Key Idea 7—Patterns/Functions:

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Overview:

Patterns help children make sense of the real world. When children are encouraged to look for patterns, they become better problem solvers. Discovering and identifying patterns in numbers and geometry help children make connections between patterns and functions and encourages good mathematical thinking. These connections support the learning of more abstract concepts in the later years. The idea of a functional relationship can be developed in the early years by exploring and observing patterns. In the middle years, the emphasis is on recognizing, describing, and generalizing functional relationships. This lays the foundation for more formal study in the high school years.

THE SNOW IS FALLING! THE SNOW IS FALLING!

Description:

Using problem simulations, students will explore the pattern of snow accumulation and possible variations to develop an understanding of functional relationships. As the activities proceed through the grade levels, various methods of describing, representing, and extending patterns will be introduced.

Elementary Performance Indicators

Students will:

- Recognize, describe, extend, and create a wide variety of patterns.
- Represent and describe mathematical relationships.
- Use a variety of manipulative materials and technologies to explore patterns.

PreK – K

1. Pose the snowfall problem as described in Activity Sheet 1 to explore the **pattern** of snow falling.
2. Have students brainstorm different ways of solving the snowfall problem. Ask "How can we figure this out?" Discuss the problem and their experiences with snowstorms. In this instance, the snow falls at a **constant rate** over a period of several hours. There is no snow accumulation when snowfall begins.
3. As a whole group, have students explore the pattern of snowfall each hour. Break it down one hour at a time: "In the first hour, how much snow fell?"
4. Model the use of manipulatives to help solve the problem: "If we use unifix cubes for the centimeters of snow, here's one cube for the first hour, here's another cube for the second hour...."
5. As a class, discuss the pattern of snowfall that the students observe.
6. Have students **predict** the amount of snow at 6 hours, 7 hours, and 8 hours if the snowfall continues at a constant rate.

Grades 1 – 2

1. Pose the snowfall problem as described in Activity Sheet 1.
2. Discuss ways to obtain a solution. Encourage students to use manipulatives and/or draw pictures.

   Example:

   1st Hour   2nd Hour   3rd Hour

3. As a whole class create a **bar graph** for each hour to show the total amount of snow that has fallen. Can students see a **pattern**?
4. Have students **predict** the amount of snow at 9 hours, 10 hours, 11 hours, and other future time periods. Assume the snowfall continues at a constant rate.
5. Vary the problem by changing the initial amount of snow on the ground and the snowfall rates.

Grades 3 – 4

1. Pose the snowfall problem as described in Activity Sheet 1.
2. Discuss ways a solution could be obtained. Encourage students to use manipulatives, draw pictures, and create tables or charts.

Hour	Total Snow
1	1 cm
2	2 cm
3	

3. Ask students to verbally describe the pattern or the **rule** (e.g., every hour another centimeter of snow falls, the total amount of snow is equal to the number of hours it snowed).
4. Repeat the same procedure with variations of the problem involving initial amounts of snow and varying snowfall rates (see Activity Sheet 2, Items 1 and 2). Have students predict the amount of snow.
5. Pose other variations of the problem (see Activity Sheet 2, Items 3 and 4) that introduce students to calculating the time the snowfall began and identifying the **rate** of the snowfall.

Intermediate Performance Indicators

Students will:

- Recognize, describe, and generalize a wide variety of patterns and functions.
- Describe and represent patterns and functional relationships using tables, charts and graphs, algebraic expressions, rules, and verbal descriptions.
- Develop an understanding of functions and functional relationships.

Grades 5 – 6

1. Pose snowfall problem as described in Activity Sheet 3, Item 1. Snow falls at a **constant rate** over a period of several hours. There is no snow accumulation when snowfall begins.
2. Have students complete a **table of values** for snowfall over a period of 5 hours, given the **data** for the first 2 hours.
3. Have students **predict** the amount of snow for various future time periods, if the snowfall continues at a constant rate.
4. Have students write the **rule** that describes the **pattern**.
5. Have students create a **graph** of the situation.
6. Repeat the same procedure with variations of the problem involving initial amount of snow and varying snowfall rates (see Activity Sheet 3, Items 2 and 3).

Grades 7 – 8

1. Pose snowfall problem as described in Activity Sheet 4, Item 1. Snow falls at a constant rate over a period of several hours. There is no snow accumulation when snowfall begins.
2. Have students create a table of values for snowfall over a period of 5 hours.
3. Have students write an **equation** to **model** the situation.
4. Have students graph the equation using their table of values.
5. Pose a variation of the problem (see Activity Sheet 4, Item 2) with a different rate of snowfall.
6. Have students write the equation that models the situation and sketch a graph.
7. Discuss the **slope** as a **rate of change** and relate the slope to both the equations and the graphs.
8. Pose a third variation of the problem (see Activity Sheet 4, Item 3) with initial snow on the ground.
9. Have students write the equation that models the situation and sketch a graph.
10. Discuss the concept of ***y*-intercept** and how it relates to both the equation and the graph.

Commencement Performance Indicators

Students will:

- Represent and analyze functions, using verbal descriptions, tables, equations, and graphs.
- Apply linear, exponential, and quadratic functions in the solution of problems.
- Translate among the verbal descriptions, tables, equations, and graphic forms of functions.
- Model real-world situations with the appropriate function.

Math A

1. Pose snowfall problem as described in Activity Sheet 5, Item 1. There is snow on the ground and snow begins to fall at a **constant** fractional **rate**.
2. Have students write an **algebraic equation** to model the situation and state the appropriate **domain**.
3. Sketch the graph of the **function**.
4. Discuss **independent** and **dependent** variables in the context of the problem.
5. Discuss the **slope** and ***y*-intercept** and explain what they represent in this situation.
6. Have students use the equation to make **predictions**.
7. Discuss the concept of **extrapolation** by having students make predictions outside the domain of the data.
8. Have students sketch a graph that appropriately **models** a particular situation involving snowfall over a period of time with varying rates (see Activity Sheet 5, Item 2).
9. Have students interpret graphs, both verbally and algebraically, involving snowfall over a period of time with varying rates (see Activity Sheet 5, Item 3).
10. Pose a situation involving two linear models of snowfall in two different towns (see Activity Sheet 5, Item 4). Have students find the **point of intersection** (same snow accumulation) both graphically and algebraically through a **system of equations**.
11. Have students use a graphing calculator to confirm their solution. (Compare data in the stat plot to the linear equation and the table.)

Math B

1. Pose snowfall problem as described in Activity Sheet 6, Item 1. Give students a **table of values** of snow accumulation over time. The pattern approximates a **linear relationship**.
2. Discuss independent and dependent variables.
3. Have students construct a **scatterplot** both on paper and on a graphing calculator and interpret the graph.
4. Have students approximate the **correlation coefficient** from the scatterplot on the basis of linearity and direction.
5. Have students find the **line of best fit** both manually and by using the graphing calculator.
6. Discuss slope and y-intercept and relate these concepts to the context of the problem.
7. Use the equation to make predictions, **interpolate,** and **extrapolate**.
8. Pose similar problem with snowfall accumulation that approximates an **exponential relationship** (see Activity Sheet 6, Item 2).
9. Have students make a scatterplot of the data and give an interpretation in terms of shape and direction.
10. Have students sketch the curve that best fits the data.
11. Have students approximate the equation of the curve in the form $y = ab^x$.
12. Discuss a and b and explain how they relate to the context of the problem.
13. Use the calculator to determine the **exponential regression** model and compare with the approximated equation.

Activity Sheet 1

It's midnight. There is no snow on the ground. The weather forecaster is predicting snow overnight. If it snows 1 centimeter each hour, how much snow will there be on the ground after 8 hours?

Activity Sheet 2

LET IT SNOW

1. There is 1 cm of snow on the ground. You want to keep track of the amount of snowfall. If it snows at a rate of 3 cm each hour, how much snow will be on the ground after 8 hours?

   Draw a picture to show what happens each hour.

   First Hour   Second Hour   Third Hour   Fourth Hour   Fifth Hour   Sixth Hour …

   Complete the chart to show the total amount of snow on the ground each hour.

Number of Hours	Total Snow (cm)
0	1
1	4
2	
3	
4	
5	
6	
7	
8	

2. There are 4 cm of snow on the ground, and it starts to snow at a rate of 3 cm per hour. How much snow will be on the ground after 8 hours?

3. There are 2 cm of snow on the ground. School A closes if there are 12 cm of snow on the roads by 6 a.m. If it snows 2 cm each hour, what time must the snow start for School A to close?

Extension:

4. There are 2 cm of snow on the ground. School B closes if there are 14 cm of snow on the roads by 6 a.m. If it starts snowing at 2 a.m., how much snow must fall each hour for School B to close?

Activity Sheet 3

THE SNOW IS FALLING

1. It's midnight. There is no snow on the ground. The weather forecaster is predicting snow overnight. It snows 3 cm each hour.

   a. Assuming the rate stays the same, use the table to find the amount of snow on the ground after 3, 4, and 5 hours.

Number of Hours	Total Snow (cm)
0	0
1	3
2	6
3	
4	
5	

   b. If the pattern continues, predict the amount of snow on the ground

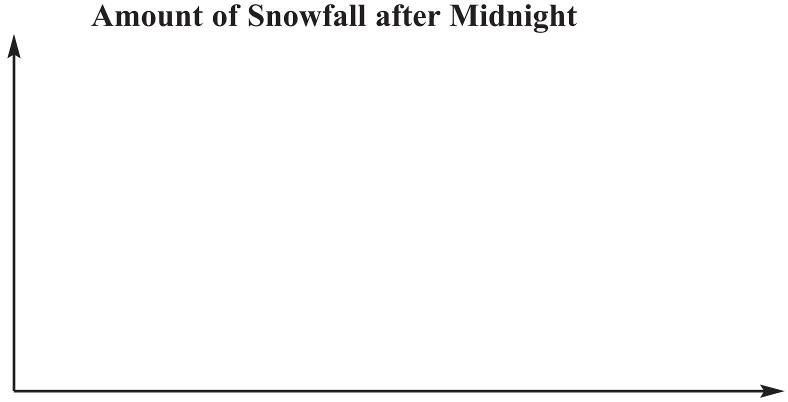
   after 8 hours? _____ after 20 hours? _____ after *n* hours? _____

   c. Write the rule that describes the pattern.

   d. Create a line graph by plotting the data on the axes below.

Amount of Snowfall after Midnight

Amount of Snowfall (cm)



Number of Hours after Midnight

2. It's midnight and there are 5 cm of snow on the ground. It begins to snow at a rate of 1 cm per hour.

Number of Hours	Total Snow (cm)
0	5
1	6
2	
3	
4	
5	

a. Complete the table.
b. Predict the snowfall after 6 hours, 10 hours, and *n* hours.
c. Write the rule to describe the situation.
d. Graph the relationship.

3. It's midnight and there are 5 cm of snow on the ground. It begins to snow at a rate of 3 cm per hour.

Number of Hours	Total Snow (cm)
0	5
1	8
2	
3	
4	
5	

a. Complete the table.
b. Predict the snowfall after 6 hours, 10 hours, and *n* hours.
c. Write the rule to describe the situation.
d. Graph the relationship.

Activity Sheet 4

SNOWING IN NEW YORK STATE

1. It's midnight in Massena and there is no snow on the ground. It begins to snow at a rate of 2 cm each hour.
   a. Create a table that shows the amount of snow on the ground for every hour after midnight until 5 a.m.
   b. If n represents the number of hours after midnight and s represents the amount of snow on the ground, write an equation to model the situation.
   c. Which part of your equation represents the rate of snowfall?
   d. Graph the equation, using your table of values.
   e. If it continues to snow at the same rate, use your equation to predict snowfall at 10 a.m.

2. At midnight in Troy there is no snow on the ground. It begins to snow at a rate of 3 cm each hour.
   a. Write an equation to model the situation, using n and s to represent the number of hours after midnight and snowfall, respectively.
   b. Graph the equation on the same set of axes as the equation for snowfall in Massena. (Make a table of values, if necessary.)
   c. Compare the two lines on your graph. What is the relationship between steepness of the line and rate of snowfall?
   d. Use your graph to predict snowfall in each town at 6 a.m., assuming a constant rate of snowfall in both towns. Extend the graph, if necessary.
   e. Which town has more snow at 3 a.m.? How much more snow does that town have? How can you use your graph to illustrate this?

3. At midnight in Poughkeepsie there are 4 cm of snow on the ground. It begins to snow at a rate of 2 cm per hour.
   a. Write an equation in terms of n and s to model the situation. Which part of the equation represents the rate of snowfall? Which part of the equation represents the amount of snow on the ground at midnight?
   b. On a separate set of axes, graph the equation. Which point represents the amount of snow on the ground at midnight?

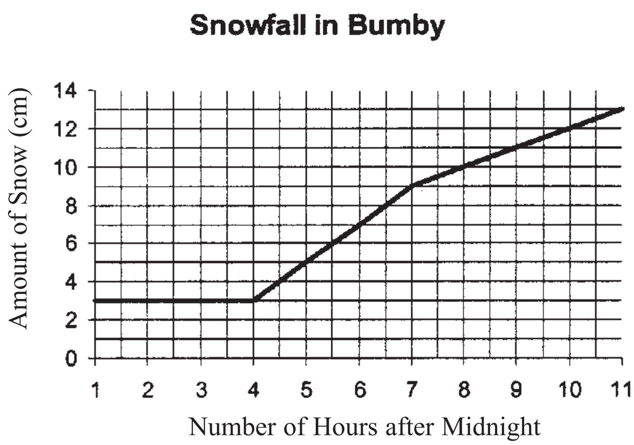
Activity Sheet 5

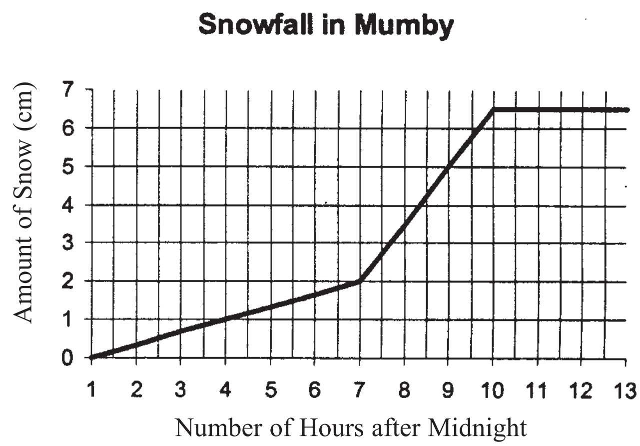
DESCRIBING SNOWFALL GRAPHICALLY

1. It's midnight and there are 4 cm of snow on the ground. It begins to snow at a rate of 2 cm every 3 hours.
   a. Write an algebraic equation to represent this situation. Define the variables. Identify the dependent and independent variables.
   b. Sketch the graph of the function over a reasonable domain.
   c. Identify the slope and *y*-intercept and explain what they represent.
   d. If the snow continued at this rate, how much snow would be on the ground at 8 a.m.?
   e. Is it reasonable to use this model to predict the snowfall in 24 hours? Why or why not?

2. There are 3 cm of snow on the ground. At 8 p.m. it starts to snow at a rate of 2 cm per hour. After 3 hours the snow stops. At 2 a.m., it starts snowing again, this time at a rate of .5 cm per hour. The snowfall continues at this rate for another hour and a half. Sketch a graph that models the situation.

3. Describe what's happening in each graph both verbally and mathematically. State the appropriate domain for all algebraic equations.





4. Massena has no snow on the ground when it begins to snow at 10 a.m. The snows falls at a rate of 3 cm every 2 hours. Troy has 6 cm of snow on the ground. At the same time snow begins to fall at a rate of .5 cm per hour. At what time will they have the same amount of snow on the ground? Solve both graphically (using a graphing calculator) and algebraically.

Activity Sheet 6

PART 1: LINEAR RELATIONSHIPS

1. It starts to snow early in the morning. Snow accumulation measurements begin at 8 a.m. The following table shows the accumulation of snow during the day.

Time	Snow Accumulation (cm)
8 a.m.	5.2
9 a.m.	6.1
10 a.m.	8.3
11 a.m.	9.3
12 noon	9.9
1 p.m.	11.9
2 p.m.	12.4
3 p.m.	13.7

a. Determine the independent and dependent variables. Make a scatterplot of the data (both by hand and with a graphing calculator). On the basis of the scatterplot, describe the relationship between time and snow accumulation. Estimate the correlation coefficient from the graph.

b. Draw a line of best fit on the scatterplot. Determine the equation of the line of best fit. Identify the slope and y-intercept of the equation. Give an interpretation of the slope and y-intercept in the context of the problem.

c. Use the calculator to determine the equation of the line of best fit. Compare your equation with the calculator's equation. State the appropriate domain for the function.

d. Use the equation to estimate the amount of snow on the ground at 1:15 p.m. Does it make sense to use the equation to predict the snow accumulation at 5 p.m.? Why or why not?

PART 2: EXPONENTIAL RELATIONSHIPS

2. It starts to snow early in the morning. Snow accumulation measurements begin at 8 a.m. The following table shows the accumulation of snow during the day.

Time	Snow Accumulation (cm)
8 a.m.	2.5
9 a.m.	3.2
10 a.m.	4.2
11 a.m.	5.3
12 noon	7.2
1 p.m.	9.5
2 p.m.	12.1

a. Make a scatterplot of the data. Describe the relationship between time and snow accumulation in terms of shape (straight or curved) and direction (increasing, decreasing, or neither).

.

b. Draw a curve on the scatterplot to approximate the shape of the data.

c. Using the model $y = ab^x$, approximate a and b from your graph. Explain what a and b represent in this situation.

d. Use the calculator to determine the exponential regression model and compare with the equation you found in part c.