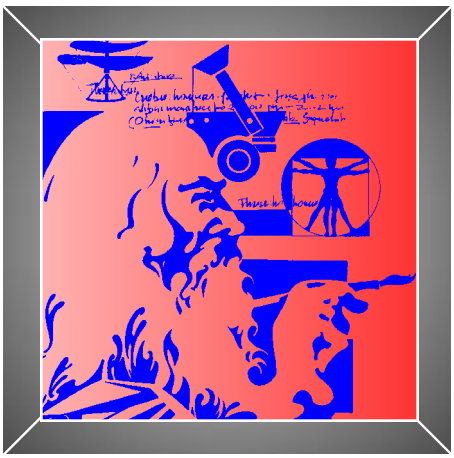

Mathematics

Resource Guide

with
Core Curriculum

THE UNIVERSITY OF THE STATE OF NEW YORK

THE STATE EDUCATION DEPARTMENT

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PREFACE

The *Mathematics Resource Guide with Core Curriculum* is one of four documents recently developed by the New York State Education Department. These documents were designed to provide guidance to districts and schools in New York State for development of local curricula, instruction, and assessment that meets local needs and resources and aids their students in achieving the mathematics standards for New York State.

The first document of the series, The *Learning Standards for Mathematics, Science, and Technology* (1996), introduced standards and benchmark performances in mathematics, science, and technology for grades four, eight, and high school. The *Mathematics, Science, and Technology Resource Guide* (1997), which is periodically updated, extended the standards into classroom practice. The *Resource Guide* contains lesson plans for mathematics, science, or technology classes as well as lessons that integrate them. The *Mathematics Test Samplers* for grades 4 and 8 were distributed in February 1998 and for Math A in May 1998. They provide a variety of assessment items similar to those which will be used in the New York State Testing Program.

The *Mathematics Resource Guide with Core Curriculum* elaborates upon the standards document and provides connections among the other three documents mentioned above. The *Core Curriculum* is not intended to be the mathematics curriculum for school districts but instead an outline of curriculum to be used to aid districts in developing local curriculum that is reasonable for their local resources and needs.

Although the *Core Curriculum* addresses only Standard 3 of the *Learning Standards for Mathematics, Science, and Technology*, a comprehensive mathematics curriculum would also include Standard 1 (inquiry, mathematical analysis, design), Standard 2 (information systems), Standard 6 (interconnectedness), and Standard 7 (interdisciplinary problem solving). The curriculum would also have connections with English language arts through the use of children's literature, reading, and writing as well as with social studies, which provides many opportunities to examine and analyze data.

The *Core Curriculum* extends the key ideas and performance indicators of Standard 3 to additional grade-level blocks of prekindergarten to kindergarten, grades 1 to 2, grades 3 to 4, grades 4 to 5, grades 7 to 8, and Math A. A draft of Math B is also included. Suggestions for grade-level content are given for each performance indicator. Suggestions of relevant assessment items or classroom activities are provided for each performance indicator. More examples of assessment items can be found in the Test Samplers and pilot tests. More examples of classroom activities can be found in the *Mathematics, Science, and Technology Resource Guide.* None of the documents just described are meant to stand alone. They should be used together.

The *Core Curriculum* is divided into three sections: Overview, Core Curriculum (Elementary, Intermediate, High School), and Reference List.

The Overview presents a discussion on processes that students use and the roles of manipulatives, technology, and assessment. Suggestions of effective instructional strategies will be discussed and each key idea of the standards will be listed and explained.

The Core Curriculum section is divided into three parts: Elementary, Intermediate, and High School. In the beginning of each part are suggestions for manipulatives. This is followed by suggestions of content for each performance indicator. Examples of assessment items are included for grade levels in which there will be State assessments. There will be suggestions for classroom activities or problems for each performance indicator for which an appropriate assessment item is not available. The assessment items and classroom activities are found at the end of each grade-level block and are listed by performance indicator (e.g., 1A. would be the first performance indicator given for Key Idea 1, which is mathematical reasoning).

The Reference List at the end of this document includes all sources used for examples of assessment items, classroom activities, and problems. Teachers may wish to refer to them for more ideas.

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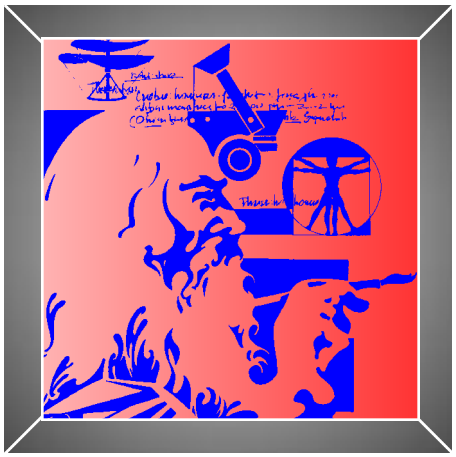


Mathematics

Core Curriculum

I. OVERVIEW

Standard 3: Mathematics

Students will understand mathematics and become mathematically confident by communicating and reasoning mathematical - ly, by applying mathematics in real-world settings, and by solving problems through the integrated study of number systems, geometry, algebra, data analysis, probability, and trigonometry.

Key Ideas

Mathematics curriculum and assessment of mathematical proficiency based on the mathematics standard of the *Learning Standards for Mathematics, Science, and Technology* revolve around the seven key ideas. The mathematics standard describes the seven key ideas listed below. Classification of mathematical content into these key ideas inevitably involves some overlap. In addition, many key ideas involve students in synthesizing knowledge across mathematical topics. A mathematics program that includes New York State's standards and reflects the revised assessments will continue to emphasize the fundamental mathematical skills and knowledge that have been traditionally expected. Students are still expected to master basic skills of arithmetic, geometry, algebra, trigonometry, probability, and statistics. The State Education Department will continue to assess these skills and concepts with tests that will be given in secure settings, and the results of these tests will be made public each year.

The seven key ideas of the learning standards are a mixture of content and process goals. They speak to the mathematics content that a student should know and, at the same time, describe the ways in which the student ought to be able to use that content in meaningful contexts. This is possible by considering the key ideas as guides for selecting appropriate content. The following brief description of each key idea may help in that selection.

1. Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Mathematical analysis is an integral part of problem solving. As a result, this key idea cuts across the content in all the other key ideas. At the elementary level, it includes the concept of pattern and at the high school level includes the concepts of logical terms such as *and, or, not, if. . .then,* as well as to what constitutes a valid argument.

2. Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Although number and numeration deal heavily with number concepts of whole numbers, fractions, decimal fractions, ratios, percents, integers, and irrational numbers, this key idea also includes procedures for ordering numbers and applying them to real-world situations.

3. Operations

Students use mathematical operations and relationships among them to understand mathematics.

Often considered to be primarily procedural, operations includes the expectation that students understand the concepts of addition, subtraction, multiplication, and division in order to be successful with problem solving. Problem solving often requires the selection of appropriate computational or operational methods. The concepts of ratio and proportion must be understood in order to recognize and solve problems that are proportional in nature.

4. Modeling/Multiple Representation

Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information, and relationships.

Most geometry concepts are included in modeling / multiple representation but the key idea also includes procedures for geometric constructions and producing graphs and tables. Modeling / multiple representation deals with many aspects of mathematical communication. As a result, it includes the use of variables, modeling relationships both algebraically and graphically, and appropriate use of functions.

5. Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Measuring is a procedure that includes the concepts of area, volume, perimeter, and circumference. It also includes the formulas that may be applied to calculate them. Much of the content of trigonometry and statistics is included in this key idea. Measurement is a major mathematics key idea that connects with science, technology, and social studies.

6. Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Estimation and probability are the major topics found in the key idea of uncertainty. Most probability concepts are found in this key idea as well as procedures for calculating probabilities. Although estimation includes number sense, estimating can be used as a problem-solving strategy. This key idea includes estimation of quantity, estimation of computations, and estimation of measurements.

7. Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

The study of patterns and functions is one of the central themes of mathematics. Patterns are a major part of mathematics study in the elementary grades. A study of patterns requires the conceptual understanding of recognizing, describing, and generalizing patterns.

The study of function helps the student to build mathematical models that can be used to predict the behavior of real-world phenomena that have observable patterns. The widespread occurrence of regular and chaotic pattern behavior can make a study of patterns and functions interesting and valuable.

Patterning and the manipulation of functions (algebraically and graphically) have procedural aspects, but pattern seeking and function finding are strategies used in problem solving. As in the key idea of mathematical reasoning, which includes looking for patterns, key idea 7 cuts across the other key ideas.

Cognitive Categories

Meaningful curriculum and assessment take into consideration the three categories of cognitive functioning that have served in shaping the National Assessment of Educational Progress (NAEP) and many other state frameworks and large-scale assessments of student learning in mathematics. These categories are ***procedural knowledge, conceptual understanding,*** and ***problem solving.*** Procedural knowledge involves "knowing when and how." Conceptual understanding involves "knowing about." They are joined together by problem solving, which involves the merging of conceptual understanding with procedural knowledge to bring students' knowledge into use in solving problems in abstract or contextual settings.[1]

Students use ***procedural knowledge*** when they:

- select and apply appropriate procedures correctly;
- verify or justify the correctness of a procedure, using concrete models or symbolic methods;
- extend or modify the procedures to deal with factors inherent in problem settings;
- use various numerical algorithms;
- read and interpret graphs and tables;
- execute geometric constructions; and
- perform non-computational skills such as rounding and ordering numbers.

Students use ***conceptual understanding*** of mathematics when they:

- recognize, label, and generate examples with a concept and without a concept;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles (i.e., valid statements generalizing relationships among concepts in conditional form);
- know and apply facts and definitions;
- compare, contrast, and integrate related concepts;
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts;
- interpret the assumptions and relations involving concepts in mathematical settings; and
- reason in settings involving the careful application of concept definitions, relations, or representations of definitions or relations.

In ***problem solving*** students are required to use their accumulated knowledge of mathematics in new situations when they:

- recognize and formulate problems;
- determine the sufficiency and consistency of data;
- use strategies, data, models, and relevant mathematics;
- generate, extend, and modify procedures;
- use reasoning (i.e., spatial, inductive, deductive, statistical, or proportional) in new settings; and
- judge the reasonableness and correctness of solutions.

Problem solving is the connecting thread through the seven key ideas of mathematics and the grade levels. Content which may require procedural knowledge for an older student may be a problem-solving task for a younger student. For example, a fourth grader can easily multiply two numbers by using a taught algorithm, but a first grader who may be unfamiliar with multiplication would have to devise a strategy to find the product, perhaps by making equal-sized groups of counters.

[1] The following is modified from the Mathematics Framework for the 1996 National Assessment of Educational Progress, NAEPMathematics Consensus Project, pp. 40-41.

The Role of Assessment

Assessment is an ongoing process and not an end in itself. A combination of individual student work, student projects, and teacher observations can be used to assess student achievement.

School districts may use assessment data to adjust curriculum and instruction. Teachers may use assessment data to strengthen the teaching and learning process in the classroom, monitor students' work and progress, and identify students' strengths and areas in need of improvement. Students may evaluate their own learning by using assessment data. Parents may be partners in assessment and receive specific information about their child's progress.

New York State Testing Program

The seven key ideas mentioned previously are one of three dimensions that are considered in the construction of State assessments. The three categories of cognitive functioning are another dimension. The third dimension of the model for specifying the nature of State mathematics assessments is the type of questions and tasks that students would be expected to complete when illustrating their competence in the key ideas and categories of cognitive functioning. These formats include multiple choice questions, short constructed response problems, and extended constructed response problems.

Multiple choice questions provide a highly reliable and efficient way of assessing students' ability to select correct answers and interpretations from a listing of possible alternatives.

Short constructed response problems provide much of the same information, but the student must develop the response. The student is also expected to create the answer, or sketch a drawing, among other possible actions. Such problems can be scored with a rubric that allows for partial credit.

Extended constructed response problems require students to develop a written description of the solution to a problem or to answer a series of subquestions. This requires the student to demonstrate greater knowledge and a necessity to communicate, in some depth, about the problem and its solution. Such tasks are usually scored with the use of a scoring rubric that also allows for partial credit.

Examples of each type of question can be found in the Core Curriculum section at the grade levels for which State pilot assessments have been administered (grades 4, 8, and Math A). There will be a mixture of question types on all the mathematics assessments. Specified percentages of items will assess each of the seven key ideas and three categories of cognitive functions.

Effective Instructional Strategies[2]

Students develop their own meanings of mathematical concepts and procedures when given the opportunity to become actively involved in learning. Teachers who use instructional strategies guided by this principle act as facilitators for children's active development of mathematics. They do not act merely as dispensers of rules and algorithms for students to memorize. Some of the instructional strategies they incorporate are the use of manipulative materials, student discussion of mathematical ideas, and small group learning. Active involvement on the part of the learner has the potential to deepen understanding of mathematics.

[2] Adapted from *Association of Supervision and Curriculum Development Curriculum Handbook.* (1995). Alexandria, VA: Association of Supervision and Curriculum Development, pp. 4.55-4.101.

Research concerning cooperative learning has indicated that students in classes using this learning model do at least as well and often better on standardized tests. Students from minority and low-income groups are frequently those who show the most improved scores. Other advantages of cooperative learning that have been found include development of thinking skills, improved self-esteem, improvement of attitudes toward minorities, and acceptance of mainstreamed students. Cooperative learning models have been shown to work well in heterogeneous classes. It must be pointed out that all "small group work" is not cooperative learning. The key elements of the cooperative learning model are that each student in the group is accountable for the final result and the students in the group must work together as a team in order to succeed.

Instruction that employs a wide range of representations and contextual environments enhance student growth in both affective and cognitive dimensions. The study of mathematics focuses on the representation and communication of numerical, spatial, and data-related relationships. Many classroom activities can support that focus; for example, students may translate their mental conceptions into symbolic forms and then provide a verbal description of the same situation. Other activities might include selecting the best model to physically explain a relationship; using technology in innovative ways to explore a problem; and writing paragraphs, letters, or journals to explain observations about mathematics.

Developmentally appropriate instruction takes advantage of what students are ready to learn. It provides classroom discourse that stimulates cognitive growth. It does not require students to memorize material that is beyond their current understanding. To make use of the notion of developmental stages, teachers observe their students closely and provide them with activities for which they are ready. Hands-on activities, as well as paired, group, and class discussions in which students develop and debate their ideas, contribute to the development of cognitive growth.

Mathematics environments that are embedded in real-world situations engage students in authentic problems that require creativity and demonstrate its uses in everyday life and careers, as well as play. When school mathematics builds on the mathematics that children developed on their own before they came to school, it becomes practical and relevant.

Role of Manipulatives

Manipulatives are physical objects that students can move around, group, sort, and use to measure as they model mathematical concepts and problems. Manipulative-rich environments may enhance understanding and achievement across a variety of mathematics topics if they are explicitly connected with the mathematical concepts and procedures they represent. Students do not automatically make connections between concrete representations of concepts or procedures and their written or symbolic forms. It is necessary for the teacher to help students make the connection.

Teachers of the early grades commonly use manipulatives but even when studying calculus, students can benefit from manipulating a physical model. Manipulatives can be elaborate and expensive, teacher-made, or simple items from home. Commercial materials often include books, games, and lessons when purchased in classroom sets. Suggestions for commonly used manipulatives will be given at the beginning of each level in the Core Curriculum section.

Role of Technology[3]

Classroom technology includes calculators, computers, videos, and multimedia. Each of these can be a valuable part of the mathematics program if used when appropriate.

Appropriate grade-level calculators should be made available to students in the classroom. They may be used to assist students in their understanding of concepts and procedures. The use of calculators should not be a substitute for a student's understanding of quantitative concepts and relationships or proficiency in basic computations. Research has shown that appropriate use of calculators in the classroom does not interfere with student knowledge of "number facts"or with their ability to perform calculations.[4] The classroom use of calculators has been shown to contribute to improved student attitudes toward mathematics and their problem-solving ability.

Four-function calculators are appropriate for instruction with elementary students although they are not permitted on the grade 4 State assessment at the present time. They might be used at this level in the role of skip counting. Skip counting by ones, using the calculator, can help young students relate counting to the symbols. For older students skip counting by 2's, 5's, or other numbers can relate repeated addition and multiplication. Integers may be explored through skip counting numbers less than zero.

Calculators that compute fractions are appropriate for the intermediate grades. Fraction calculators that simplify fractions through a series of stages, for example, may be used to help students speculate on the algorithm the calculator uses to put fractions in simplest form. The discussion may include concepts of common factors that are prime numbers and may lead to prime factoring as an algorithm for determining the simplest form of a fraction. Calculators will not be permitted on the multiple choice portion of the grade 8 State assessment but will be required for the rest of the test.

Scientific and graphing calculators are appropriate for secondary mathematics students. Graphing calculators demonstrate multiple ways of representing functions: graphically, symbolically, and in tabular form. Calculators that combine geometric sketching programs and symbolic manipulator software may be appropriate for high school students. Calculators will be required on the Math Aand Math B assessments. (See *calculator requirements* at the beginning of *High School: Math Aand Math B.*)

Computer software that allows students to create mathematics, as well as to explore, conjecture, and investigate mathematics, provides unique opportunities for students learning mathematics. Teachers should take full advantage of this resource, if available.

Young students can benefit from computer explorations that model manipulatives. The graphics, color, and sound capabilities of the microcomputer make for a dynamic model of the manipulative on the screen. Students may explore a concept, make mistakes for concept clarification, and even pose problems. The computer program often provides the student with instant help and feedback that are not possible when the student is working only with a manipulative. Studies concerning students using geometry construction software demonstrate that students develop a deeper understanding of explored concepts and, with proper instruction, develop independent inquiry skills associated with mathematical thinking.[5]

More recent developments in microcomputer capabilities have increased its ability to aid students in the study of real-world data. Internet capacity allows students to communicate across vast distances and provides a whole other dimension to mathematics experiments and data collection. Handheld computer-based laboratory systems which include probes facilitate the collection and analysis of real-world data and can be used to connect mathematics, science, and technology.

[3] Adapted from *Association of Supervision and Curriculum Development Curriculum Handbook.* (1995). Alexandria, VA: Association of Supervision and Curriculum Development, pp. 4.51, 4.59.
[4] Henbree, R. and D.J. Dessart. (March 1986). "Effects of Hand-Held Calculators in Precollege Mathematics Education: A Meta-Analysis." *Journal for Research in Mathematics Education.* 17(2), pp. 83-99.
[5] Mathis, J. (1991). *Support Materials for Geometry Education Software* (Unpublished NSF Report ISI-9060238). Washington, DC: National Research Council.

Yeryshalmy, M., D. Chazan, M. Gordan, and R. Houde. (1987). *Guided Inquiry and Technology: A Year-Long Study of Children and Teachers Using the Geometric Supposer* (Technical Report 88-6). Cambridge: Education Technology Center, Harvard Graduate School of Education.

Elementary
Prekindergarten to Kindergarten
Grades 1 to 2
Grades 3 to 4

Some Manipulatives

- Simple counters
- Clocks
- Tangrams
- Base 10 blocks
- Dice
- Sandbox
- Rulers*
- Measuring cups and spoons
- Pan balance scales
- Thermometers
- Square tiles*
- Connecting cubes
- Spinners
- Water table
- Tape measures
- Pentominoes
- Blocks of different sizes
- Pattern blocks
- Fraction models
- Geoboards
- 3-dimensional solids
- Meter sticks
- Pattern blocks*

*Punch-outs of these are used on the grade 4 State assessment.

Calculator
Four-function (optional parenthesis keys). Not permitted on grade 4 assessment.

Note
The elementary level of the Core Curriculum that follows is separated into grade-level blocks of prekindergarten to kindergarten, grades 1 to 2, and grades 3 to 4.

Students in grade 4 are expected to demonstrate proficiency with all the elementary performance indicators as given in Standard 3 of the *Learning Standards for Mathematics, Science, and Technology*. The grade 4 State assessment may test any of the topics listed in the Core Curriculum with each performance indicator. The examples of assessment items for grades 3 to 4 were taken from the 1998 Test Sampler. Suggestions for classroom activities are substituted for any performance indicator which was not represented in the Test Sampler.

Assessment items are not provided for prekindergarten to kindergarten or grades 1 to 2 because there are no State assessments at those levels. Suggestions for possible classroom activities or problems are given to provide clarification of each performance indicator.

Key ideas and performance indicators have been adapted and in some cases eliminated for grade-level blocks prekindergarten to kindergarten and 1 to 2 to provide an example of how district curriculum might provide a scope and sequence for the elementary level of their curriculum. Topics in these cases are labeled **MAY INCLUDE,** which is meant to indicate that school districts may arrange curricula in other ways to fit their own needs and resources.

Grades: PreK-K

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their reasoning.	• Prepare for number concept development through sorting and classifying activities, using concrete objects such as buttons, blocks, and bottle tops. • Integrate the comparison of sets and counting with other activities such as cooking, housekeeping, stories, games, and block building. • Participate in sorting and classifying activities with blocks, toys, and cookies, observing likenesses and differences. Use two categories at a time. • Explore likenesses and differences (color, shape, size. . .).	See Classroom Idea 1A.
1B. Use patterns and relationships to analyze mathematical situations.	• Begin to recognize a number sequence. • Relate counting to repeated patterns.	See Classroom Idea 1B.
1C. Explain their answers and solution processes.	• Describe their rationale for grouping or sequencing objects in a given manner.	See Classroom Idea 1C.

Key Idea 2
Number and Numeration

Grades: PreK-K

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
2A. Use whole numbers to determine number positions and quantify groups of objects.	• Develop the idea that if two sets can be matched or put into one-to-one correspondence, then they are equivalent. • Develop the idea that the last number counted in a set indicates how many things there are in a set (*cardinal number*). Do this gradually, first for the numbers 1-5, then for the numbers 1-10. • Use ordinal number names from *first* to *tenth*. • Observe numerals (names for numbers) in the environment.	See Classroom Idea 2A.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions.	• Provide repeated opportunities to proceed from concrete manipulation to pictorial and symbolic representation of numbers. • Develop an awareness of fractions in daily use. • Explore the fraction concept by introducing the words *whole* and *half*. • Participate in sharing experiences that show that a whole (such as a cake, an apple, or an orange) may be divided into equal parts.	See Classroom Idea 2B.
2C. Relate counting to grouping and place value.	• Explore, through counting, the cardinal numbers of a multitude of sets and collections of real objects, such as boys and girls, cookies, milk cartons, toy trucks, mittens.	See Classroom Idea 2C.
2D. Recognize the order of whole numbers.	• Develop the concept of *first, last,* and *middle*. • Use a number line to count forward and backward. • Discuss and use the concepts *before, after, following,* and *between* in classroom conversations. • Illustrate such ideas as "Awhole is *more than* a half" and "Ahalf is *less than* a whole."	See Classroom Idea 2D.

Grades: PreK-K

Key Idea 3
Operations

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
3A. Develop strategies for selecting the appropriate computational and operational methods in problem solving.	• Share sets of objects (cookies, toys, crayons). • Investigate various numerical problems that arise in the classroom, such as attendance, milk orders, and bus loadings. • Sets of objects (such as a bag of candy or a set of blocks) can be divided into equal parts, beginning with halves, without the total number of objects changing.	See Classroom Idea 3A.
3B. Develop readiness for single-digit addition and subtraction facts.	• Practice the skill of counting on. • Given a group of objects, find one more or one less. • Put two simple sets together to produce a new set, the cardinal number of which is less than 10.	See Classroom Idea 3B.
3C. Understand the commutative and associative properties.	• Engage in numerous individual manipulative experiences to perceive that the cardinal number of a set remains the same no matter how the elements are arranged.	See Classroom Idea 3C.

Key Idea 4
Modeling/Multiple Representation

Grades: PreK-K

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
4A. Use concrete materials to model spatial relationships.	• Create geometric pictures and designs from cut-out shapes. • Understand positions described by *top, middle, bottom, inside,* and *out - side* while building with blocks.	See Classroom Idea 4A.
4B. Construct charts and graphs to display and analyze real-world data.	• Line up children with other children according to gender, eye color, hair color. • Introduce the use of blocks and other concrete objects to represent real-world data.	See Classroom Idea 4B.
4C. Use multiple representations (manipulative materials, pictures, diagrams) as tools to explain the operation of everyday procedures.	• Develop an awareness of the concepts, words, and symbols related to numbers as used in daily living.	See Classroom Idea 4C.
4D. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.	• Order sets of objects (dolls, dishes, blocks) from smallest to largest, and from largest to smallest. • Introduce names of shapes. • Fold various geometric shapes (circles, squares, rectangles) into halves. • Explore the environment and note objects with geometric shapes (windows, doors, pictures, signs, floor and wall tiles, tables, etc.). • Collect boxes, bricks, and other three-dimensional shapes. Relate counting activities to geometric activities, such as finding out how many faces, edges, or corners a cracker box has. • Use a geoboard to make different shapes. Discuss the fact that position and orientation of shapes do not change their names. (i.e., ▲ and ▼ are both triangles).	See Classroom Idea 4D.

Grades: PreK-K

Key Idea 5
Measurement

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5A. Select appropriate standard and nonstandard measurement tools in measurement activities.	• Participate in activities that involve weighing, first in the hands and then on balance scales. • Use an egg timer, or an hourglass, to compare the duration of eating time, story time, (e.g., takes *longer, less time than*, or *as long as*). • Explore various nonstandard units of measure, such as blocks, books, children's feet, handspans, and bodies for length and distance; buttons, blocks, and bottle tops for weight; cupfuls, bowlfuls, and handfuls for capacity; and faucet drips for time.	See Classroom Idea 5A.
5B. Understand the attributes of length, capacity, weight, time, money, and temperature.	• Extend quantitative comparisons in many different situations to terms such as *bigger than, greater than, same size as, less than, equal to.* • Compare materials in terms of more, less and the same amount. • Use such terms as *heavier* or *lighter than* and *weighs more, less*, or *the same as.* • Compare the length, height, and width of various objects such as ribbons, toys, blocks, and of other children (e.g., their hands and feet). • Use sand or water to compare the capacity of containers (the pail, cup, holds *more, less*, or *the same as*). • Use real money for activities such as shopping, to learn the names of bills and coins.	See Classroom Idea 5B.

Key Idea 5
Measurement

Continued

Grades: PreK-K

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5C. Estimate measures such as length and volume, using both standard and nonstandard units.	• Use terms like *longer than, taller than, smaller than, shorter than, as long as*. Then compare distances, using such terms as *farther* and *nearer*. • Estimate in terms of *less than, bigger than, greater than, equal to, more, the same as. . .*	See Classroom Idea 5C.
5D. Collect and display data.	• Have children line up to show their preferences of things like different types of juices, games, types of apples, etc. Then pile blocks in stacks to represent the children's preferences. • Gather data relating to familiar experiences by counting, tallying, and using stickers, post-it notes, pictures, etc.	See Classroom Idea 5D.
5E. Use statistical methods such as graphs and charts to interpret data.	• Discuss graph data in terms of *most, least, more than, less than, the same.*	See Classroom Idea 5E.

Grades: PreK-K

Key Idea 6
Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
6A. Recognize situations in which only an estimate is required.	• Participate in activities which involve anticipating outcomes, such as stacking blocks until the pile falls or how far up the cup or bowl water will be when ice melts and then guessing (estimating) the results of a repeat experiment.	See Classroom Idea 6A.
6B. Develop a variety of estimation skills and strategies.	• Practice estimating (guessing) sizes, using phrases like *about as long as, almost as long as, wider than. . .* • Predict the number of colored objects (e.g., beads, buttons, blocks) in a container and count the actual number to check the prediction. • Use string or arms to measure the circumference of trees or pumpkins.	See Classroom Idea 6B.
6C. Predict experimental probabilities.	• Use colored spinners for decision making in games and for choosing activities. • Discuss the certainty and uncertainty of events such as whether a beanbag will land on the number 3, whether a blindfolded classmate will pick a blue block from a box, whether a dog will fly.	See Classroom Idea 6C.

Key Idea 7
Patterns/Functions

Grades: PreK-K

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
7A. Recognize, describe, extend, and create a wide variety of patterns.	• Observe patterns. • Attempt to extend patterns. • Describe patterns. • Follow directions to copy a pattern. • Explore creating patterns.	See Classroom Idea 7A.
7B. Represent and describe mathematical relationships.	• Use the term *set* synonymously with terms such as *group, lot, pile, bunch.* • Identify number names orally through 15. • Develop gradually the ability to read and write numerals from 0 to 10.	See Classroom Idea 7B.
7C. Use a variety of manipulative materials to explore patterns.	• Engage in pattern-forming activities with colored blocks, puzzles, buttons, pattern blocks, square tiles, etc.	See Classroom Idea 7C.
7D. Interpret graphs.	• Look for patterns in picture, bar, and concrete graphs.	See Classroom Idea 7D.
7E. Explore and develop relationships among two- and three-dimensional geometric shapes.	• Make prints from geometric solids to show the shape of the faces.	See Classroom Idea 7E.
7F. Discover patterns in nature, art, music, and literature.	• Create sound patterns with hand clapping and foot stamping. • Have a wide variety of children's literature and picture books available for motivation and mathematical topics. • Go for nature walks, looking for patterns in nature like: numbers of petals on different flowers, geometric shapes. On the walks children can also look for human-made examples of different shapes. • Examine quilt patterns for different geometric shapes.	See Classroom Idea 7F.

Grades: PreK-K

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

1A.

Provide students with a variety of seashells to sort, classify, and count by shape, color, size, etc. in an activity center.

1B.

Provide the child with five cards with dots on them (one dot, two dots, three dots, four dots, five dots). Have the child identify the card with one dot, then the card with one more than that, then the card with one more and so on until all five cards are in line from one dot to five dots. Then mix the cards up and give them back to the child to line up from the smallest to the largest by herself. [Also 7A.]

1C.

Provide students with buttons and ask that they sort them. After they have done so, ask them how they decided to group the buttons. If they have difficulty, show them a new button and ask which group it belongs to and why.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

Grades: PreK-K

2A.

During group or small group have children count how many mats have been set out for the group activity. After the children are seated, ask whether there are the same number of mats as children, and count to be sure.

2B.

Participate in sharing experiences that show that a whole graham cracker may be divided into smaller parts. Talk about a whole. Then have children share half a graham cracker. Have children see that the whole is more than the half.

2C.

Use students to show the number being studied. For example, to represent seven petals on a flower, have seven students lie down on the floor to make the flower.

2D.

Incorporate words that describe order into conversations with groups of children and individuals; e.g., *before, after, following, between*.

Grades:

PreK-K

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

3A.

Make a stack of two chips for each child and let the children count the chips by taking them off one by one. Ask them to stack the chips again. Now have them add one. They will see that the stack now has three. Let them try this procedure a few times until they see that adding one more chip brings the stack to the next higher number.

3B.

Have children solve classroom problems. For instance, we all have to sit at the table to draw in our books. Can we all fit at the table? What can we do so that everyone can sit at a table to draw in our books?

3C.

Have students work in pairs, with one on each side of a desk. Give each pair six blocks. Ask one student to divide the blocks into two groups and total them from left to right. That is, 4 and 2 equals 6. The student on the other side says 2 and 4 equals 6. Have students find all possible solutions.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

Grades: PreK-K

4A.

Have students work in pairs. Give each pair of students three plastic connecting links or linking cubes in three different colors. Explain that one child will tell the other how to make the three-link chain or rod by naming the color that should be used at the top, the middle, and the bottom of the chain (rod). Children should hold up their chains (rods) to identify positional terms. Have the children alternate roles. [Also 4D.]

4B.

Start with having children line up to show their preferences, and then make picture graphs comparing characteristics of the two groups. Ask question such as: Are you left-handed? Did you walk or ride to school? Then have them make graphs comparing three groups. Ask: Are you wearing pants, a skirt, or a dress? Do you go home for lunch, bring your lunch, or buy your lunch? Then try picture graphs comparing four groups, and ask: What shape cracker did you choose? Would you prefer a peanut butter sandwich, a hot dog, a hamburger, or spaghetti for lunch? What color life-saver did you eat—red, green, yellow, or orange? [Also 5E.]

4C.

Have students decide how many napkins, snacks, and drinks are needed for the class. To figure out the answer, have them draw a picture of how they will distribute the snack materials [Also 2A., 2B.]

4D.

Provide children with blocks. Their continual experimentation with blocks will help them find out what surfaces fit together well, which blocks make the best walls or roofs, and which surfaces balance best. [Also 5B., 5C., 5D.]

Grades: PreK-K

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

5A.

Children have items such as marbles or blocks. Have them select a container that will hold all their items.

5B.

Paste a strip of masking tape on a blank wall, reaching from the floor and extending two meters. Each child will need a separate strip. Each child stands against the wall in front of the tape. Place a mark on the tape at the top of the child's head. The child stands back and sees how tall he is. Mark the date next to the line. Repeat this activity periodically so that the children can see how much they have grown. By using a meterstick or tape measure, you may also wish to mark the child's height in centimeters. Say, "Today you are 120 centimeters tall." Show the 100 centimeter mark on the meterstick or tape measure. [Also 3C.]

5C.

Use *Okay Everybody* by Karla Kuskin as a basis for comparing and measuring activities.

Okay Everybody
Okay, everybody, listen to this:
I am tired of being smaller than you
And them
And him
And trees and buildings.
So watch out
All you gorillas and adults
Beginning tomorrow morning
Boy
Am I going to be taller.

5D.

Take a poll of the class, giving the children two choices (which food they like best, what toy they would rather play with) represented by a picture of the choice. Give each child a block to represent the choice. Have them place the block by the picture of their choice. Look at the blocks to guess which item had the most blocks. Count the blocks and talk about what the blocks represent.

5E.

Make a large pictograph with the months of the year. Put each child's name with the month in which he/she was born. Talk about the information shown on the graph.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

Grades: PreK-K

How many blocks will it take to make a road? Count the blocks, and then estimate how many it will take to build a road to a more distant destination.

6B.

Have the children use a balance scale to guess how many beans/seeds weigh as much as a teddy bear counter. [Also 6A., 7B.]

6C.

Provide students with a "more-less" spinner, linking cubes, and a "more-less" card. The spinner has two equal sectors; one says "more" and the other says "less." There are two cards, one labeled "more" and the other labeled "less." The children play this game in pairs. Each child snaps a train of cubes together. The trains' lengths are matched with one another so each child starts with the same number of cubes. Then each child breaks his or her train into smaller stacks. These stacks can be any size the child desires as long as the children have two different sizes. To begin, each child puts one stack on his or her work mat and puts the appropriate "less-more" card with the stack. One of the children spins the spinner. If the spinner indicates "more," the child with more cubes on his or her mat wins and takes both stacks. If the spinner says "less," the child with less cubes wins and takes both stacks. Repeat the activity until one child runs out of stacks. The student with all the cubes is the winner. [Also 5A., 5C., 5D.]

Grades: PreK-K

CLASSROOM IDEAS

EXAMPLES FOR

GRADES PREKINDERGARTEN - KINDERGARTEN

7A.

The teacher claps a pattern and the children join in. The children's suggestions for movement can be incorporated into the rhythmic clapping as the activity proceeds.

7B.

Set up a flannel board with many felt shapes. Put up two groups of shapes, asking questions like: "Are there as many ____ as ____? Which set has more? How many of each are there?" The children can point, tell with words, and move the pieces around to show that they understand the idea.

7C.

Place a few familiar manipulative objects on the tables in the math center and paper, crayons, and felt-tip pens in the art center. Children can use a variety of equipment and materials found in the room to make patterns. The teacher can ask children questions such as: "Where does your pattern begin? What comes next? Where does your pattern end? How would your pattern look if it kept going on and on?" [Also 1C.]

7D.

Once children start making graphs, they often think of problems to solve on their own. The following are some comparisons that might be of interest to young children:

- Number of brothers and sisters
- Hair color, eye color, clothing colors
- Kinds of pets children have
- Heights of children in the class
- Number of children in class each day
- Sizes of shoes
- Favorite TV programs or characters
- Favorite foods
- Favorite colors
- Favorite storybooks
- Type of weather each day for a month
- Number of cups of sand or rice that will fill different containers
- Comparison of the number of seeds found in an apple, an orange, a lemon, and a grapefruit
- Comparison of the number of different items that are placed in a balance pan to weigh the same as a standard weight (like plastic teddy bears).

[Also 5E.]

7E.

In the sandbox use several blocks to make footprints in the sand. The children identify the blocks that were used to make the footprints. Before beginning this activity, children need some experience with the three-dimensional figures being used, perhaps by sorting them or building with them. [Also 4D.]

7F.

Children can find patterns outdoors as well as indoors. Place a few sheets of paper on each paper plate, fasten it with a clothespin, and distribute these *notepads* to the children along with crayons and pens. When they are on the playground, ask them to look for and describe the patterns they see. Four- and five-year-olds can find patterns in fences, bricks, climbing apparatuses, windows, bike tires. Have them draw the patterns they see outdoors on the papers on their notepads. After they have made their discoveries, bring them inside and encourage them to talk about their patterns. [Also 4D.]

Key Idea 1
Mathematical Reasoning

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Grades: 1—2

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their reasoning.	• Categorize objects, using attributes such as likenesses and differences in color, shape, size, etc. • Observe likenesses and differences, using at least two categories at a time. • Draw pictures and use manipulatives to represent problems.	See Classroom Idea 1A.
1B. Use patterns and relationships to analyze mathematical situations.	• Patterns for sums and differences, using concrete materials, tables, calculators, and number lines. • Patterns of numbers that add up to a specific sum (e.g., all combinations of numbers that add up to 6). • Use patterns and relationships to discover commutative and associative properties and identity elements.	See Classroom Idea 1B.
1C. Justify their answers and solution processes.	• Clarify problems, using discussions with the teacher or knowledgeable others. • Explain to others how he/she went about solving a numerical problem. • Use concrete materials to justify solutions. • Use patterns and relationships to justify solutions. • Most topics can be used in problem solving.	See Classroom Idea 1C.
1D. Use logical reasoning to reach simple conclusions.	• Use concrete objects, pictorial representations, tables, and number lines to represent and solve problems. • Brainstorm possible strategies before starting a problem.	See Classroom Idea 1D.

Grades: 1—2

Key Idea 2
Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.	• Arrangement of elements does not change the cardinal number—i.e., elements can be matched in a one-to-one correspondence. • Identify number names orally through 100. • Ordinal numbers *first* to *thirty-first* and beyond. • Count occurrences with tallies.	See Classroom Idea 2A.
2B. Use concrete materials to model numbers and number relationships for whole numbers and fractions including decimal fractions.	• Count forward by 1's, 2's, 3's, 4's, 5's, 10's in various ways and backward by 1's, 2's, 5's, and 10's, possibly using a calculator to skip count. • Represent two- and three-digit numbers up to 999, using concrete models such as bundles of ten sticks. • Identify even and odd numbers. • Halves (thirds, etc.) of a whole are equal to each other. • Unit fractions 1/2, 1/3, 1/4, 1/5, 1/8, 1/10 as part of a whole or part of a collection of things. • Aset of objects can be divided into equal parts.	See Classroom Idea 2B.
2C. Relate counting to grouping and place value.	• Use metric measures and money problems to reinforce place value. • Recognize dollars and cents notation to ten dollars. • Regroup ones and tens. • Recognize the meaning of zero in the place value system. • Meaning of digits in three-digit numbers. • Expanded notation for two- and three-digit numbers (e.g., 27 = 2 tens + 7 ones = 20 + 7).	See Classroom Idea 2C.
2D. Recognize the order of whole numbers and commonly used fractions.	• Understand that in sharing or measuring things there is sometimes a need to use numbers between whole numbers. • Count forward and backward. • Use words that describe order such as *first, last, before, after, between,* and *middle.* • Order relatively small sets of numbers.	See Classroom Idea 2D.
2E. Demonstrate the concept of ratio through problems related to actual situations.	• Relate many-to-one in preparation for the concept of ratio.	See Classroom Idea 2E.

Key Idea 3
Operations

Grades: 1—2

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
3A. Add and subtract whole numbers.	• Demonstrate with manipulatives how addition and subtraction are opposite operations. • Add and subtract up to three-digit numbers with no regrouping. • Add and subtract up to two-digit numbers requiring regrouping. • Combine sets to produce a new set. • Develop addition strategies such as doubles, doubles-plus-one, and number families.	See Classroom Idea 3A.
3B. Develop strategies for selecting the appropriate computational and operational method in problem solving.	• Solve real-world problems involving addition and subtraction of whole numbers. • Explore division as a process for finding the number of equivalent subsets in a given set. • Explore division as a process of sharing cookies, crayons, etc. as it comes up in classroom activities. • Explore multiplication as a way of determining how many are needed of something for each student to have a given amount. • Special role of zero in addition.	See Classroom Idea 3B.
3C. Know single-digit addition and subtraction facts and develop readiness for multiplication and division facts.	• Relate multiplication to repeated addition and counting by 2's, 3's, 4's, etc. • Explore division as a process of sharing. • Use manipulatives to relate division to repeated subtraction. • Special role of zero. • Master addition facts with sums 0-18 and subtraction with differences 0-9. • Readiness activities with rectangular arrays of objects. • Repeated addition or counting activities. • Use manipulatives to explore multiplication and division facts through 25.	See Classroom Idea 3C.
3D. Understand the commutative and associative properties.	• Associative property of addition. • Commutative property of addition.	See Classroom Idea 3D.

Grades: 1—2

Key Idea 4
Modeling/Multiple Representation

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
4A. Use concrete materials to model spatial relationships.	• Make geometric pictures and designs, using geometric shapes. • Make designs, using congruent and noncongruent shapes.	See Classroom Idea 4A.
4B. Construct charts and graphs to display and analyze real-world data.	• Record information with tallies, blocks, and pictographs.	See Classroom Idea 4B.
4C. Use multiple representations (manipulative materials, pictures, diagrams) as tools to explain the operation of everyday procedures.	• Compare dimensions of various objects, using terms like *longer than, taller than, smaller than, shorter than, as long as, farther, nearer.* • Concepts of *more, less, the same*.	See Classroom Idea 4C.
4D. Use variables such as height, weight, and hand size to predict changes over time.	• Compare heights over time in general terms.	See Classroom Idea 4D.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.	• Understand that shapes such as circles, squares, rectangles, and triangles can be found in nature and in things that people make and that these shapes can be used to describe many things. • Compare attributes of objects—size, shape, weight, texture, etc. • Draw symmetrical designs. • Examine bilateral symmetry by paper folding or mirror activities.	See Classroom Idea 4E.

Key Idea 5
Measurement

Grades: 1—2

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5A. Understand that measurement is approximate, never exact.	• Compare (weather, time, temperatures) in general terms. • Develop an understanding of the need for standard units of measure.	See Classroom Idea 5A.
5B. Select appropriate standard and nonstandard measurement tools in measurement activities.	• Explore various nonstandard measurement tools such as blocks, books, children's feet, bowlfuls. • Relate measurement of temperature to different thermometers. • Translate time between analog and digital clocks. • Use a ruler to measure to the nearer centimeter. • Using a balance scale, determine mass (weight) of familiar objects.	See Classroom Idea 5B.
5C. Understand the attributes of area, length, capacity, volume, weight, time, temperature, and money.	• Compare sets of objects, using terms: *more than, bigger than, greater than, less than, one more than, the same size, equal to, before, after,* and *between*. • Compare the capacity of containers, using sand and water. • Weighing experiences, using the terms *heavier than* or *lighter than*. • Measure time to the day, month, and year, using a calendar. • Measure time in half hours, quarter hours, and minutes. • Weigh objects, using grams. • Make change for amounts of money up to $1.00, using pennies, nickels, dimes, quarters, and half-dollars. • Introduce the kilogram and liter. • Basic understanding of the concept of temperature and how it is measured.	See Classroom Idea 5C.

Grades: 1-2

Key Idea 5
Measurement

Continued

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
5D. Estimate measures such as length, perimeter, area, and volume, using both standard and nonstandard units.	• Measure objects, using nonstandard units. • Estimate sizes, using phrases like *about as long as, almost as long as, wider than. . .* • Use meter, centimeter, and decimeter for measuring length. • Discuss English system of measure if mentioned by students.	See Classroom Idea 5D.
5E. Collect and display data.	• Collect objects of all sorts. • Collect data concerning body measurements and other things of interest to the students. • Simple bar graphs, using stacks of blocks.	See Classroom Idea 5E.
5F. Use statistical methods such as graphs, tables, and charts to interpret data.	• Compare data in terms of number, equality, inequality, similarities, differences. • Understand that simple graphs can help one to understand observations.	See Classroom Idea 5F.

Key Idea 6
Uncertainty

Grades: 1—2

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
6A. Make estimates to compare to actual results of both formal and informal measurement.	• Estimate quantities. • Make quantitative estimates of familiar lengths, widths, and time intervals and check them against measurements.	See Classroom Idea 6A.
6B. Make estimates to compare to the actual results of computations.	• Estimate answers before solving problems and compare estimates with solutions.	See Classroom Idea 6B.
6C. Recognize situations in which only an estimate is required.	• Investigate various numerical problems that arise in school.	See Classroom Idea 6C.
6D. Develop a wide variety of estimation skills and strategies.	• Use manipulative materials for estimating quantity.	See Classroom Idea 6D.
6E. Determine the reasonableness of results.	• Anticipate outcomes by guessing or estimating and compare guess or estimate with outcome.	See Classroom Idea 6E.
6F. Predict experimental probabilities.	• Discuss certainty or uncertainty of events. • Understand that some events are *more likely* to happen than others. • Discuss fairness of a game. • Predict outcomes of coin tosses. • Record data from experiments, using spinners and colored tiles/cubes, and use the data to predict which of two events is more likely to occur if the experiment is repeated.	See Classroom Idea 6F.
6G. Make predictions, using unbiased random samples.	• Understand that one can find out about a group of things by studying just a few of them.	See Classroom Idea 6G.
6H. Determine probabilities of simple events.	• Perform experiments with three or more likely outcomes. • Use language such as “one chance in three.” • Solve problems such as how many different pairs of numbers have a sum of 6. • Concept of *combination* or *arrangement*. • Solve problems such as: “How many different sets of three numbers will add up to 12?” or “How many different ways can you rearrange the letters of your name?"	See Classroom Idea 6H.

Grades: 1—2

Key Idea 7
Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	MAY INCLUDE	EXAMPLES
7A. Recognize, describe, extend, and create a wide variety of patterns.	• Recognize, describe, and extend number sequences and patterns in the range of 1-1000. • Recognize, describe, extend, and create patterns with geometric shapes.	See Classroom Idea 7A.
7B. Represent and describe mathematical relationships.	• Introduce symbols <, >, =. • Read and write numerals 0-100.	See Classroom Idea 7B.
7C. Explore and express relationships, using variables and open sentences.	• Concepts of equality and inequality, using numbers. • Write open sentences like 3 + ❍ = 5.	See Classroom Idea 7C.
7D. Solve for an unknown, using manipulative materials.	• Use counters to explore or explain commutative and associative properties of addition. • Use counters to find missing values as in open sentences like 3 + ❍ = 5.	See Classroom Idea 7D.
7E. Use a variety of manipulative materials and technologies to explore patterns.	• Terms *inside*, *outside*. • Discover properties of 3-D shapes. • Do skip counting with manipulatives. • Use manipulative materials to explore symmetry. • Use manipulative materials to explore linear patterns such as ▼●❋▼●❋.	See Classroom Idea 7E.
7F. Interpret graphs.	• Readiness activities for mode, median, and mean.	See Classroom Idea 7F.
7G. Explore and develop relationships among two- and three-dimensional geometric shapes.	• Understand the basic properties of and similarities and differences between circles, squares, rectangles, and triangles.	See Classroom Idea 7G.
7H. Discover patterns in nature, art, music, and literature.	• Identify symmetry in nature, art, and music. • Utilize children's literature for motivation, exploration, and problem solving.	See Classroom Idea 7H.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

Grades: 1—2

The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

1A.

Provide students with a bag of small toys—perhaps a bag of plastic farm animals or a mixture of domestic and wild animals, a bag filled with toy cars, boats and planes, or buttons or shells or stones. Let students work with a partner or in a group to sort their pieces in an open-ended activity. With younger children, you may want to be sure they are comfortable with simple sorting activities before they work with the animal pieces. For example, sorting materials with different shapes, colors, or sizes provides good preparation. Instruct students to sort their pieces into different groups and provide a good name for each group. Then they should share what they did with their partner. Mix up the pieces and let the partner sort the pieces into new groups. They should check each other's work, talk about the sorting with each other, and find a way to record their groupings. Let students describe their groupings with other members of the class. A follow-up might be to challenge the students to put their sorted groups in horizontal or vertical graphing grids. [Also 4B., 5E., Science: Living Environment Concept 1)

1B.

Provide students with a specified number of two-color counters in a bag. Students shake and then spill the counters, keeping track of different combinations of red and yellow. For example, a child may be given six counters. After the counters are spilled, the counters may look like this:

4 + 2 = 6

❍❍❍❍●●

Children record the different combinations they find and stop when they believe they found all of them. Explore with the entire class whether 4 + 2 and 2 + 4 are the same combination. [Also 3A., 3C., 3D.]

1C.

Provide students with color tiles to solve the following riddles. After solving the riddle, have students discuss how they found the answer and explain why they believe their solution is correct.

- There are ten tiles in two colors. Each color has an odd number of tiles. How many tiles of each color are there?
- There are an odd number of tiles. There are more than four tiles but less than ten tiles. You can use all the tiles to make three towers, all the same size. How many tiles are there?
- There are an even number of tiles. There are more than eight tiles but less than 22. You can use all the tiles to make four trains, all the same length. How many tiles might there be? [Also 1D., 2B.]

1D.

Have students use counters to solve the following problem. A pet store owner sold only birds and cats. One day he asked his clerk to count how many animals there were in the store. The clerk told him he counted 18 legs. How many cats and birds might there have been? Could there be more than one combination of animals in the store? [Also 3A., 3D.]

Grades: 1 - 2

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

2A.

Send students home with blank books that may be made from a few sheets of stapled newsprint. With a parent, the child tours the house to count various common features such as the number of windows in the house or the number of bicycles. They create one page for each feature such as:

In my house there are _____ bicycles.

Books can be shared with reading buddies and kept in the book corner. A "super" counting book could be created, using tallies to keep track of all the numbers of each feature mentioned by more than one student. [Also 2C.]

2B.

Utilize class routines to practice number relationships. For example, when students leave the classroom in a group, coach them to line up in two equal rows. After determining the number of students who are absent that day, ask questions such as:

- How many students will be lined up today?
- Can we have an equal number in each row?
- How many in each row? (If there is an odd number of students, the teacher can be included to pair up with one of the students.)

Then count off by twos to check. As the year progresses, this activity provides reinforcement in learning the concepts of even and odd numbers, equality, one-to-one correspondence, subtraction, addition, division by two, and counting by twos. [Also 3A., 3B.]

2C.

Provide students with linking cubes and place value charts. Using a spinner with numbered sectors 1-9 or 0-9, the teacher spins a number and students represent the number with linking cubes, write it on their place value chart, and write the numeral. As teacher spins, students count on more linking cubes and are directed to always make a rod of 10 when they can. On their place value mats they will represent the new number and write the new numeral. Students could also be asked to show the new numbers in expanded notation such as:

1 ten + 1 one.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

Grades: 1—2

2D.

Have five students act out the following poem:

Five little pumpkins are sitting on a gate.
The first one said, " Oh my, it’s getting late."
The second one said, "There are witches in the air."
The third one said, "But we don’t care."
The fourth one said, "Let's run and run and run."
The fifth one said, "I'm ready for some fun!"
"Whoooo-oo-oo" went the wind and out went the lights
And the five little pumpkins rolled out of sight.

Ask questions like:

- What happened first in the story? What happened last?
- Which one comes after the second pumpkin?
- The last pumpkin is in what place?
- If Sally is the third pumpkin, which one will come next? [Also 2A.]

2E.

Using pattern blocks such as a green triangle, blue rhombus, red trapezoid, and yellow hexagon, have students discover the quantity of triangles needed to cover the blue rhombus, the red trapezoid, and the yellow hexagon.

Grades:

1—2

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

3A.

Provide students with coffee stirrers or craft sticks, rubber bands, and a place value chart. Have them represent problems like 203 + 123, using single sticks for the units, a bundle of ten sticks for ten, and a bundle of ten bundles of ten for hundreds. Students combine the coffee stirrers to find the sum and show the addition on the place value chart.

3B.

Use the story *A Doorbell Rang* by Pat Hutchins to help students understand division as a process of sharing. The book begins with two children who are about to share 12 cookies. Just as they are about to share the cookies, the doorbell rings and two friends join them. Now there are four people to share 12 cookies. Then two more friends arrive and now there are six children to share the cookies. The doorbell rings again and six more children are there. Now there are 12 children sharing 12 cookies. Have students use their counters to demonstrate each situation and decide how many cookies each child will get in each situation.

3C.

Give students specified numbers of color tiles—for example, 18 tiles. Have them make as many rectangles as possible out of the tiles and record each rectangle on a piece of graph paper, noting the number of rows and columns of each rectangle, to find all the multiplication facts for the given number.

3D.

Students can demonstrate the commutative property of addition by using color tiles to make, for example, two trains of three tiles. The first is red, red, blue; the second is blue, red, red.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

Grades: 1—2

4A.

Read *Grandfather Tang's Story* by Ann Tombert, showing the students the tangram pictures. Provide the children with tangrams so they can make the animals in the story, using outlines of the animals from the book.

4B.

Have students use linking cubes to make a bar graph showing the colors of their shoes. They can answers questions like:

- How many colors of shoes are we wearing?
- Which color is most popular?

4C.

Students are given a number of two-color counters. They spill them and record which color comes up more or if an equal number of each color comes up. They keep track by making tallies under categories of

MORE RED SAME MORE YELLOW

4D.

Students plant bean seeds and measure the growth at the end of every week. [Also 5C., 5E., Science: Living Environment Concept 4]

4E.

Have students explore symmetry by using mirrors with pattern blocks or by folding paper or by making ink blot designs. Students find the lines of symmetry in the letters of the alphabet and in numerals. They can fold paper and cut out geometric designs.

Grades:

1—2

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

5A.

Give students a variety of objects in pairs and have them use balance scales to determine which of the two objects is heavier.

5B.

Have students trace two copies of their foot and cut them out. Have them lay the “feet” end to end to measure the length of the room. Have them compare their measurements. Discuss why the measurements were not the same.

5C.

Give students a variety of different-sized and -shaped jars. Ask them to put the jars in order from the one that holds the least to the one that holds the most. After the students have put their jars in order, have them test their order by pouring rice (sand or water) from the smallest jar to the next-sized jar. If the rice spills out of the second jar, then the order was not correct at this point. Have them continue the process through their entire series of jars.

5D.

Let students work in pairs to tear off lengths of adding machine tape (which can be purchased in most office supply stores) equal to their height and then tear off other pieces for their arms, legs, and any other body parts that they may wish to include. The teacher can ask them to compare the different parts of their body: “Is your arm about as long as your leg?” etc. Students can put the body parts together to make a copy of themselves.

5E.

Every month the children can make a class graph for which they must make a decision such as “Would you rather make a jack-o’lantern with a happy face or a sad face?” They could use self-drawings on the graph each month and place their picture over their preference. The picture graphs can be reinterpreted as bar graphs, using connecting cubes that can then be counted by grouping cubes into groups of ten to reinforce place value concepts. [Also 2B., 2C.; Science: Living Environment Concept 4]

5F.

Let students bring their favorite books to class and show them to each other. Have them discuss ways in which the books are similar or different. Have them choose one category of difference and sort the books on those categories. The books can be put on a large floor grid and the teacher can then ask questions about the number of books in each category.

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

Grades: 1—2

6A.

Show students a jar filled with linking cubes. Have them estimate how many cubes are in the jar. Have them put their estimates on post-it notes and create a class graph of their estimates. When counting out the number of connecting cubes in the jar, link them in rods of ten to reinforce place value concepts. [Also 2C., 5E.]

6B.

Introduce front end rounding for addition and multiplication. For example, in the example 24 + 32, the student estimates that 20 + 30 is 50 so the answer must be greater than 50. Have them discuss why the sum is greater than 50 and not less than or equal to 50.

6C.

Have students discuss questions such as:

- Would it be all right to estimate the number of milk orders today?
  Why or why not?
- Would it be all right to estimate how many students are going on the class field trip?
  Why or why not?
- Would it be okay to estimate how many cupcakes to bring to a party?

[Also 1A.]

6D.

When estimating how many candies are in a jar, let students count how many are on the bottom of the jar and then estimate how many layers there are. They can perform repeated addition to find an estimate of the number of candies in the jar. [Also 6A.]

6E.

Have students guess how many cut-out ladybugs they could put on a small leaf and then check by putting as many as possible on the leaf. Give them larger ladybugs and ask them if they would be able to use the same number, less, or more of the larger ladybugs to completely cover the leaf and explain their reasoning. [Also 1C., 5D., 6A., 6D.]

Grades:

1—2

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

6F.

Let students examine paper cups to decide how they might land when dropped—on their sides, on their tops, or on their bottoms. Have them predict which outcome is most likely and test it. [Also 6E., 6H.]

6G.

Give students bags with different-colored jellybeans in them. Have them reach in the bag, pull out a jellybean, record its color, and then replace it. After about ten samples, have the student decide which color occurs most often in the bag.

6H.

Have students predict how many times tails will come up in ten tosses of a coin and explain their reasoning. Then each student tosses a coin ten times and records with tallies the number of times heads or tails came up. Have students share their results with the class and tell why they believe the results agreed or did not agree with their prediction. [Also 2A., 5E., 6F.]

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

Grades: 1 - 2

7A.

Using a hundred board and colored transparent chips, give the students the first three numbers of a pattern, e.g., 1, 3, 5 or 6, 16, 26. Have them put a chip on each of the three numbers, showing the first three numbers of the pattern. Then ask the students to continue the pattern by showing the next three numbers. Help the students verbalize a rule for each pattern. [Also 1B., 2B., 2D., 3C.]

7B.

Use masking tape to make a large square on the classroom floor. Divide the square into four equal parts and label each with a number from 1 to 4. Have children stand behind a throw line six feet from the target and take turns tossing a beanbag. The child who tosses must read the number and then perform some designated action such as stamping, clapping, hopping, or jumping that number of times. [Also 2A.]

7C.

Assign two children to work together, using a mathematical balance. Place a weight on any number, on one side of the balance. By using two weights on the other side of the balance, how many ways can the scale balance? Children should write the equations they discover on the balance. [Also 3A., 3C.]

Using a mathematical balance, the teacher discusses with the children the analogy of the balance fulcrum with the equal symbol in an equation. Have children correlate their equations with the balance. Give them open sentences such as 5 + ❒ = 9. Have them represent it with connecting cubes to find the missing addend and check their answer on the mathematical balance by placing a weight on the 4 on the same side as the 5 and seeing if it balances. [Also 1A., 1C., 1D., 3C., 4C.]

7E.

Provide students with a blank 100s grid, counters, and a calculator. Have them determine how many counters fit across the grid. At the same time have them count by ones on the calculator. Ask them what patterns they notice on the calculator as they fill up the grid. Have them hold as many beans as they can in their hand, then count how many they could hold. Have them use the calculator and the grid to determine how many they could hold in two hands. [Also 1B., 2B., 2C., 2D., 3A., 5B.]

Grades:

1—2

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 1 - 2

7F.

Have students write their first name on a post-it note and count the number of letters in their name. Have the students come to the board, where the teacher has written different numbers (starting with 2), and place their post-it note over the number which is the same as the number of letters in their name. Ask questions about who has the greatest number of letters, who has the least, who is in the middle, what is the most common number of letters in a name, etc. [Also 4C., 5E., 5F.]

7G.

Provide each group of students with some geometric solids. Ask them to identify and share with the rest of the class the solids that answer each of these questions. Think of a square. Find a block with a square face (cube and rectangular prism with square bases). Think of a rectangle. Find a block with a rectangular face (any prism). How many different kinds of rectangular faces can you find? Think of a triangle. Find all the blocks you can with triangular faces (triangular prism, pyramid, tetrahedron). How many triangular faces does each one have? [Also 4E.]

7H.

Give students an opportunity to look at a number of quilt patterns. Consider reading the book *The Keeping Quilt* by Patricia Polacco as an introduction. Let students choose one quilt to study and write about the pattern they notice. Using construction paper to make geometric shapes, students should create a block of their own quilt pattern. [Also 4A., 4E.]

Key Idea 1
Mathematical Reasoning

Grades: 3-4

Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
1A. Use models, facts, and relationships to draw conclusions about mathematics and explain their thinking.	• Study factor and product relationships, using number lines and arrays. • Statements that use *and*, *or*, and *not*. • Draw pictures, diagrams, and charts to represent problems. • Clarify problems, using discussions with peers.	See Assessment Example 1A.
1B. Use patterns and relationships to analyze mathematical situations.	• Addition, subtraction, multiplication in number patterns. • Patterns in sequences of numbers such as triangular and square numbers. • Symmetry or patterning in number tables. • Money as related to fractions and decimals.	See Assessment Example 1B.
1C. Justify their answers and solution processes.	• Verify an answer to a problem. • Use estimation, number relationships, and mathematical checks to justify answers.	See Assessment Example 1C.
1D. Use logical reasoning to reach simple conclusions.	• Use concrete objects, diagrams, charts, tables, and number lines to help solve problems. • Use open sentences, patterns, relationships, and estimation as strategies to solve problems. • Identify missing information in a story problem.	See Assessment Example 1D.

Grades: 3—4

Key Idea 2
Number and Numeration

Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
2A. Use whole numbers and fractions to identify locations, quantify groups of objects, and measure distances.	• Read and write whole numbers to hundred millions. • Use ordinal numbers through 500th. • Relate fractions and decimals to the monetary system and to metric measure. • Identify use of fractions and decimals in daily life.	See Assessment Example 2A.
2B. Use concrete materials to model numbers and number relationships for whole numbers and common fractions, including decimal fractions.	• Manipulatives: base 10 blocks, abaci, chip trading for place value in whole numbers and decimal fractions to hundredths. • Odd and even numbers as a result of addition, subtraction, multiplication. • Prime numbers. • Skip counting. • Various ways a figure can be divided into equal parts, using terms *numerator* and *denominator*. • Order unit fractions and decimals and use < and > signs utilizing concrete materials. • Find equivalent fractions. • Number line and coordinates with positive and negative numbers.	See Assessment Example 2B.
2C. Relate counting to grouping and to place value.	• Place value concepts extended to millions and hundredths.	See Assessment Example 2C.
2D. Recognize order of whole numbers and commonly used fractions and decimals.	• Whole numbers to millions. • Fractions with denominators 2, 3, 4, 5, 6, 8, 10, 12. • Decimals to hundredths.	See Assessment Example 2D.
2E. Demonstrate the concept of ratio and percent through problems related to actual situations.	• Percents that are multiples of 5. • Concept of ratio in real-world situations.	See Assessment Example 2E.

Key Idea 3
Operations

Grades: 3—4

Students use mathematical operations and relationships among them to understand mathematics.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
3A. Add, subtract, multiply, and divide whole numbers.	• Addition and subtraction of whole numbers less than one million. • Subtraction with zeros in the minuend. • Multiply three-digit numbers by two-digit numbers. Multiplication by multiples of 10. • Division of three-digit dividends by one- and two-digit divisors (quotient and remainder).	See Assessment Example 3A.
3B. Develop strategies for selecting the appropriate computational and operational method in problem-solving situations.	• Use diagrams, charts, and tables to help understand problem information. • Use open sentences to model problems. • Use commutative, associative, distributive, inverse properties. • Look for patterns. • Break problem into parts.	See Assessment Example 3B.
3C. Know single digit addition, subtraction, multiplication, and division facts.	• Inverse relationships of operations. • Special role of zero. • Multiplication and division facts through 144. • Application of identity elements of addition and multiplication in learning and understanding number facts.	See Classroom Idea 3C.
3D. Understand the commutative and associative properties.	• Commutative property: addition, multiplication. • Associative property: addition, multiplication.	See Assessment Example 3D.

Grades: 3—4

Key Idea 4
Modeling/Multiple Representation

Students use mathematical modeling / multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
4A. Use concrete materials to model spatial relationships.	• Properties of plane figures. • Properties of solid figures (vertices, line segments, edges, and angles). • Designs and patterns with geometric figures.	See Classroom Idea 4A.
4B. Construct tables, charts, and graphs to display and analyze real-world data.	• Ordered pairs on a grid (coordinate plane). • Graphs and charts of real-world data and data in other subject areas. • Draw conclusions and make predictions from graphs.	See Assessment Example 4B.
4C. Use multiple representations (simulations, manipulative materials, pictures, and diagrams) as tools to explain the operation of everyday procedures.	• Perimeter, area, and volume by counting units. • Circumference of circles by measuring with string. • Area of circles by counting units in a grid. • Volume by "filling space" with standard-sized objects to build a foundation for the volume formula.	See Assessment Example 4C.
4D. Use variables such as height, weight, temperature, and hand size to predict changes over time.	• Compare temperatures and heights over time.	See Classroom Idea 4D.
4E. Use physical materials, pictures, and diagrams to explain mathematical ideas and processes and to demonstrate geometric concepts.	• Terms: *polygon, chord, radius, face, edge, vertex, angle, line segment, point, parallel, perpendicular, intersecting*. • Common plane and solid geometric figures in the environment and drawings. • Straightedge rulers, protractors, and compasses to construct circles, squares, etc. • Lines of symmetry.	See Assessment Example 4E.

Key Idea 5
Measurement

Grades: 3—4

Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
5A. Understand that measurement is approximate, never exact.	• Identify appropriate metric units for measuring the area, mass, perimeter, and volume of a variety of objects.	See Classroom Idea 5A.
5B. Select appropriate standard and nonstandard measurement tools in measurement activities.	• Identify equivalent measures within the metric system of measure. • Relate decimal concepts to metric measurement tools. • Relate the clock face to fractions of a circle.	See Classroom Idea 5B.
5C. Understand the attributes of area, length, capacity, weight, volume, time, temperature, and angles.	• Study time to five-minute, one-minute, and one-second intervals. • Find the area and volume of specific figures by counting units. • Explore connections between factors and multiplication facts and area and volume. • Measurement problems related to other areas such as literature, science, and social studies.	See Assessment Example 5C.
5D. Estimate and find measures such as length, perimeter, area, and volume, using both nonstandard and standard units.	• Select and use appropriate metric measurement tools. • Compare equivalent measures within the metric system. • Perimeter of polygons. • Find the circumference of circles by measuring with string.	See Assessment Example 5D.
5E. Collect and display data.	• Graphs of statistical data drawn from newspapers, magazines, polls, charts, surveys, etc. • Make frequency tables from tallied data. • Organize data with graphs, models, pictures, lists.	See Assessment Examples 5E-F.
5F. Use statistical methods such as graphs, tables, and charts to interpret data.	• Use concrete materials to develop the concept of average or arithmetic mean. • Find the range and the mean in a collection of organized data.	See Assessment Examples 5E-F.

Grades: 3—4

Key Idea 6
Uncertainty

Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6A. Make estimates to compare to actual results of both formal and informal measurement.	• Rounding numbers, using number lines and measuring instruments (meterstick, thermometer). • Estimate measurements before measuring.	See Assessment Example 6A.
6B. Make estimates to compare to actual results of computations.	• Estimate the outcomes of problems/experiments, complete the task, and compare the results with the prediction.	See Classroom Idea 6B.
6C. Recognize situations in which only an estimate is required.	• Explore the meaning of large numbers through such activities as estimating the grains of rice in a coffee can, the number of letters on a page, ways that newspapers report large numbers. • Discuss real-world examples of when estimating would be acceptable and when it would not. • Explore quantitative information that will help to relate personal experiences to the meaning of *million*.	See Assessment Example 6C.
6D. Develop a wide variety of estimation skills and strategies.	• Round numbers to nearest tenth, whole number, hundred, and thousand. • Develop a variety of strategies for estimating addition, subtraction, multiplication, and division. • Develop a variety of strategies for estimating quantities. • Develop strategies for estimating measurements.	See Assessment Example 6D.

Key Idea 6
Uncertainty

Continued

Grades: 3-4

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
6E. Determine the reasonableness of results.	• Develop orderly ways to determine the number of possible arrangements and combinations (e.g., tree diagrams). • Estimate the result of computations before using a calculator, especially in computations with decimals. • Make generalizations about the difference between products of numbers greater than one and when one number is less than one. • Estimation strategies for multiplication and division such as: when the divisor is greater than one, the quotient will be less than the dividend; and when it is less than one, the quotient is greater than the dividend.	See Assessment Example 6E.
6F. Predict experimental probabilities.	• Conduct and predict outcomes of various experiments, using unequally as well as equally likely outcomes. • Recognize events that are certain and events that have no chance of occurring. • Explain why a game is fair or unfair.	See Assessment Example 6F.
6G. Make predictions, using unbiased random samples.	• Collect statistical data from newspapers, magazines, polls. • Use spinners, drawing colored blocks from a bag, etc. • Explore informally the conditions that must be checked in order to achieve an unbiased random sample (i.e., a set in which every member has an equal chance of being chosen) in data gathering and its practical use in television ratings, opinion polls, and marketing surveys.	See Assessment Example 6G.
6H. Determine probabilities of simple events.	• Determine the number of ways an event can occur. • Use fractional notation to express the probability of an occurrence. • Explore problems that involve a systematic identification of ordered arrangements, using models, pictures, lists, or tree diagrams.	See Assessment Example 6H.

Grades: 3—4

Key Idea 7
Patterns/Functions

Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7A. Recognize, describe, extend, and create a wide variety of patterns.	• Number patterns and sequences. • Repeated patterns (abab, etc.). • Design patterns.	See Assessment Example 7A.
7B. Represent and describe mathematical relationships.	• Use symbols <, >, ≤, ≥. • Terms *at most* and *at least*. • Present division facts in more than one way, such as, 18 ÷ 3 = 18/3. • Describe number sequences. • Investigate relationships between addition and subtraction; addition and multiplication; subtraction and division; and multiplication and division. • Relate fractional notation for tenths, hundredths, thousandths to decimal fraction notation. • Consider, discuss, and predict whether the sum, difference, or product of two numbers is odd or even. • Relate area and volume formulas to counting squares or blocks.	See Assessment Example 7B.
7C. Explore and express relationships, using variables and open sentences.	• Solve open sentences with missing information. • Use open multiplication and division sentences in situations of equality and inequality. • Use formulas to find perimeter and area of geometric shapes.	See Assessment Example 7C.
7D. Solve for an unknown, using manipulative materials.	• Use counters to solve division problems to find the number of groups possible when each group is a given size, and the number of objects in each group when the number of groups is known. • Use counters to explore number patterns like triangular numbers and square numbers. • Use counters to help solve problems that can be summarized with open sentences. • Use counters to explore or explain commutative and associative properties of addition and multiplication.	See Assessment Example 7D.

Key Idea 7
Patterns/Functions

Continued

Grades: 3-4

PERFORMANCE INDICATORS	INCLUDES	EXAMPLES
7E. Use a variety of manipulative materials and technologies to explore patterns.	• Use manipulatives or computer programs that allow students to manipulate geometric shapes such as tangrams and pattern blocks. • Use manipulatives or calculators to skip count and relate to multiplication. • Use manipulative materials and relevant computer software to explore symmetry. • Use manipulatives or relevant computer software to explore linear patterns.	See Assessment Example 7E.
7F. Interpret graphs.	• Find mode, median, mean, and range of a set of data. • Compare frequencies within a bar graph or histogram. • Describe trends in bar graphs and line graphs.	See Assessment Example 7F.
7G. Explore and develop relationships among two- and three-dimensional geometric shapes.	• Identify the geometric shapes of the faces of prisms, pyramids, cones, and cylinders. • Identify different types of prisms and pyramids.	See Assessment Example 7G.
7H. Discover patterns in nature, art, music, and literature.	• Find two- and three-dimensional shapes in nature, art, and human-made environment. • Find examples of tessellations in the real world. • Identify examples of symmetry in nature, art, and music. • Relate the concept of fraction to beat value of notes in music (whole note = one beat, half note = one-half beat, etc.). • Relate examples of children's literature to mathematics for motivation, exploration, and problem solving.	See Assessment Example 7H.

Grades: 3—4

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 3 - 4

1A.

Alice started with 8 marbles. She lost 2 marbles. Then she bought 6 new marbles. Which of these statements is true about the number of marbles Alice has now?

A. Alice has fewer marbles than when she started.
B. Alice has 4 more marbles than when she started.
C. Alice has 6 more marbles than when she started.
D. Alice has the same number of marbles as when she started.

1B.

In these number sentences, the same shape always stands for the same number.

❍ + ❍ = 8
Δ + ❑ = ❍
Δ + ❍ = 5

Part A

Use the number sentences to find which numbers the Δ, ❑, and ❍ stand for.
Write the correct number in each shape above.

Part B

On the lines below, explain the steps you used to find the answer.

_____

_____

1C.

Tony and Mara saw 13 clowns at the circus. Each clown was wearing either red, yellow, or green shoes.

- Twice as many clowns were wearing red shoes as yellow shoes.
- There were 4 clowns wearing green shoes.

When they got home from the circus, Mara said there were 6 clowns wearing yellow shoes. Explain whether or not Mara is correct. Include the number of clowns wearing each shoe color.

_____

_____

You may use counters to help you solve this problem.

Tina has 3 craft sticks. Tina and Dan together have 5 craft sticks. Dan and Sally together have 7 craft sticks. How many craft sticks does Sally have?

A. 5 C. 3
B. 2 D. 4

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 3 - 4

Grades: 3—4

2A.

Label the cookies in the pan to show that
1/3 of them are Oatmeal (O)
1/2 of them are Butter (B), and
the rest are Sugar (S).
What fraction of the cookies on the pan are Sugar?

Answer_____

2B.

Use your pattern blocks to help you solve this problem.

How many of the green triangles would be needed to cover exactly 2/3 of this shape?

A. 1
B. 3
C. 6
D. 9

2C.

The students are collecting craft sticks. The pictures below show the number of sticks they collected each week.

How many sticks do the students have after collecting them for 3 weeks?

A. 809
B. 890
C. 806
D. 980

2D.

Which number would be served next?

F

G

H

J

2E.

One hundred students were asked to name their favorite sport. The table below shows the results of the survey.

FAVORITE SPORT

Sport	**Number of Students**
Baseball	20
Basketball	15
Soccer	35
Football	30
TOTAL	**100**

What percent of students chose soccer as their favorite sport? **Answer**_____%

What percent of the total group of students did *not* choose soccer as their favorite sport? **Answer**_____%

Show your work.

ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 3 - 4

Grades: 3—4

3A.

You may use counters to help you solve this problem.

2 c. flour
1 c. sugar
3 eggs
1 t. baking soda
6 oz. chocolate chips
4 T. butter

The list shows how much of each item is needed to make one batch of Chocolate Chip Cookies.

You want to make ***2 batches*** of Chocolate Chip Cookies.

Complete the list below to show how much of each item will be needed.

_____ c. flour
_____ c. sugar
_____ eggs
_____ t. baking soda
_____ oz. chocolate chips
_____ T. butter

3B.

Pat, Chris, and Jessica are skating to raise money for the school band. The table below shows how many laps each student skated and how much each student earned per lap.

SKATING FOR THE BAND

Student	Number of Laps Skated	Amount Earned Per Lap	Amount Raised
Pat	10	$0.75	
Chris	16	$0.50	
Jessica	13	$0.50	

Total raised by all 3 students _____

Complete the table to show the amount of money each student raised and the amount of money the 3 students raised altogether.

3D.

Shelly, Mike, and Jason looked at the stars through a telescope every night for a week. Shelly saw 6 shooting stars, Mike saw 9, and Jason saw 11. They wondered in what order they should add up the numbers to get the highest total. Does it matter? On the lines below, explain why or why not.

_____

_____

_____

ASSESSMENT EXAMPLES

EXAMPLES FOR

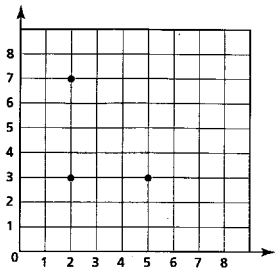
GRADES 3 - 4

4B.

Alissa plotted the first 3 corners of a rectangle on the grid.

What are the coordinates of the 4th corner?

F. (5, 7)
G. (5, 6)
H. (6, 5
J. (7, 5)



4C.

In the library, there are 2 round tables. There are 3 students sitting at each table. Each student has 4 books.

In the space below, draw a diagram or model to represent this information.

4E.

Use your pattern blocks to help you solve this problem.

In the space, trace aound two pattern blocks that show quadrilateral shapes.

On the lines below, explain what makes these shapes ***quadrilaterals.***

5C.

Use your pattern blocks to help you solve this problem.

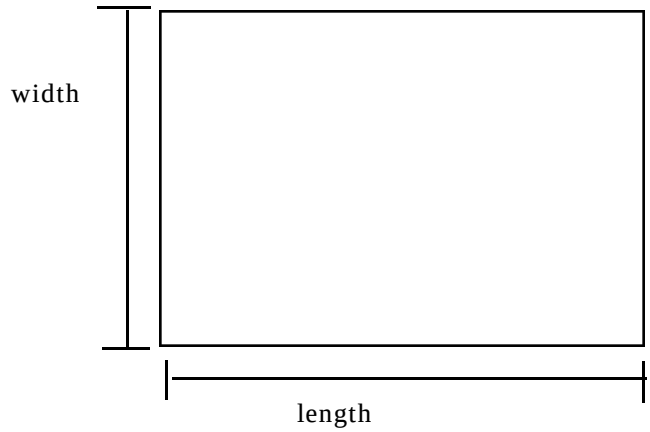
In the space below, trace one shape that has only **right** angles.

In the space below, trace all shapes that have any angle **larger** than right angles. Place an X on all of the angles **larger than** right angles.

5D.

Use your centimeter ruler to solve this problem.

Tim cut rectangles out of construction paper to make a picture.
The first rectangle he cut is shown here.



Part A
What is the perimeter in centimeters of Tim's first rectangle?

Show your work.

First Perimeter _____ centimeters

Part B
The second rectangle Tim cut has the same width as the first, but the length is 2 centimeters longer. What is the perimeter in centimeters of Tim's second rectangle?

Second Perimeter _____ centimeters

On the lines below, explain in words how you found the perimeter of Tim's second rectangle.

_____

_____

_____

ASSESSMENT EXAMPLES

EXAMPLES FOR GRADES 3 - 4

5E-F.

Mr. Jacobs asked his first-grade students to choose their favorite crayon colors. The results are shown in the table below.

FAVORITE CRAYON COLORS

Eric	Red
Shannon	Purple
Jill	Yellow
Keisha	Red
Allison	Green
Christine	Blue
Nicole	Purple
Roosevelt	Blue
Stacy	Red
Marie	Green
Mel	Blue
Peter	Green
Julie	Red
Luis	Blue
Ramon	Blue
Paul	Red
Marcus	Blue
Ali	Green
Ken	Blue
Leon	Yellow

On the grid below, make a bar graph showing the number of students who prefer each crayon color. Use the information from the table to help you.

Be sure to:
- title the graph
- label the axes
- graph all the data.

Using the information from your graph, write ***one*** statement comparing the crayon colors.

ASSESSMENT EXAMPLES

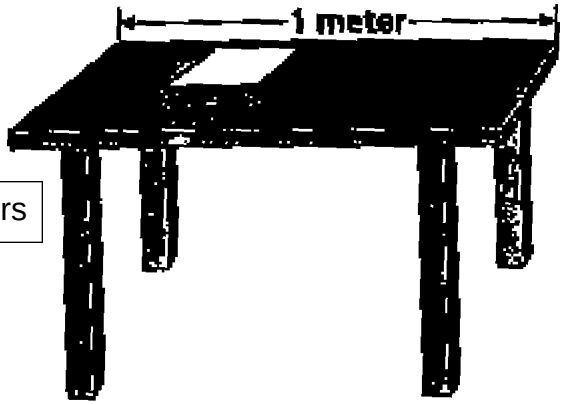
EXAMPLES FOR

GRADES 3 - 4

Grades: 3 - 4

6A.

Look at the picture. The table is 1 meter long.
Estimate the width of the piece of paper in centimeters.



1 Meter = 100 centimeters

Estimate _____

6C.

The Lee family is standing in line for the Turtle Olympics. The family is trying to figure out the following four things:

- How much money will it cost to get in?
- How long will they have to wait in line?
- How many tickets will they need?
- What time will the Olympics start?

Which of these things will the Lee family most likely have to estimate?
Explain why you would use an estimate or why an estimate would not be used on the lines below.

_____

_____

6D.

This jar has about 40 marbles in it.

Which of the jars has the amount closest to 120 marbles?

6E.

Amy says that a spaceship traveling at 7,200 miles per hour would go 15,000 miles in 8 hours. Sue disagrees and says that the spaceship would travel 56,000 miles in that amount of time.

Which student has the closer estimate to the actual distance? Explain why.

_____

_____

_____

6F.

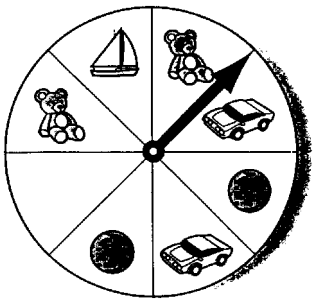
For the spinner to have an equal chance of landing on each prize, which prize should go in the blank space?

F.

G.

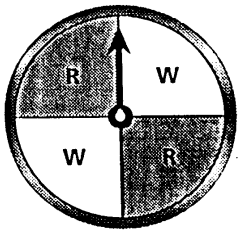
H.

J.

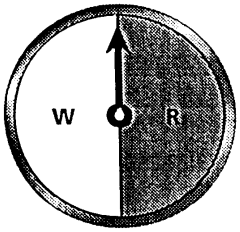


6G.

Kim and Lisa are playing a game. Each player will use a different spinner shown below. Each player will spin their own spinner 10 times and score 1 point each time the arrow on her spinner lands on R. The player with the most points wins.



Kim's Spinner



Lisa's Spinner

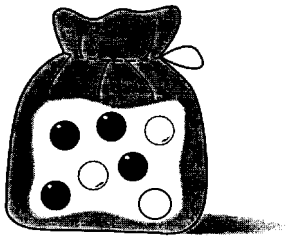
Use the two spinners to explain why both players do or do not have a fair chance of winning the game.

_____

6H.

Jack will choose one marble from the bag without looking.
What is the probability he will choose a black marble?

A. 1/7 C. 3/7
B. 2/7 D. 4/7



ASSESSMENT EXAMPLES

EXAMPLES FOR

GRADES 3 - 4

Grades: 3 - 4

7A.

Study the pattern.

What is the next shape in the pattern?

7B.

In a computer game, a ball was dropped from a height of 512 centimeters. The picture shows how high the ball bounced.

Part A
If the pattern continues, how high will Bounce 3 be?

Answer _____ cm

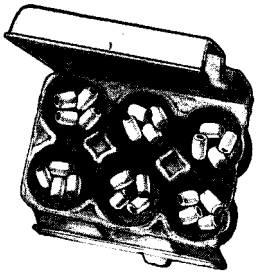
Part B
On the lines below, describe how the height changes from one bounce to the next.

_____

_____

Part C
Predict how high Bounce 6 will be. **Answer** _____ cm

7C.



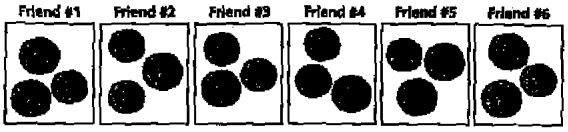
David put 4 beads in each section of the egg carton shown.

Which of these number sentences could not be used to find the total number of beads?

F. 4 X 6 = ❑ H. 4 + 4 + 4 + 4 + 4 + 4 = ❑
G. 6 X 4 = ❑ J. 6 + 6 + 6 + 6 + 6 + 6 = ❑

7D.

You may use your counters to help you solve this problem.



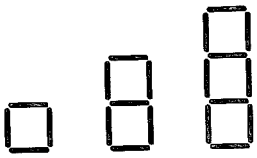
Erin has 18 cookies. She gave 3 to each of her 6 friends.

Al has 12 cookies, and wants to divide the cookies equally among each of his 3 friends. How many cookies will each of Al's friends receive?

F. 6 H. 3
G. 2 J. 4

7E.

Look at the pattern below. Each figure is made with craft sticks.



How many craft sticks are needed to make the next figure in this pattern?

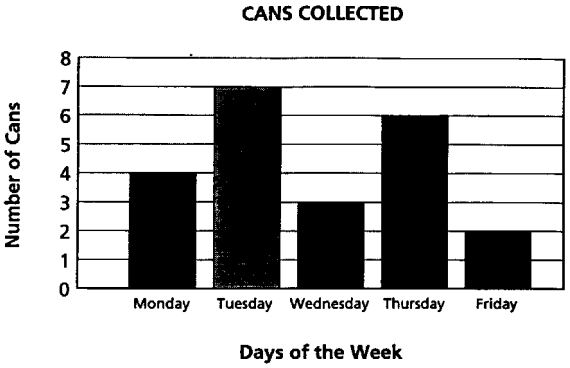
A. 3
B. 4
C. 12
D. 13

7F.

The graph shows the number of canned goods that Mr. Ruiz's class collected. Study the graph. Then answer the questions.

Part A

Mr. Ruiz promised to reward his class with animal stickers when the total number of cans reached 15. The bar graph shows the number of cans the class brought in each day.



What day did Mr. Ruiz give his class the animal stickers?

A. Tuesday
B. Wednesday
C. Thursday
D. Friday

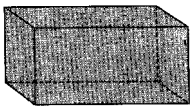
Part B

How many days did the students collect more than 5 cans?

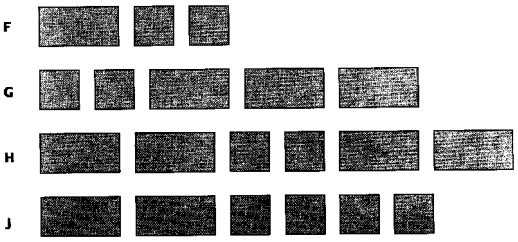
F. 5
G. 2
H. 3
J. 4

7G.

Look at this rectangular solid (box).



Which of these groups shows all the sides of the rectangular solid (box) above?

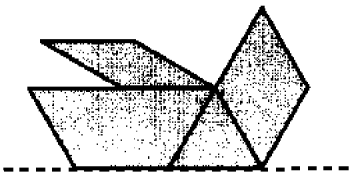


7H.

Use your pattern blocks to help you solve this problem.

Dan used pattern blocks to draw half of a design.

Complete the design below so that the dotted line is a line of symmetry. Trace around your pattern blocks to show the other half of the design.



CLASSROOM IDEAS

EXAMPLES FOR

GRADES 3 - 4

Grades: 3 - 4

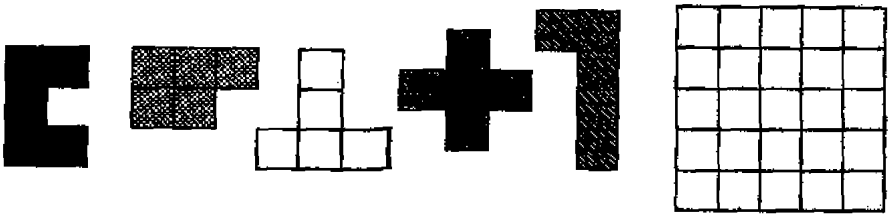
The following ideas for lessons and activities are provided to illustrate examples of each performance indicator. It is not intended that teachers use these specific ideas in their classrooms; rather, they should feel free to use them or adapt them if they so desire. Some of the ideas incorporate topics in science and technology. In those instances the appropriate standard will be identified. Some classroom ideas exemplify more than one performance indicator. Additional relevant performance indicators are given in brackets at the end of the description of the classroom idea.

3C.

Tell students, "Your classmate says that any number times zero starts with the number you are multiplying and ends with a zero (e.g., 5 x 0 would be 50)." Have students, with their partners, prepare an explanation for why this is not correct. Let students share their explanations and decide which correct explanation they feel is the clearest. [Also 1A., 1C.]

4A.

Show how these five puzzle pieces can be made to fit into the 5 x 5 grid below. Your final result will be a square. Keep moving the pieces until you can find an arrangement that works. Describe how you found the solution. [Also 1D.]



4D.

Thermometers placed outdoors on the north, south, east, and west walls of the building and also inside the building *can be used* to collect temperatures at different times of the day for several days. Students could be asked to choose a graph to display their recorded data. Topics for discussion include: comparisons of the warmest and coolest parts of the day, the warmest and coolest locations inside and outside, and reasons for these temperature differences. [Also 4B., 5C., 5E., 5F.]

Grades: 3 - 4

CLASSROOM IDEAS

EXAMPLES FOR

GRADES 3 - 4

5A.

Have students work in groups to create their own measurement scale, using as units handspan, book, length of string, etc. Have them measure different objects in the classroom with their measurement scale and also with metric measurements and compare the results. They could use marbles to develop a measurement scale in a cylinder and compare it to the graduated markings in milliliters as to its accuracy in measuring by displacement. [Also 5B., 5C., 5D., 6A.]

5B.

Give students magazines or catalogs. Have them cut out various pictures of objects and glue them to poster board. Underneath the picture they should write what unit of metric measure would be used to measure the object (millimeter, centimeter, meter, kilometer, milligram, gram, kilogram, milliliter, or liter) and write a sentence underneath explaining why they feel the unit of measure chosen is the most appropriate.

6B.

Students explore various methods of rounding numbers to estimate computation. For example, in addition or multiplication, round each number to the nearer thousand and add the rounded numbers. Or, use the numbers in the thousand place and add for a lower limit and then raise all the numbers to the next thousand for the upper limit. The actual sum will be between the two values. Have students compare the two methods. For example:

Method 1		Method 2	
3569	4000	3000	4000
+2712	+3000	+2000	+3000
	7000	5000	7000

The sum is greater than 5000 but less than 7000.