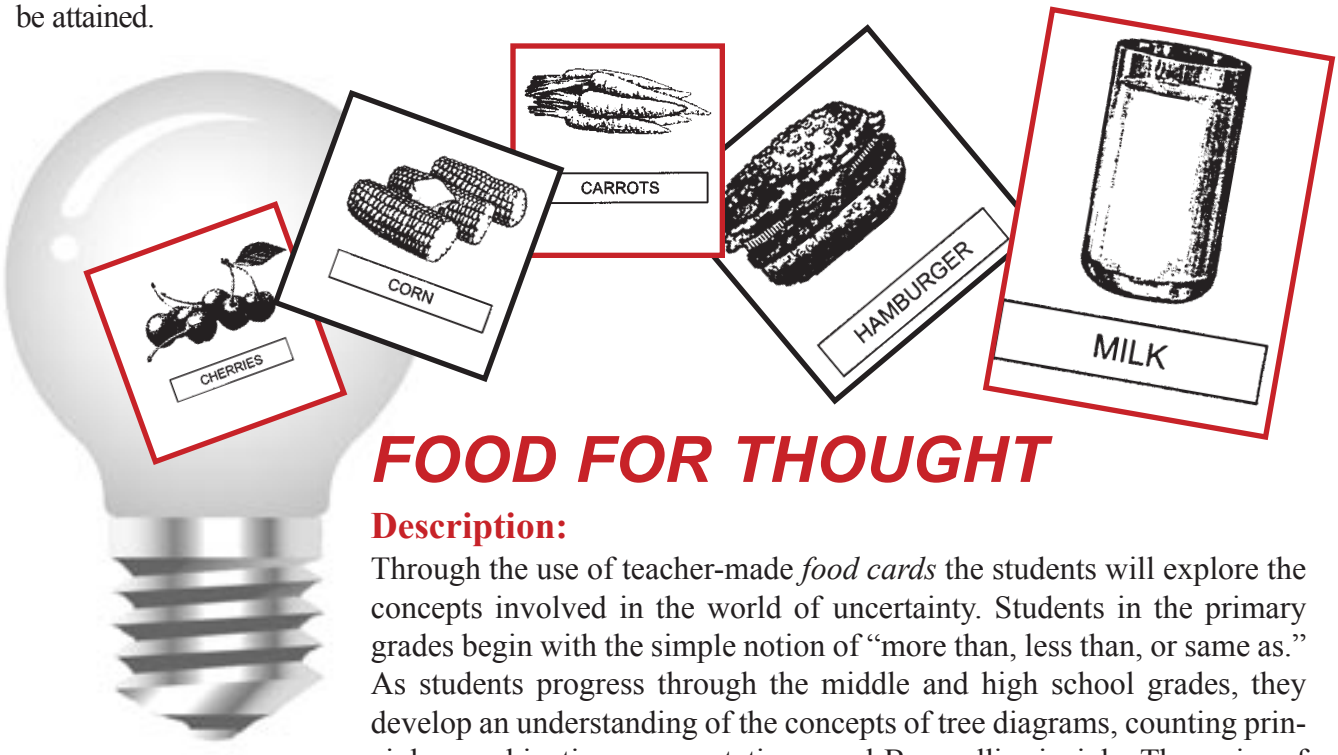
Key Idea 6—Uncertainty:

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Overview:

Uncertainty incorporates estimation as well as methods for calculating probabilities and predicting outcomes of future events. Students can perform experiments that examine possible outcomes involving repeated trials. Empirical and theoretical probability, along with estimation, are major components of the mathematical prekindergarten to grade 12 experience with uncertainty. Activities about uncertainty investigate problems incorporating manipulatives, including number cubes, spinners, cards, marbles, and coins. Students learn how to set up tables, document outcomes, and tally results. Use of frequency tables and practice in expressing probability as a fraction, decimal, or percent are all a part of the curriculum in prekindergarten to grade 6. In the intermediate levels, opportunities for experiments in uncertainty are extended. Arrangements involving order become more prevalent. Empirical versus theoretical probability is explored. Tree diagrams introduce students to uncertainty activities and provide clues for identifying relationships among independent events. As the student progresses into high school, technical terms specific to uncertainty (combinations, permutations, counting principle, Bernoulli experiment) can be developed within the context of activities. Through the use of experimentation with tasks increasing in difficulty, a deeper understanding of the topic of uncertainty can be attained.



FOOD FOR THOUGHT

Description:

Through the use of teacher-made *food cards* the students will explore the concepts involved in the world of uncertainty. Students in the primary grades begin with the simple notion of "more than, less than, or same as." As students progress through the middle and high school grades, they develop an understanding of the concepts of tree diagrams, counting principle, combinations, permutations, and Bernoulli principle. The series of activities presented will help students develop a deeper understanding of probability and the place it has in their lives. Food cards are used as the theme for these activities, but any topic can easily be substituted to fit the needs of an individual classroom. (Possible food cards are included. See Activity Sheet 1.)

Elementary Performance Indicators

Students will:

- Make estimates to compare to actual results of both formal and informal measurements.
- Make estimates to compare to the actual results of computations.
- Recognize situations in which only an estimate is required.
- Develop a variety of estimation skills and strategies.
- Determine the reasonableness of results.
- Predict experimental probabilities.
- Develop a wide variety of estimation skills and strategies.
- Make predictions using unbiased random samples.
- Determine probabilities of simple events.

PreK – K

1. The teacher will distribute sets of 20 food cards (10 turkeys and 10 carrots) to each pair of students.
2. Working in pairs, students will take turns drawing from a shuffled, face-down deck until all 20 cards are distributed.
3. Students will then sort their own cards into the two food types and arrange the cards in rows to compare the lengths. Have students discuss the results using the terms **more than**, **less than**, and **same as**.
4. Again through observation, the students pairs will compare cards of each food type, noting who has more than, less than, or the same as the other person.

Grades 1 – 2

1. The teacher will create a set of 4 meat and 4 vegetable cards for each pair of students. There should be 2 different meats and 2 different vegetables (2 copies of each type).
2. Have the students try to **predict** the number of different possible **arrangements** of one meat and one vegetable (see Activity Sheet 2, Part 1).
3. Have the students use the cards to determine the actual number of different arrangements that can be made. A preprinted 2 x 2 matrix might be helpful (see Activity Sheet 2, Part 2).
4. Have the students discuss their observations and make predictions about the number of arrangements that can be made with other size populations (e.g., 3 meats and 2 vegetables).
5. Have the students sort their cards into the two food types. Using only the vegetable cards, ask questions such as:

   What is the **probability** of drawing a meat card from this deck? (impossible or 0)

   A vegetable card? (always or every time)

   Discuss this in terms of **certainty** and **uncertainty** (see Activity Sheet 2, Part 2).
6. Collect all cards and place them together in a deck. Have the students discuss what is **more likely/less likely/equally likely** to happen—drawing a meat card or a veggie card? Have them use experimentation to test their predictions. A **tally sheet** can be used to record the **outcome** (the card drawn). Replace the card in the deck and perform the experiment 10 times.
7. Have the students form a deck containing 3 meat and 4 vegetable cards. Repeat discussions of more likely, less likely, and equally likely as in the previous activity.
8. Discuss the **fairness** of playing a game using the various deck combinations.

Grades 3 – 4

1. The teacher will create a deck of 6 food cards (3 different meats and 3 different vegetables) for each pair of students.
2. Have the students try to predict the number of arrangements or **combinations** of one meat and one veggie that can be made, using the cards as visuals. List all the combinations.
3. This concept can then be extended to drawing **tree diagrams** (see Activity Sheet 3). The teacher should lead discussion about what the tree diagram depicts and how it relates to the **counting principle**.
4. Using different numbers of meat and veggie cards to make up a deck (e.g., 2 meat and 1 vegetable), explore the **probability** that a single event will occur and express the probability as a fraction and a ratio (e.g., $P(\text{veg}) = \frac{1}{3}$ or 1:3).

Intermediate Performance Indicators

Students will:

- Use estimation to check the reasonableness of results obtained by computation, algorithms, or the use of technology.
- Estimate the probability of events.
- Use simulation techniques to estimate probabilities.
- Determine probabilities of independent events and mutually exclusive events.

Grades 5 – 6

1. The teacher will create a deck of 10 food cards (3 meats, 2 fruits, and 5 vegetables) for each pair of students.
2. Have the students **predict** (using the cards, if necessary) how many **combinations** of one meat, one fruit, and one vegetable can be made.
3. Have the students construct **tree diagrams** of these combinations to determine the accuracy of their predictions (see Activity Sheet 4).
4. Using the 3 meat, 2 fruit, and 5 vegetable cards, experimentation and discussion can take place regarding the **probability** that an **event** will occur, such as $P(\text{meat}) = \frac{3}{10}$. These probabilities can be expressed as fractions, decimals, and percents. The teacher should show how these probabilities are related to the tree diagram.
5. The teacher should lead student exploration of the concepts of **experimental probability** (number of times a desired event occurs/total number of trials) and **theoretical probability** (number of desired outcomes/total number of possible outcomes).

Grades 7 – 8

Part 1:

1. The teacher will create a deck of 20 food cards, any combination of 3 or 4 food types (e.g., 4 turkey, 2 steak, 5 corn, and 9 carrots) for each student pair. Larger quantities of cards are suggested for advanced classes.
2. Students should NOT look at the cards! Have one student draw a card, record the food type in a **frequency table**, replace the card, and shuffle the deck. Repeat this process for a total of 10 trials. The students will then calculate the **empirical** (experimental) **probabilities**, *P*(E), of each food type from their tables and express these results as a fraction, decimal, and percent.
3. Have the students predict the makeup of the 20-card deck on the basis of their tabulated results. Have the groups compare their predictions.
4. The whole process should be repeated using a **larger sampling size** (e.g., 36 draws). Their predictions should be modified if desired.
5. The class counts can then be combined to get a very large sampling, and the class can once again modify their prediction, if needed.
6. Have the students take an actual count of the deck after all sampling size trials have been completed. Discussion should take place about why the small sampling might have given different results from the larger sample sizes.
7. Class discussions should include choosing appropriate **sample sizes**, **cumulative** and **relative frequency**, empirical versus theoretical probability.
8. This problem can lead to discussions of how **polls** can be fairly accurate.

Part 2:

9. Each student receives 4 cards (1 turkey, 1 broccoli, 1 pasta, and 1 strawberry). Students should discuss how many ways these four cards can be arranged.
10. Investigate, using the cards, the actual number of ways this can be accomplished. Students should be recording the ways as they manipulate the four cards.
11. The students should discuss what they discovered; discussion should lead to the concept of **factorial** and its symbol. **NOTE:** Students should practice using this symbol (**!**) on a calculator.
12. The students should discuss the number of ways these four cards can be arranged if they can use only 3 cards. What happens if they can use only 2 cards?
13. Have the students investigate these scenarios using their four cards (recording their results as they go).
14. Have the students share their results; this should lead to a discussion of **permutations** (order matters) versus combinations (order doesn't matter).

Commencement Performance Indicators

Students will:

- Judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability and statistics.
- Use experimental and theoretical probability to represent and solve problems involving uncertainty.
- Determine probabilities, using permutations and combinations.

Math A

1. The teacher will create a deck of 32 food cards containing one meat, one vegetable, one dairy, and one fruit (8 copies of each). Working in pairs, have the students shuffle the deck, remove one card, record its type, and replace it. The students will then draw a second card and record it as the second card of an ordered pair (e.g., turkey, corn). The students should repeat this process several times, recording the ordered pairs formed and always replacing the card removed. Discuss the concept "**with replacement**."
2. Have the class discuss what patterns they notice.
3. Have the students investigate all the possible **outcomes** by laying out the pairs, using their 32 cards; then have students record the results on paper.
4. Discuss the students' findings. Possible questions include: What is *P*(meat **and** vegetable)? P(meat **or** vegetable)? *P*(~ meat)? Discuss **combinations** versus **permutations** and introduce the formulas. Calculate the probabilities both manually and by using the calculator.
5. Discuss how they think things would be different **without replacement**. In this discussion, the concept of **mutually exclusive** should be investigated.

Math B

1. The teacher will create sets of 10 food cards containing one meat and one vegetable—5 copies of each card (e.g., 5 turkeys and 5 carrots).
2. There should be a review of the concepts **and**, **or**, **not**, with replacement, without replacement, and **factorial**.

Part 1:

3. Have the students remove 2 vegetable cards and 3 meat cards from the deck. Using the cards as visuals, the students will display all the ways to get "**exactly** 2 vegetable cards if five cards are being drawn." *At this time don't discuss the probability aspect—just the possible arrangements* (e.g., MMVVM, MVMVM, etc. [10 possible arrangements]).
4. Repeat this activity, using a different number of desired vegetable cards (e.g., "exactly 1 vegetable card if 5 cards are drawn").
5. Show the students that the combination formula gives them those numbers ($_5C_2$ and $_5C_1$).
6. Repeat, using different situations (e.g., "exactly 2 meats in 4 choices," etc.).

Part 1:

7. Have the students make a "square" game board by using 1 meat and 3 vegetable cards. (Put the other cards aside. They will not be used at this time.) Before starting, have the students decide what the rules for a penny dropping game should be, including "do-over" situations like landing on a line.
8. Discuss the probability that various events will occur. "What is the probability that if a penny is dropped onto the game board, it will land on a vegetable card *P*(V)?" "What is the probability that it will land on a meat card *P*(M)?" Discuss the concept that *P*(M) = *P*(~V). Discuss the probability of one of the scenarios discovered in Part 1. "What if you were to drop the penny 5 times and you wanted to get exactly 2 vegetables?" One possibility is MVMVM.

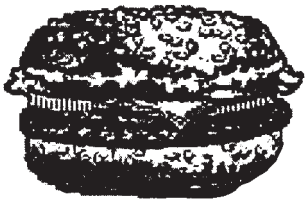
$$P(\text{MVMVM}) = \left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right)\left(\frac{3}{4}\right)\left(\frac{1}{4}\right) = \frac{9}{1024}.$$

9. Continue discussion with regard to the fact that this scenario, "exactly 2 vegetable cards in five tries," occurs 10 times (as determined in Part 1).

   Therefore, multiply $\frac{9}{1024}$ by 10 to get the probability $\frac{90}{1024}$.

10. Repeat this activity, using "exactly one vegetable in 5 tries" or "exactly 2 meats in four tries." This should lead to the **Bernoulli formula:** $_nC_rp^rq^{n-r}$.
11. Expand this activity to explore the concepts **at least** and **at most**.

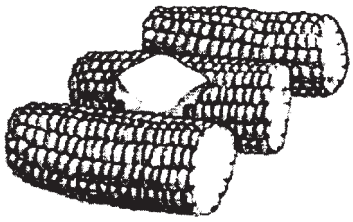
Activity Sheet 1



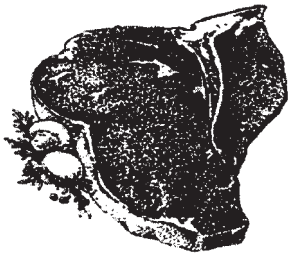
HAMBURGER

LETTUCE

TURKEY



CORN

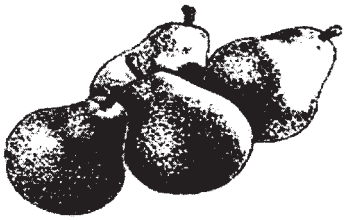


STEAK

BROCCOLI

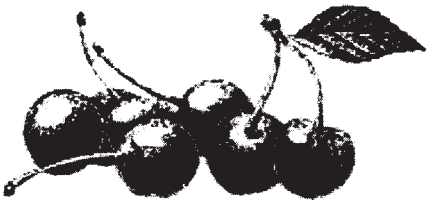
CARROTS

PEAS



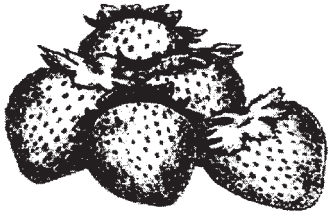
PEARS

POTATOES

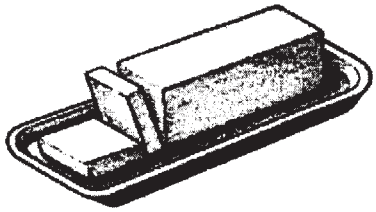


CHERRIES

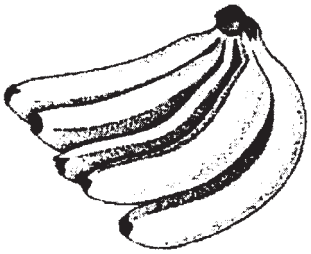
MILK



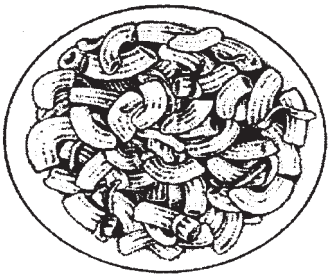
STRAWBERRIES



BUTTER



BANANAS



PASTA

Activity Sheet 2

FOOD COMBOS
PART 1

Predict the number of different possible combinations of one meat and one vegetable. _____

List possible combinations.

Compare your prediction to your list of combinations. Was your prediction accurate? _____

PART 2

Sort your cards into the two food types. Use <u>only the vegetable cards</u>. Put your meat cards aside.

What is the probability of drawing a meat card from this deck? _____

What is the probability of drawing a vegetable card from this deck? _____

Form a deck of 3 meat and 4 vegetable cards.

What is more likely to happen—drawing a meat card or a vegetable card? _____

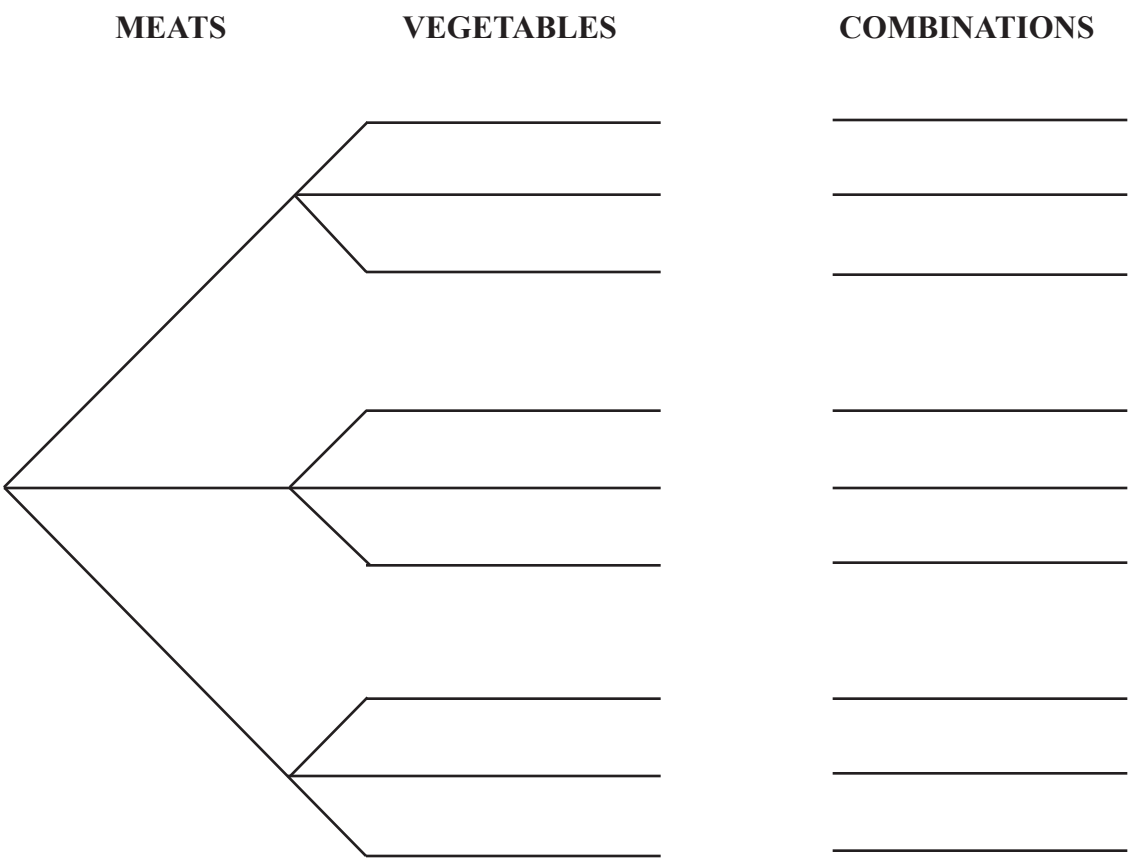
What is less likely to happen—drawing a meat card or a vegetable card? _____

What is equally likely to happen—drawing a meat card or a vegetable card? _____

	VEGETABLE 1 _____	VEGETABLE 2 _____
MEAT 1 _____		
MEAT 2 _____		

Activity Sheet 3

A TREE DIAGRAM MODEL



1. How many possible combinations are there? _____

2. Is the number of combinations greater than, less than, or equal to the number of branches?

   _____

3. What computation could you use to get the same number as your answer to question 1? Explain.

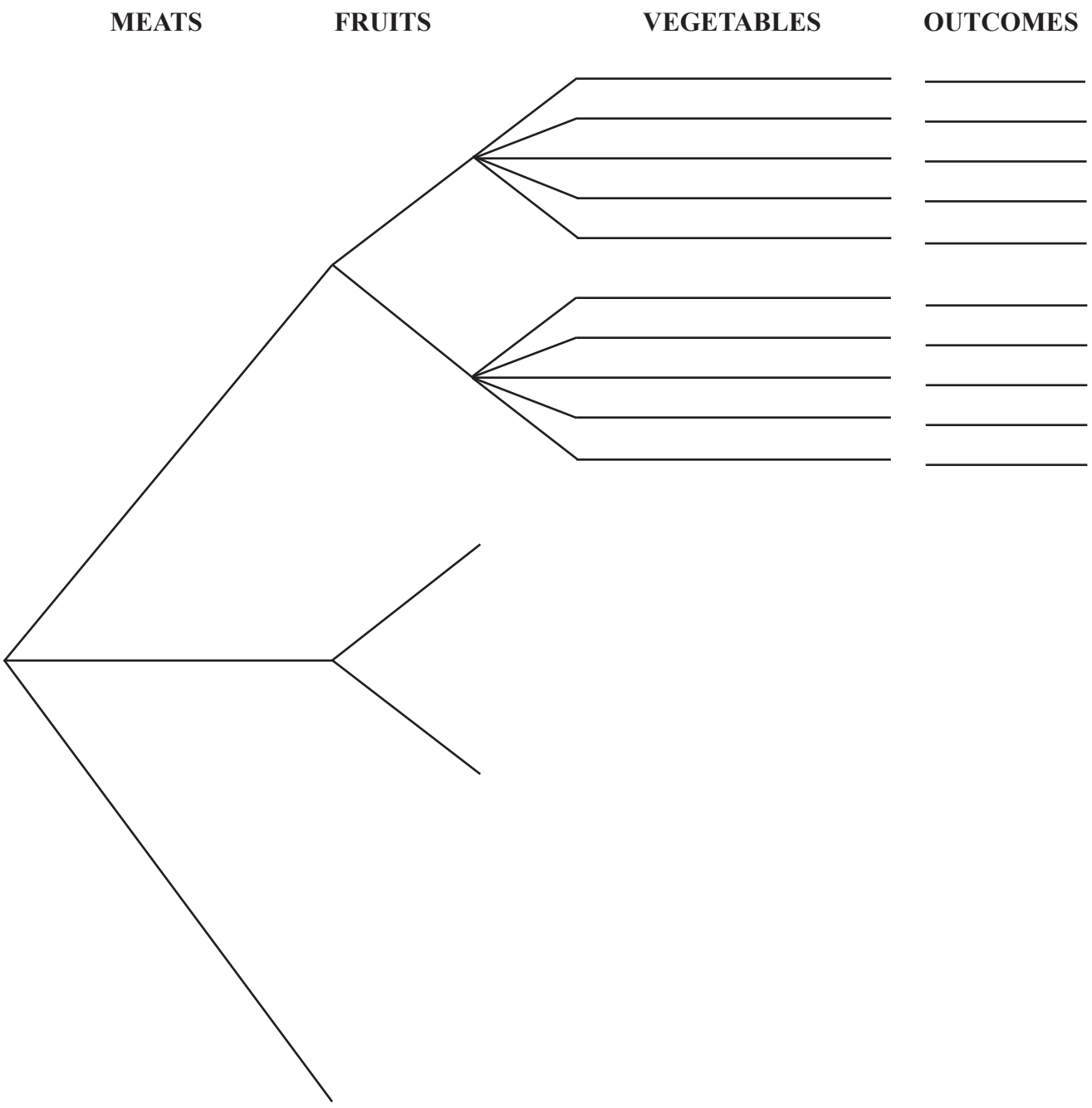
   _____

   _____

Activity Sheet 4

A TREE DIAGRAM GROWS WITH YOU

This is the beginning of the tree diagram needed to complete the activity. Complete the tree diagram.



1. How many possible outcomes are there? _____

2. What is the probability of getting a meal with a potato? Express the answer as a

   fraction _____ decimal _____ percent _____

3. If the probability of an event is more likely when it is closer to 1 than to 0, which event, getting a meal with turkey, a meal with pasta, or a meal with corn, is more likely to happen? (Hint: Calculate the probabilities and arrange the fractions or decimals in order.)

4. Describe how the tree diagram and probabilities would change if there were 2 meats, 3 fruits, and 4 veggies.

5. What computation could you use to get your answer?