Mathematics, Science & Technology

PART III.1

NOTE: This document is a work in progress. Parts II and III, in particular, are in need of further development, and we invite the submission of additional learning experiences and local performance tasks for these sections. Inquiries regarding submission of materials should be directed to: The Mathematics, Science, and Technology Resource Guide, Room 681 EBA, New York State Education Department, Albany, NY 12234 (tel. 518-474-5922).

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Proposed State Assessments for Mathematics Science, Technology

As a new assessment system for mathematics, science, technology comes closer to reality in New York State, the following proposals for revising existing assessments are being discussed. Your participation in this dialogue is encouraged. Please use the response form at the end of this Teacher Resource Guide to share your comments with the New York State Education Department.

Common Understandings

Assessments will:

- be mapped directly to the science standards with attention to mathematics and technology and especially to inquiry, information systems, the interconnectedness of common themes, and interdisciplinary problem-solving
- assess a broader range of skills than is assessed by the present State tests
- test content matter and require that students construct meaning given unique situations
- all include both an on-demand test (administered to all students during a common time period) and an extended task (a standardized task or set of tasks completed over a longer period of time)
- all have a laboratory requirement as a prerequisite at the commencement level
- be scored by New York State teachers using scoring guides that describe students' work at different levels of performance

The following descriptions of the new assessments are based on the results of pilot tests and the recommendations of several groups including teachers, administrators, and mentors. They represent current thinking and suggest the direction of the assessments, but exact specifications for each test will continue to evolve as teachers and test developers contribute to further refinement.

Descriptions:

Proposed Elementary Science Assessment

The Elementary Science Program Evaluation Test (ESPET) will continue to consist of both an on-demand component and a performance component made up of several stations. At each of these stations students will be asked to use inquiry methods to answer questions and / or address problems posed to them.

School districts will continue to be encouraged to use the results of this test to evaluate their science program by identifying areas which are particularly effective and those which are in need of strengthening.

There will be a mathematics, science, and technology curriculum-embedded task at the elementary level which will require students to apply their knowledge in those three areas to design a solution to a given situation. This task is intended to be a long-range problem in which students work together in groups to develop a solution.

Proposed Intermediate Science Assessment

The proposal for the intermediate assessment includes an exam offered at grade eight comprised of an on-demand component assessing content, skills, and application and a laboratory task.

There may be a mathematics, science, and technology curriculum-embedded task at the intermediate level which will require students to apply their knowledge in those three areas to design a solution to a given situation. This task will be a long-range problem in which the students work together in groups to develop a solution.

Proposed Commencement Science Assessment

The current proposal for commencement level includes Regents exams in Earth science, biology, chemistry, and physics. These exams will be revised and aligned to reflect the learning standards. They will be comprised of an on-demand portion assessing content, skills, and applications to real-world situations; a laboratory practical to occur before the exam; and a long-range project which incorporates all the mathematics, science, and technology standards.

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5 6 7

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Assessment System Overview

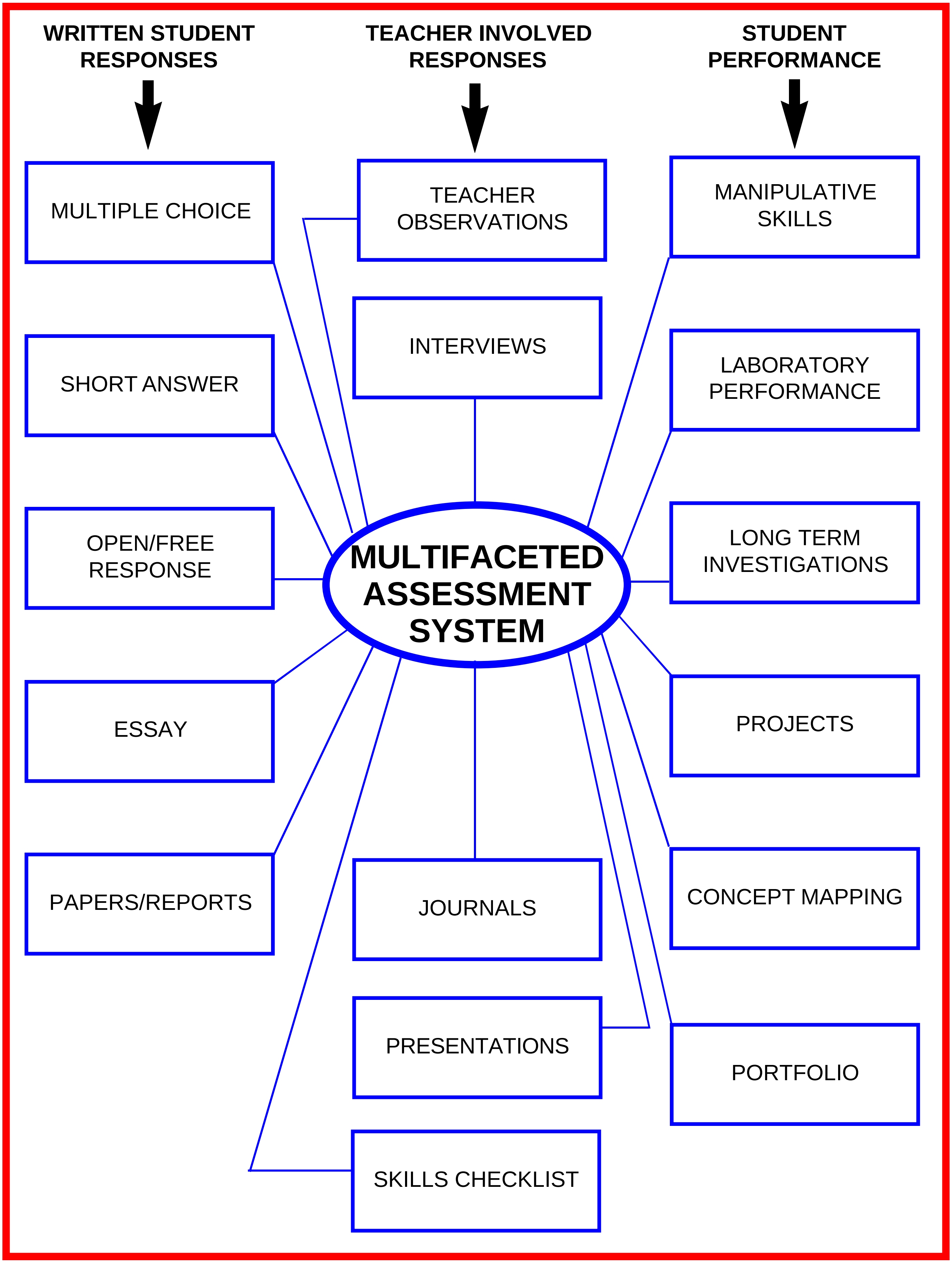
The following two charts provide an overview of the kinds of activities that can be used as assessments. In planning an assessment for a particular purpose, teachers should consider various options and how they might enhance student learning. Examples of many of these assessment types can be found in Part II of this teacher resource guide.

The chart showing a Multifaceted Assessment System suggests the kind of assessment system that we hope to develop in New York State. It includes short answers and performance-based questions; it provides opportunities for student and teacher reflection; it reminds teachers that both on-demand and extended task components will be included.

Some Assessment Options

True-False Item			
Multiple-Choice Item	**Responses**	**Objectively**	**Conventional**
Matching	**Selected**	**Scored**	**Assessments**
Modified Objective			
Completion			
Short Answer			
Essay			
Papers			
Lab Reports			
Observations	**Constructed**	**Subjectively**	**Alternative**
Discussion	**Responses**	**Scored**	**Assessments**
Interviews	**• open**		
Skills Check-List	**• free**		
Performance Testing			
Lab/Field Practicals			
Projects			
Poster-Board Session			
Portfolios			
Self Rating			
Peer Rating			

Source: Reynolds, Douglas S., Doran, Rodney L., Allers, Robert H., and Agruso, Susan A. *Alternative Assessment in Science: A Teacher's Guide*, State Education Department, University at Buffalo, 1996.


WRITTEN STUDENT RESPONSES
TEACHER INVOLVED RESPONSES
STUDENT PERFORMANCE
MULTIPLE CHOICE
TEACHER OBSERVATIONS
MANIPULATIVE SKILLS
INTERVIEWS
SHORT ANSWER
LABORATORY PERFORMANCE
OPEN/FREE RESPONSE
MULTIFACETED ASSESSMENT SYSTEM
LONG TERM INVESTIGATIONS
ESSAY
PROJECTS
PAPERS/REPORTS
JOURNALS
CONCEPT MAPPING
PRESENTATIONS
PORTFOLIO
SKILLS CHECKLIST

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Designing Equitable Assessments

Although this statement was written in light of changing mathematics assessments, it is valid for assessments in all disciplines. Those who design assessments for student use will want to be conscious of the Principle of Equity.

The Principle of Equity asserts that all students—not only a talented few—can learn mathematics to the extent required to live and work effectively in the 21st century. Assessments must be designed with this principle in mind and will:

Determine:

- what students have learned
- what students still need to learn
- further educational opportunities for all students.

Require:

- rethinking of what and how to assess
- reflecting on how individuals and groups are affected by assessment design and procedures
- designing flexible tasks that challenge students and allow them to demonstrate accomplishment.

Contribute to:

- opportunities for students to learn
- higher expectations for all students
- better instructional practices
- increased effort on the part of students to achieve at higher levels.

Adapted from: *Measuring What Counts: AConceptual Guide for Mathematics Assessment*. Copyright 1993 by the National Academy of Sciences. Courtesy of the National Academy Press, Washington, DC.

Curriculum, Instruction, and Assessment in Mathematics:

Changes Necessary to Reflect New York State Learning Standards and Assessment

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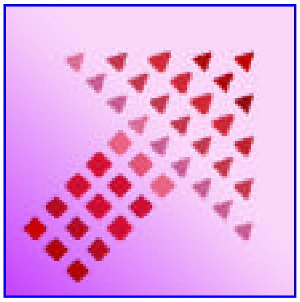
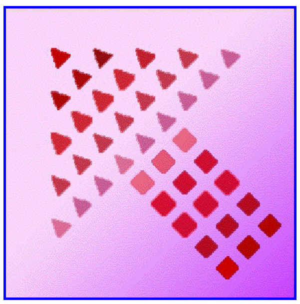


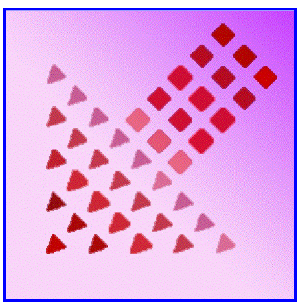
Curriculum:

Mathematics content is closely aligned to both national and State standards. Little substantive change in content is anticipated.



Instruction:

Teachers will need to help students make connections of mathematics to science, technology, and other disciplines. They will teach students how to write effective explanations for their responses and how to work collaboratively with other students to accomplish projects. Teachers will guide students in real-world explorations of a mathematical nature.



Assessments:

Revised assessments at all levels will include extended tasks or projects performed individually, in small groups, or as a whole class to accomplish the tasks. Students will individually communicate their results to the teacher. The number of student constructed responses of various lengths, as well as short multiple-choice items, will be increased. Students will be expected to show how they arrived at their answers by writing sentences which explain their reasoning. Such questions will be phrased in the context of a real-life application of the skill or concept being assessed. Skill in writing effective explanations will thus be demonstrated on the extended task as well as in the greater number of student constructed items on tests.

Mathematics A Regents Assessment Specifications

To conduct a meaningful assessment of mathematics proficiency, it is necessary to measure a student's proficiencies for the seven key ideas for as described in the *Learning Standards for Mathematics, Science, and Technology.* These key ideas are: mathematical reasoning, number and numeration, operations, modeling / multiple representations, measurement, uncertainty, and patterns / functions. Classification of mathematical topics into these key ideas may not be exact and inevitably involves some overlap. In addition, many of the key ideas can be assessed by tasks which involve students in synthesizing knowledge across mathematical topics.

The seven key ideas mentioned above will be only one of three dimensions considered in the construction of the assessment. The other two dimensions will include three process or cognitive levels, and three categories of questions and tasks.

and Tasks					
Aspect	**Range***	**Aspect**	**Range***	**Aspect**	**Range***
Mathematical Reasoning	5-10%	Procedural Knowledge	25-40%	Multiple Choice	20-35%
Number and Numeration	10-15%	Conceptual Understanding	25-35%	Short Constructed Response	20-35%
Operations	10-15%	Problem Solving		Extended Constructed Response	30-45%
Modeling /Multiple Representations	20-30% 15-20%				
Measurement	5-10%				
Uncertainly	15-20%				
Patterns/ Functions					

*The percents indicated are in terms of the total number of points available on the test.

The ranges for each dimension were established by a panel of consultants composed of New York State high school mathematics teachers. In addition to the dimensions listed above, the panel also decided on a range for the amount of questions that are to be in context. Apreliminary estimate for the percentage of the assessment that will either be an application or be given in a contextual setting is 50-60 percent.

The tentative architecture of the assessment will include a Part I, which will contain multiple choice questions that are scored either correct or incorrect with a value of two points each; a Part II, where students will be required to display their solutions and will be scored with a two or three point rubric; and a Part III, where students will be expected to show, demonstrate, or explain their responses to more complex problems and applications that will be scored with a four point rubric. Students may have a choice on Part III of the assessment.

One possible design for the Mathematics ARegents Test is:

Part I	20 questions	(2 points each)	Total 40 points
Part II	8 questions	(2 points each)	Total 16 points
	8 questions	(3 points each)	Total 24 points
Part III	10 questions*	(4 points each)	Total 40 points
	* Possible choice of 10 out of 14 questions for Part III		
			Total Test - 120 points

In addition to the Mathematics ARegents Test described above, students may be given an extended task or project as part of their course work. Some of the questions contained in the on-demand portion of the assessment would be based on what the students are expected to learn from performing the extended task. Details of the extended task or project are still under discussion.

The mathematics content included in these specifications constitutes a subset of the commencement level performance indicators published in standard 3 - Mathematics of the *Learning Standards For Mathematics, Science, and Technology* (MST - March 1996 edition). In some cases, the performance indicators have been clarified by the use of more specific examples of content.

Although the goal of the Regents is to eventually have every student accomplish all of the commencement performance indicators listed in the MST document, it was felt that it was unreasonable to expect that level of performance at this time. The transition to an all Regents curriculum and the institution of MST standards-based assessments in grades four and eight will take several years to affect all of our students.

The Regents' commitment is to review the specifications often and amend them, as appropriate, in order to continue to raise the performance expectation for all students until the commencement level goal is reached.

Mathematical Reasoning

1. Students use mathematical reasoning to analyze mathematical situations, make conjectures, gather evidence, and construct an argument.

Students should be able to:

* construct simple valid arguments

* follow and judge the validity of arguments.

Note: Formal Euclidean proof is not required.

Numbers and Numeration

2. Students use number sense and numeration to develop an understanding of the multiple uses of numbers in the real world, the use of numbers to communicate mathematically, and the use of numbers in the development of mathematical ideas.

Students should be able to:

- understand and use rational and irrational numbers
- recognize the order of the real numbers
- apply the properties of real numbers to various subsets of numbers; in particular the use of closure, commutativity, associativity, distributivity, identity element, and inverses.

Operations

3. Students use mathematical operations and relationships among them to understand mathematics.

Students should be able to:

- use addition, subtraction, multiplication, division, and exponentiation with real numbers and algebraic expressions
- use integral exponents on integers and algebraic expressions
- recognize and identify symmetry and transformations on figures
- use field properties to justify mathematical procedures.

Modeling/Multiple Representation

4. Students use mathematical modeling/multiple representation to provide a means of presenting, interpreting, communicating, and connecting mathematical information and relationships.

Students should be able to:

- represent problem situations symbolically by using algebraic expressions, sequences, tree diagrams, geometric figures, and graphs; in particular the classification of triangles and quadrilaterals (parallelogram, rectangle, rhombus, square, and trapezoid), polygons of 5, 6, 8, 10, and 12 sides, congruence and similarity, and solids (prism, rectangular solid, pyramid, right circular cylinder, cone, and sphere)
- justify the procedures for basic geometric constructions
- use transformations in the coordinate plane
- develop and apply the concept of basic loci to compound loci
- model real-world problems with systems of equations and inequalities.

Measurement

5. Students use measurement in both metric and English measure to provide a major link between the abstractions of mathematics and the real world in order to describe and compare objects and data.

Students should be able to:

- apply formulas to find measures such as length, area, volume, weight, time, and angle in real-world contexts
- choose and apply appropriate units and tools in measurement situations
- use dimensional analysis techniques

- use statistical methods including the measures of central tendency to describe and compare data; in particular the use of histograms, bar graphs, circle graphs, line graphs, stem and leaf plots, box and whisker plots, scatter plots
- use trigonometry as a method to measure indirectly in particular the use of sine, cosine, and tangent
- apply proportions to scale drawings and direct variation
- relate absolute value, distance between two points, and the slope of a line to the coordinate plane
- explain the role of error in measurement and its consequence on subsequent calculations
- use geometric relationships in relevant measurement problems involving geometric concepts; in particular the use of ratios of perimeters and areas in similar polygons and with circle circumference and area, angle classification and measurement (acute, right, obtuse, straight, supplementary, complementary, vertical, corresponding, alternate interior, or exterior) and sum of the interior and exterior angles of polygons.

Uncertainty

6. Students use ideas of uncertainty to illustrate that mathematics involves more than exactness when dealing with everyday situations.

Students should be able to:

- judge the reasonableness of results obtained from applications in algebra, geometry, trigonometry, probability, and statistics
- use experimental and theoretical probability to represent and solve problems involving uncertainty
- use the concept of random variable in computing probabilities
- determine probabilities using permutations and combinations.

Patterns/ Functions

7. Students use patterns and functions to develop mathematical power, appreciate the true beauty of mathematics, and construct generalizations that describe patterns simply and efficiently.

Students should be able to:

- represent and analyze functions using verbal descriptions, tables, equations, and graphs
- apply linear and quadratic functions in the solution of problems
- translate among the verbal descriptions, tables, equations, and graphic forms of functions
- model real-world situations with the appropriate function.

Note: Use of the quadratic formula is not required.

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Old Paradigm/New Paradigm

Dr. Courtney Young of the Nanuet Union Free School District, took questions from old Regents exams and re-wrote them to embody the new standards. The new versions require students to explain their mathematical manipulations in real-world contexts.

Original

The frequency table below shows the distribution of time, in minutes, in which 36 students finished the 5K Firecracker Run.

Interval (minutes)	Frequency
14-16	2
17-19	6
20-22	9
23-25	8
26-28	7
29-31	4

a How many students finished the race in less than 23 minutes? [2]

b Based on the frequency table, which interval contains the median? [2]

c On your answer paper, copy and complete the cumulative frequency table below. [2]

Interval	Cumulative Frequency
14-16	2
14-19	
14-22	
14-25	
14-28	
14-31	

d On graph paper, using the cumulative frequency table completed in part c, construct a cumulative frequency histogram. [4]

Alternative

a) Who won? Who needs to train more and perhaps lose weight?

b) Were there good times? (Assume a level course and good weather) Discuss your conclusion.

c) Calculate the number of miles in the 5000 meter course. Estimate the number of football fields (100 yds) that would equal.

d) Which interval contains the median? What is the range of times?

e) Complete this table and compute the average time.

Mid Point	Frequencies	Totals
15	2	30
18	6	‘
21	9	‘
‘	‘	‘
‘	‘	‘
‘	‘	‘
30	4	120

f) Describe which is the best indicator of performance and why you think so: the mean, median or mode.

g) Complete the cumulative frequency table.

h) What percent of the runners finished in less than 23 minutes?

i) Construct a cumulative frequency histogram.

j) Plot the mean, median, and mode on the vertical axis.

k) Discuss the general shape of your histogram.

Source: Young, Dr. Courtney D. Jr. Nanuet High School., Nanuet Union Free Schools.

Original

Let p represent: “The stove is hot.”
Let q represent: “The water is boiling.”
Let r represent: “The food is cooking.”

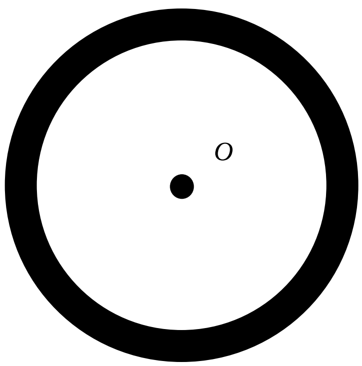
a Write in symbolic form the converse of the statement: “If the stove is not hot, then the water is not boiling.” [2]

b Write in sentence form: $\sim r \rightarrow \sim q$ [2]

c Write in sentence form: $p \vee \sim r$ [2]

d *On your answer paper*, construct a truth table for the statement $p \wedge \sim q$. [4]

In the accompanying diagram, both circles have the same center



The radii of the circles are 3 and 5.

a Find, in terms of π, the area of the shaded region. [4]

b What percent of the diagram is unshaded? [2]

c A dart is thrown and lands on the diagram. Find the probability that the dart will land on the

(l) shaded area [2]

(2) unshaded area [2]

Alternative

p: The “Series” is close!
q: The ‘Fans’ are cheering!
r: The “Yankees” are scoring!

a) Converse of: If the series is not close, then the Fans are not cheering.

b) Sentence form: $\sim r \rightarrow \sim q$ Write an equivalent sentence. Prove they are equivalent.

c) Sentence form: $p \vee \sim r$ Translate to an If-then sentence. Prove they are equivalent.

A small Spruce tree is centered at ◯

Its branches spread out to a 3 ft. radius.
The surrounding mulch has a 5 ft. radius.
The mulch is surrounded by concrete.

a) Find the area of the shaded region (the mulch).

b) What percent of the total area is covered by the tree?

c) If the tree needs a rain gathering area of mulch equal to 300 percent of the area covered by the tree in order to survive, will this tree live? Explain. If not, by how much should the radius of the mulch be increased?

d) Assuming rain falls vertically, find the probability that rain will land on.

i) the mulch (shaded) area.

ii) the branches of the Spruce.

Original

The ages of three children in a family can be expressed as consecutive integers. The square of the age of the youngest child is 4 more than 8 times the age of the oldest child. Find the ages of the three children. [l0]

Solve the following system of equations algebraically or graphically and check:

$$3y = 2x - 6$$
$$x + y = 8$$

[8.2]

Alternative

Three teachers plan to retire this year: Mr. A, Mrs. B, and Ms. C. Mrs. B's class decides to have a party for them, and during their planning they invent a math problem to give all three teachers in a contest setting. Are they going to get even or what! Here is the contest problem:

* The ages of 3 retiring teachers are 3 consecutive integers. Mr. Ais younger than Mrs. B who is younger that Ms. C. The square of the age of the youngest is 400 years more than 48 times the age of Mrs. B. Find all 3 ages.

a) Show how the problem can be solved.
b) "Extra credit" will be awarded to the teacher who can find a second method of solution. Prepare an answer "key" for a second method.

The Graphing Calculator Club is small, (only 8 members) but they have big plans. In order to earn money for a "T192" the members of one group managed to raise $20.00 each. The rest of the members purchased reference material to use with the new machine; they spent $30.00 a piece. Overall they still had a surplus of $60.00 to apply toward the new calculator.

How many members were in the money raising group?

a) Set up a table to analyze the situation. Use expressions for "total members" and for "money surplus" in your table's heading. Identify the "solution" row of your table.
b) Write and solve a pair of equations to verify your work.
c) Graph your pair of equations and identify the "solution."

Original

In ΔABC, AB is $\frac{3}{5}$ of the length of $\overline{BC}$, and AC is $\frac{4}{5}$ of the length of $\overline{BC}$. If the perimeter of ΔABC is 24, find the lengths of $\overline{AB}$, $\overline{AC}$, and $\overline{BC}$. [*Only an algebraic solution will be accepted*.] [5.5]

a) On graph paper draw the graph of the equation $y = x^2 - 6x + 8$ for all values of x in the interval $0 \leq x \leq 6$.
b) On the same set of axes draw the image of the graph drawn in part a after a translation of (x - 3,y + l) and label it b.
c) Write an equation of the graph drawn in part b.

Alternative

Al, Bill, and Carla live near Albany, Buffalo, and Canton respectively. They use ham radios to communicate. Let triangle ABC represent the situation. A storm is centered in upstate New York on Monday (the interior of triangle ABC) and they are just barely able to make contact with each other. The maximum effective ranges are as follows. From Al to Bill is 3/5 the range from Bill to Carla, and from Al to Carla is 4/5 the range from Bill to Carla.

a) Label your triangle (A,B,C,) and use expressions to label the effective ranges AB, AC, BC.
b) Suppose the minimum perimeter for effective communication between the three is 1200 miles. Study some possible range values by completing this table:
c) Represent the distances in terms of one variable and find the ranges given the minimum perimeter of 1200 miles
d) Compute the area of Monday's communication triangle. The storm passes by Wednesday and this area doubles. What are the dimensions of the new triangle? **Explain your work.** Can Al now reach Chicago? Discuss!

AB	AC	BC	Perimeter
120	160	200	480
		250	
		300	
		350	
		400	
		450	
		500	
		550	
		600	

An olympic diver leaves an 8 meter platform at time 0, enters the water at time = 2 seconds, reaches the bottom of her arc at time = 3 seconds, and emerges at the surface at time = 4 sec.

a) Develop a quadratic model of her path based on a y - intercept of 8 (meters) and t intercept of 2 and 4 (seconds). Draw the graph for $0 \leq t \leq 6$ (seconds).
b) Use the same set of axes to draw the image after a translation of (t - 3,y + 1) and write its equation.
c) Interpret this change in terms of a diving situation.

Original

The length of a rectangle is 4 less than twice its width. If the area of the rectangle is 20, find the width of the rectangle to the nearest tenth.

In the accompanying diagrams of parallelogram MATH, $\overline{AH}$ is a diagonal, altitude $\overline{AV}$ is drawn to side $\overline{MH}$, AT = 18, VH = 10, and m M = 42.

a) Find AV to the nearest tenth.
b) Find the area of parallelogram MATH to the nearest integer.
c) Find the perimeter of parallelogram MATH to the nearest integer.

Solve the following system of equations algebraically and check:

$$x^2 + y = 100$$
$$y = x - 2$$

Alternative

We need 20 square yards to store our new Bass Boat! The shape of our garage is about twice as long as wide and we want the storage space approximately in that proportion. If we can "save" 4 yards in front of the boat, we can store the motor and other accessories also. How can we determine the dimension of the storage area?

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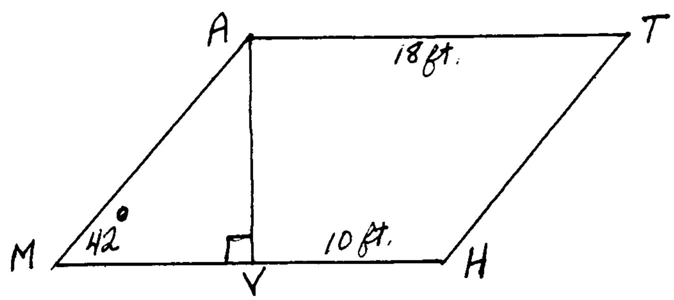
a) The Math Club is allocated space at the school fair as shown in the diagram:
b) Find the maximum allowable width for a display table.
c) Find the length of rope needed to cordon off the entire space while the floor is being painted.



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a) Assume that Jupiter has a circular orbit and lies 10 astronomical units from the Sun. Place the Sun at the origin of your coordinate system and write an equation modeling Jupiter's path. Draw the graph.
b) The "Floater" species on Jupiter (very intelligent) has discovered that an alien ship has entered the solar system and is traveling the path modeled by $y = x - 2$. Graph their path on your coordinate system.
c) Find, in quadrant one, the coordinates of the point where the alien ship will intersect Jupiter's orbit.
d) Assume Jupiter is located at (10,0) when the alien ship crosses its orbit. If the floater species sends a signal to the aliens, how far will the signal travel?
e) If the alien computer takes 3 minutes to decode the message and encode a reply, how long will the Floaters wait? Explain your reasoning.

Original

Given: $\triangle$ EAD, $\overline{ABCD}$, $\overline{AB} \cong \overline{DC}$ and EBC ≅ ECB.

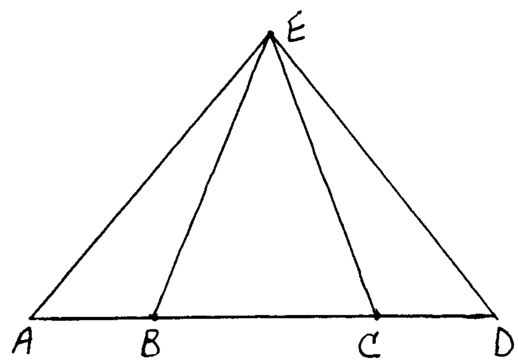
Prove: $\triangle$ EAD is isosceles.

Quadrilateral ABCD has vertices A(-8,2), B(0,6), C(8,0) and D(-8,-8). Prove that quadrilateral ABCD is an isosceles trapezoid.

Alternative

Our Family tent has enough headroom but we need more sleeping area. My little brother suggested that we use the floor material to expand the size. (We can sew in a new floor later.)

In our tent currently EBC ≅ ECB and we want to make the new sleeping spaces $\overline{AB} \cong \overline{CD}$. Will our "new" tent be isosceles? Write out a discussion proof.



Four "Sport" fishing boats are spying on a renegade whaler. The whaler is located at W(0,0). The "Fishing" boats ABLE, BAKER, CHARLIE, and DELTAare located at (8,-2), (0,6), (8,0) and (-8,-8) respectively.

a) If ABLE sends a visual signal to BAKER, can DELTA use the same direction to send a signal to CHARLIE? Why or Why not?

b) ABLE and DELTAare towing a sonar rig. BAKER and CHARLIE tow one also. Each team must maintain a fixed distance so as to avoid damage to the sonar equipment. Are they? (the rigs are the same size). What is the current distance (AD, BC)?

c) What is the current shape of the array ABCD? Discuss how you know this.