



Mathematics, Science & Technology

PART II.6

Inverse (Indirect) Machines.....	2
Statistics of The M&M Candy.....	8

NOTE: This document is a work in progress. Parts II and III, in particular, are in need of further development, and we invite the submission of additional learning experiences and local performance tasks for these sections. Inquiries regarding submission of materials should be directed to: The Mathematics, Science, and Technology Resource Guide, Room 681 EBA, New York State Education Department, Albany, NY 12234 (tel. 518-474-5922).



Inverse (Indirect) MACHINES

Levers: Simple Machines



- MST 1**
- ▲ compare data
 - ▲ evaluate conjectures
 - ▲ apply skills to solutions

- MST 3**
- ▲ ratios/proportions
 - ▲ graphing utilities
 - ▲ proportions to scale
 - ▲ curve fitting

- MST 6**
- ▲ revise a model
 - ▲ collect information
 - ▲ mathematical models
 - ▲ predictions

- MST 7**
- ▲ work effectively
 - ▲ gather/process information
 - ▲ generate/analyze ideas
 - ▲ observe common themes
 - ▲ realize results
 - ▲ present results

Prior Knowledge
 Students should be familiar with the solution of proportions and with writing equations and expressions from verbal descriptions.

Patricia Frey-Mason

Buffalo Academy for Visual and

Performing Arts

333 Clinton Street

Buffalo, NY 14204

(716) 851-3868

freymp@aol.com

Grades 9 & 11

Students need to recognize relationships between quantities. Most relationships studied in the early years of algebra are either direct or inverse (indirect) relationships. Mathematics classes spend a large part of their curriculum learning to manipulate mathematical models of direct relationships. Although listed as part of the *NYS Sequential Mathematics Course 1* curriculum, students do not generally spend very much time with inverse (indirect) relationships and seldom have an opportunity for a hands on investigation of the same. The study of levers as simple machines is no longer contained in the junior high school science curriculum. Thus, the first encounter a student may have with the study and /or use of levers is in a physics course, if (s)he chooses to take such a course.

After a demonstration and discussion of the three different classes of levers, students will be assigned to groups of three or four to complete the following activities:

Classes of Levers

There are three (3) classes of levers. You can tell them apart by observing the relative positions of the **fulcrum**, the **load**, and the **effort**.

Fulcrum: The pivotal point or hinge about which the lever rotates.

The **Effort:** The force applied to the lever, usually in the form of a push or a pull.

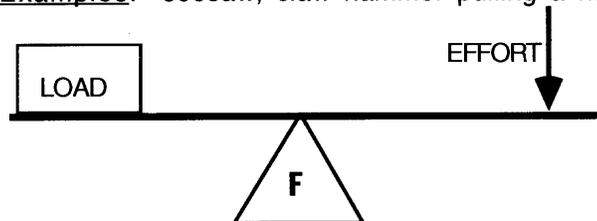
The **Load:** The resistance to be overcome or the weight to be lifted.

Ask the following questions to decide which type of lever you have.

- a. Is the fulcrum between the load and the effort?

If YES, then you have a **First Class** lever.

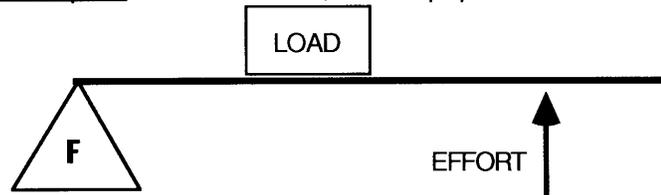
Examples: seesaw, claw hammer pulling a nail



- b. Is the load between the fulcrum (F) and the effort (E)?

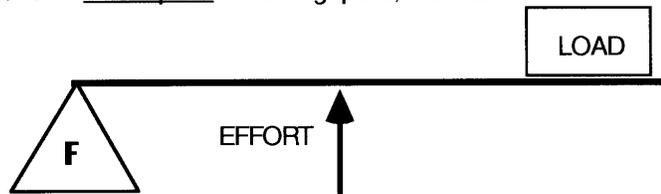
If YES, then you have a **Second Class** lever.

Examples: wheelbarrow, office paper cutter



- c. If NOT EITHER of the above, then you have a **Third Class**

lever. Examples: fishing pole, human forearm



Day 1

- With the entire class, introduce the terms: levers, fulcrum, effort, and load.
- In small groups of three or four, have the students build each type of lever and “play” with it. What similarities/differences do the students observe? Record these and use in a full class discussion.
- Have the students identify levers in their daily surroundings and identify each lever’s class.

HOMEWORK: Students will draw or cut out a picture as an example of each type of lever. Students will label the appropriate parts of the picture: fulcrum, effort, and load

Day 2

- Complete **Lab A**.

Using the *Lego dacta™ Technic I* kit, students will build a class one (1) lever and determine the relationship between weights placed on the lever and its distance from the fulcrum.

Students will be asked to express the observed relationship in an algebraic form. As part of **Lab A**, students will be making predictions and testing them. They will record only those predictions, regarding the number and position of the *bricks*, which yielded a balance between the load and the effort.

HOMEWORK: Students will complete the *Conclusions* section of **Lab A**.

Day 3

- The full class will report and discuss the results of **Lab A**.
- The class will determine, from among the group responses and further discussions, an appropriate algebraic representation for the relationship between weight (load) and distance from the fulcrum for a First Class lever.

HOMEWORK: Students will make corrections and adjustments to their work on **Lab A**.

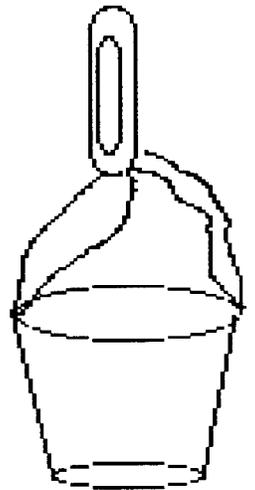
Day 4

- Complete **Lab B**.

Using the *Lego dacta™ Technic I* kit, adjust your lever so that the fulcrum is placed at the third hole from one end of the lever. A specified weight, a brick from the *Lego dacta™ Technic I* kit, should be placed on the short end of the lever. Construct a *bucket* utilizing two rubber bands joined together for the handle and attached to a small paper cup. The *bucket* will be suspended from the opposite end of the lever by hanging it from a paper clip through the last hole (#10) of the lever. Predictions should be made regarding the number of paper clips needed to cause the lever to change orientation (from raised to dropped).

- Test your observation using the knowledge of relations and algebraic expressions developed in **Lab A** by adding paper clips to the bucket until the lever changes orientation. Were your predictions correct? If not, why?
- By averaging the results of several groups, or several trials, you will obtain data which you should graph.

HOMEWORK: Graph the data and determine a line of best fit with the equation for that line. (Uncooked spaghetti can be used to help find the line of best fit.) Use that equation to determine the weight of the *brick*.



Day 5

- Full class reporting and discussion of the results of **Lab B**.
- What were the limitations of this experiment? How could the results of the experiment be made more exact? What modifications to the **Labs** could be made to yield “better” data? Can you think of other situations/experiments which would also illustrate inverse (indirect) variation?
- Complete *Pair Practice Sheet* identifying direct and inverse (indirect) variation relationships.

HOMEWORK: Review information regarding direct and inverse (indirect) variation in preparation for a quiz.

Day 6

- Traditional pencil and paper assessment, identifying direct and inverse (indirect) variation relations.

One Week Later

- Students will be randomly divided into groups of four as they arrive in the classroom. They will be given the following situation and asked to propose a workable solution.

A school bus has taken a group of students on an outing to a remote pond for the purpose of collecting science data. The bus acquires a flat tire, and the substitute bus driver discovers that the company has forgotten to include a jack for the bus in the tool compartment. He does have all the other necessary tools and a spare tire. How can the students help the bus driver?

Note: The class will first have to determine an approximate weight of the bus. This could be done by estimation based upon the comparative weights of other large objects, i.e., cars [In this case students would be working with direct proportions to obtain an estimate of the bus's weight.], or by contacting a local school bus company.

- Each group must write up a viable solution which will include a diagram, a list of necessary tools and/or materials, and step-by-step instructions for solving the bus driver's dilemma. Solutions will be holistically ranked and the better solutions will be presented to the class the following day.

Cooperative Learning

- The teacher will organize groups of three to four students each.
- Groups will reflect gender and ethnic balances.
- Each group member will have a specific role in the group.
- Social skills, such as explaining ideas, encouraging team members, asking for clarification, giving constructive criticism, and keeping an open mind will be encouraged.
- Peer review and responsibility for the group's success will be emphasized.



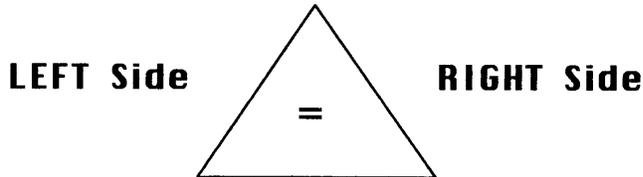
Gender and Equity Issues

- Gender equity will be considered when forming the groups.
- Activities will be both gender-free and bias-free.
- All students will be expected to know how to use the materials and will have an opportunity to do so.
- Through the cooperative groups, students with different skill levels will be able to get help from their peers in addition to assistance from the teacher.

Course 1 Lab A Variation

DISTANCE: Count the number of "pegs" from the center, including the "peg" on which the stack of "bricks" rests

WEIGHT: Count the number of "bricks" in the stack



	Weight (W_L)	Distance (D_L)	Weight (W_R)	Distance (D_R)
1.	1	7	3	2
2.	3	1	1	4
3.	3	4	2	6
4.	2	5	3	3
5.	1	5	3	1

Lab A:

Using the *Lego dacta™ Technic I* kit students will build a class one(1) lever and determine the relationship between weights placed on the lever and its distance from the fulcrum. Students will be asked to express the observed relationship in an algebraic form.

What do you notice about the relationship between the Distance and the Weight?

I noticed that a magnitude of the distance between is 3. The product of ^{the distance or weight on} both sides is close to equal

Using W_L , D_L , W_R , and D_R , write an algebraic expression which shows that relationship.

$W_L * D_L = W_R * D_R$ $\frac{W_L}{D_R} = \frac{W_R}{D_L}$

INDIRECT OR INVERSE VARIATION

Lab B:

Using the *Lego dacta™ Technic I* kit, students will adjust their lever so that the fulcrum is placed at the third hole from one end of the lever. A specified weight, a *brick* from the *Lego dacta™ Technic I* kit, will be placed on the short end of the lever. The students will construct a *bucket* utilizing two rubber bands joined together for the handle and attached to a small paper cup. The *bucket* will be suspended from the opposite end of the lever by hanging it from a paper clip through the last hole (#10) of the lever. Students will test their observations and algebraic expressions developed in **Lab A** by adding paper clips to the *bucket* until the lever changes orientation. By averaging the results of several groups, or several trials, students will obtain data which should be graphed. A line of best fit should be determined and the equation for that line written. Students should recognize that the equation can be used to determine the weight of the *brick*.

ASSESSMENT



- Written work in student portfolio
- Student-generated graph
- Student-generated algebraic expressions and equation for the student generated data
- Rationale for observed patterns (collected orally by teacher while circulating among groups)
- Teacher observed data, actions, and interactions.

NAME Vessica HR Period

Scoring Rubric for Inverse Variation

Assessment Scale: **3 Acceptable** - Your work fulfills all of the objectives of this portion of the activity.

1 Unacceptable or Missing - Your work is either incomplete or requires major revisions.

2 Minimally Acceptable - Your work is acceptable, but needs minor revisions or improvements.

35
36

Part I: Lab A

- | | | | |
|---|---|---|-----|
| 1. Have the 5 trials been completed? | 1 | 2 | (3) |
| 2. Is the described relationship correct & accurate? | 1 | 2 | (3) |
| 3. Is the algebraic expression a correct interpretation of the <u>description</u> ? | 1 | 2 | (3) |

Part II: Lab B

- | | | | |
|--|---|-----|-----|
| 4. Is the data chart complete? | 1 | 2 | (3) |
| 5. Are the computed averages correct? | 1 | 2 | (3) |
| 6. Is the hole # vs. # of paper clips graphed correctly? | 1 | 2 | (3) |
| 7. Is the algebraic expression for completing the brick's weight accurate? | 1 | 2 | (3) |
| 8. Is the estimate for the brick's weight reasonable? | 1 | 2 | (3) |
| 9.* Is the estimate for the brick's weight accurate? (within 10 grams) | 1 | (2) | 3 |

Part III: Pictures/Drawings of Classes of Levers

Accurate drawing and correct labeling for:

- | | | | |
|------------------------------------|---|---|-----|
| Class 1 Lever- | | | |
| 10. Fulcrum, Effort, Weight (Load) | 1 | 2 | (3) |
| Class 2 Lever- | | | |
| 11. Fulcrum, Effort, Weight (Load) | 1 | 2 | (3) |
| Class 3 Lever- | | | |
| 12. Fulcrum, Effort, Weight (Load) | 1 | 2 | (3) |

* extra credit for computing the brick's weight within 5 grams

Statistics of The M&M Candy

INTERMEDIATE

Standards & Performance Indicators

MST

1

- ▲ inductive reasoning
- ▲ solve problems

MST

3

- ▲ reasoning strategies
- ▲ conjectures/arguments
- ▲ conclusions using inductive
- ▲ simple/compound statements
- ▲ equivalent forms
- ▲ ratios/proportions/percents
- ▲ order relations
- ▲ estimate/make/use
- ▲ standard/nonstandard measurements
- ▲ display/describe/compare
- ▲ critical judgement
- ▲ check reasonableness
- ▲ solve problems
- ▲ estimate probability
- ▲ simulation techniques
- ▲ determine probabilities

I didn't know there was this much math in a bag of M&M's.

Student

Materials:

- rulers and/or Vernier calibrators (one for each pair of students)
- triple beam balance (one for each group of four)
- three 1-pound bags on M & M candies - two plain and one peanut; Note: One bag of the plain candies is used in estimating and then as simple rewards. The second bag is used in calculating the mass.
- a small package of M & M candies (one for each individual student)

Basically, this unit introduces the students to estimation, measurement (linear and mass), and experimental and theoretical probability using a bag of M&M candies. The students conduct surveys in each class to determine the team's favorite color and compare their results to the company's research using various charts and graphs on the computer. They are also informally introduced to ratio, proportion, and percentage.

Maureen Gipp

Conrack School District

Carlewood Middle School

1200 Carll's Straight Path

Dix Hills, NY 11746

(516) 595-2784

Grade 7

Each student is expected to complete the worksheets. The candies in the containers are used as rewards in class and each student is required to place tally marks on charts set up in the classroom to record the results. When all the candies are given out, the frequency of each color is tallied and recorded.

ACTIVITY 2: SURVEY, TALLY, AND FREQUENCY

A quick discussion, led by the teacher, is held about which color(s) appeared to be the most frequent in the clear container.

STUDENT NAME Lisa

SURVEY, TALLY AND FREQUENCY

The M&M company will be conducting a survey to determine which color M&M is the favorite choice. To help them out we will conduct a survey in the classroom to see the favorite color of the class.

COLOR	TALLY	FREQUENCY
Red		7
Orange		1
Green		3
Yellow		2
Brown		3
Blue		17

A survey of the students, and any adults, regarding their favorite M&M color is then conducted by the teacher. The teacher can have a transparency of the chart, modeling for the class how to deal with the tally marks. These results are then compared to their observation of the clear container.

Questions that can be addressed:

- Are the results similar? How?
- Who was included in the sample surveyed?
- How would this compare to a survey of adults?
- What would be the favorite color of the entire team? Why?

Each student is expected to complete the worksheets.

Note: Activities 3 and 4 can be done independently in science and mathematics or combined in a double science/mathematics period.

good,
but confusing at
bottom. I can't figure
out which were

~~Crossed out~~
or were
the #'s that
you wanted

to put
down

(33)

1. Based on our survey results answer the following questions:

- which color was picked the most? blue
- Which color was picked the least? orange
- Were any colors picked approximately the same number of times? yes If yes, which colors? green and brown

2. If you owned the company, how many of each color would you put in a bag? (Assume you have 100 M & M's)

Red	25	 	Yellow	10	
Orange	10	 	Brown	5	
Green	15	 	Blue	25	

confusing

ACTIVITY 3: MEASURING M&M CANDIES

In science class (or during a double period) each student will work with a partner to measure the diameter of one M&M candy using a metric ruler or Vernier calibrator. The students repeat the process using 20 candies laid in a straight line. They compute the average and compare it to their first measurement.

The second part of the activity deals with mass measurement and the students use a triple beam balance to compute the mass of one candy and then 20 candies. The students receive 20 candies in a sandwich bag. An average mass is then calculated.

Using this information, the student is expected to calculate how many candies are in a 1-pound bag. Each student will then review their observation estimate and their measurement estimate and write their conclusions.

MEASURE GRAPHIC

STUDENT NAME: LISA

MEASURING M&M CANDIES

1. LINEAR MEASUREMENT

What is the approximate diameter of a plain M&M to the nearest tenths? Be sure to include the unit of measure.
1/2 in

What tool did you use? ruler Do you feel the measurement is accurate? Why? we lined up the m+m with the ruler.

Lay 20 plain M&M candies in a straight line. Measure this distance to the nearest tenths? Include the unit of measure. 10 in

What tool did you use? ruler

Find the average diameter of one plain M&M candy. 1/2 in

Compare the measures of the two diameters. Are they equal? Which measurement do you feel is more accurate? Explain.
they are equal. paper 2

2. MASS MEASUREMENT

What is the mass of one M&M candy to the nearest tenths? Be sure to include the unit of measure. 9.99 g

What tool did you use? TBB scale Do you feel that this measurement is accurate? yes Why? because we used the TBB scale.

Find the mass of 20 M&M candies to the nearest tenths. 20 g Include the unit of measure.

What tool did you use? TBBs Do you feel that this measurement is accurate? yes Why? because we used the TBB scale.

Find the average mass of one M&M candy. 7 g

Compare the two masses. Are they equal? yes Which measure do you feel is the most accurate? yes Why? because we used the TBB scale for both

ACTIVITY 4: STATISTICAL PROBABILITY OF M&M CANDIES

In mathematics class (or as part of a double period) each student is given a small bag of M&M candies. Each student records how many of each color of the candies is in the bag. The students are now informally introduced to writing a ratio. A discussion is led by the teacher to see how many of each color the students had. Since each student's bag is different, the need for percent is introduced and the students are instructed on how to do the conversion on a calculator.

4. Place all of your M&M's (NO CHEATING!) back in the bag. Pick an M&M from the bag and note its color in the chart below using tally marks. Place the M&M back in the bag. Repeat this procedure **twenty** times. Then compute the frequency, probability, and rate of percent.

EXPERIMENTAL PROBABILITY

COLOR	TALLY	FREQUENCY	PROBABILITY	PERCENT
RED		7	$\frac{7}{59}$	11%
ORANGE		3	$\frac{3}{59}$	5%
GREEN		1	$\frac{1}{59}$	1%
YELLOW		1	$\frac{1}{59}$	1%
BROWN		8	$\frac{8}{59}$	13%
BLUE	0	0	$\frac{0}{59}$	0%

5. Compare your theoretical percents and your experimental percents. Are they different? Are they equal? How does the number of each color affect its probability? If the two rates of percent are relatively close then your experiment worked the way it was supposed to work. Explain your results. (Write a minimum of 5 complete sentences.)

The colors (Red + Brown) that had the most showed up the most the ~~ones~~ colors that had the least showed up the least

STUDENT NAME Lisa

STATISTICAL PROBABILITY OF M&M CANDIES

1. What is the mass of one M&M candy? 1g
What is the mass of your bag of M&M candies? 46.5
Using the space below estimate how many M&M candies are in your bag of M&M's.
65

2. Using your bag of M & M's, fill in the tally of each color in the chart below. Compute the frequency of each color, its simple probability and rate of percent.

THEORETICAL PROBABILITY

COLOR	TALLY	FREQUENCY	PROBABILITY	PERCENT
Red	 	15	$\frac{15}{65}$	23%
Orange		5	$\frac{5}{65}$	07%
Green		6	$\frac{6}{65}$	09%
Yellow		3	$\frac{3}{65}$	4%
Brown	 	30	$\frac{30}{65}$	46%
Blue	0	0	$\frac{0}{65}$	0%

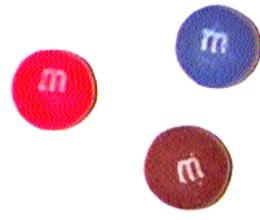
TOTAL NUMBER OF M & M'S

3. Compare your prediction in question #1 to the actual number of M&M's you have in your bag. Did you overestimate or underestimate? Would you consider your estimate to be "good"? Explain.

good over estimate
the actual is 59

The second part of the activity is the experiment. Each student places the candy back in the bag or some other container, retrieves one candy, records its color, returns it to the bag and repeats this procedure 20 times.

The final activity requires students to compare their two results and draw conclusions.



ACTIVITY 5: M&M CONCLUSIONS

The concluding worksheets were designed to allow the students to summarize some of the data and to draw some conclusions from this data. The concepts of range, median, mode, and mean are also reinforced in this activity.

M&M BONUS ACTIVITIES:

These are extended activities for the student to explore. A choice is given and students are encouraged to come up with their own ideas. These activities are optional and count as extra credit.

M & M CONCLUSIONS

1. The first activity of the M & M Performance Task was to estimate how many candies were in a one pound bag. Your estimate was 560. Based on all the activities do you feel you estimated exactly, overestimated or underestimated? Explain why. (Write a least three complete sentences.)

2. In the second activity a survey was conducted in each class to determine which color each student preferred. In your class the color with the highest frequency was blue and the color with the smallest frequency was orange.

Complete the table below based on the overall team's opinion.

Nice Chart

COLOR	FREQUENCY OF EACH CLASS PERIODS					TOTAL	PERCENT
	3	4	5	7	8		
Red	6	7	1	5	7	26	21%
Orange	1	1	0	0	0	2	1%
Green	3	3	2	4	4	16	13%
Yellow	2	2	5	0	0	9	7%
Brown	1	3	0	1	1	6	5%
Blue	12	17	9	10	12	60	50%
TOTALS	25	33	17	20	24	119	→

3. Based on the results of the entire team survey, answer the following questions.

a) Which color was picked the most? _____

b) Which color was picked the least? _____

incomplete

ASSESSMENT



Many group discussions are held as the project evolves. All of the questions are designed to further their critical thinking skills. Since the students sit in cooperative learning groups they are able to further clarify any discussions that take place.

The worksheets themselves are graded by the student, another student, and the teacher according to a scoring rubric. These scores are averaged and a grade is assigned. The grade is counted as a project.

Students are also encouraged to do an extra credit task from bonus activities which are suggested. However, student-generated ideas are readily accepted. New additions this year were designing an M&M pillow and a wood candy dispenser as well as other ideas for surveys.

M & M Candy Rubric

- A+**
1. All measurements and calculations are correct.
 2. Clear, logical reasoning is demonstrated.
 3. All procedures are clearly described.
 4. All information is given.
- A**
1. There is a minor error in the calculations *or* minor information is missing.
 2. Clear, logical reasoning is demonstrated.
 3. All procedures are clearly described.
- B**
1. There are several computational mistakes *or* some information is missing.
 2. Clear logical reasoning is demonstrated.
 3. All procedures are clearly described.
- C**
1. Some major information is missing.
 2. There are minor flaws in reasoning.
 3. An explanation of reasoning is attempted but lacks development.
- D**
1. There are many flaws in the reasoning process that impede a logical process.
 2. There is little explanation.
 3. There is lack of completion.
- F**
1. Answers can not be found.
 2. No explanations of reasoning are given.

EVALUATION AND REFLECTION: What grade should you get on this project? Explain why you should receive this grade. What did you enjoy most about this project? What was the most difficult part of this project? What did you learn?

LISCJ
Good job

Checked by:
Jessica
Mull

M & M CANDY RUBRIC

Comments
inside

SCORING GUIDELINES

1. ESTIMATION AND PREDICTIONS

- The observation table is filled in completely.
- All questions are answered.
- In question 3 the sum of their results equals 100.
- The ratio and percent is calculated for each color.
- The computations are correct.

2. SURVEY, TALLY AND FREQUENCY

- The survey table is filled in completely.
- All questions are answered.
- In questions 2, 3 and 4 the sum of their results equals 100.
- A clear explanation (at least one sentence) is written in question 4.
- In question 5 one type of graph is indicated.
- A clear explanation (at least one sentence) is written in question 5.

3. MEASURING M & M CANDIES

- All the questions in the linear and mass measurement sections are completed.
- All measures are to the nearest tenths.
- Units of measure are included.
- Each explanation is a clear, complete sentence.
- In question 3 all work is shown clearly and neatly.
- The computations are correct.
- The student understood what they were doing.
- The conclusions are complete.
- Words like equal, greater or less are used in the written explanation.
- One prediction is picked and a clear explanation (at least one sentence) is written.

4. STATISTICAL PROBABILITY OF M & M CANDIES

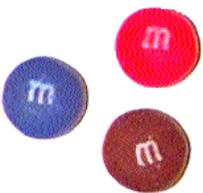
- In question 1 all answers are complete and all work is shown clearly.
- The theoretical probability chart is complete.
- The computations are correct.
- It is clearly stated that the estimate is an overestimate or an underestimate.
- A clear explanation (at least one sentence) is written.
- The experimental probability chart is complete.
- The computations are correct.
- In question 5 a clear explanation is written.

5. M & M CONCLUSIONS

- Each question is answered completely.
- All tables are complete
- The computations are correct.
- Explanations are written in clear, complete sentences.
- Knowledge about probability and statistics is demonstrated.

STUDENT	GRADER	TEACHER
Assign 0, 1, 2 or 3		
3	3	
2	2	
2	2	
2	2	
1 1/2	2	

REFLECTION:



I have found that almost all learners are able to successfully complete these activities to some degree. With the lowest functioning students, a lesson in simple probability might prove useful. These students also require more involvement from the teacher whereas the other students work independently.